Mathematics

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Table of Contents

Arithmetics and Number System

Numbers

Numbers are fundamental concepts in mathematics that represent quantities, measurements, or values. They form the basis for arithmetic, algebra, and more advanced mathematical fields. Numbers can be classified into various types based on their properties, such as whether they include negative values, fractions, or non-repeating decimals. Understanding these types helps in solving problems across disciplines like science, engineering, and finance.

Types of Numbers

Numbers are categorized into several sets, each building upon or extending the previous ones. These sets include natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers.

Natural Numbers

Definition:

Natural numbers are the positive counting numbers starting from 1 and extending infinitely. They are used for counting objects and ordering. Note: Some definitions include 0 as a natural number, but traditionally (especially in set theory), natural numbers start from 1.

Properties:

- Closed under addition and multiplication (i.e., adding or multiplying two natural numbers results in another natural number).
- Not closed under subtraction or division (e.g., 3 - 5 = -2, which is not natural).
- They are infinite and have no upper bound.
- The smallest natural number is 1.
- They are all positive and have no fractional or decimal parts.

Examples:

- 1, 2, 3, 4, 5, ...
- Counting apples: If you have 5 apples and add 3 more, you get 8 apples (addition).
- Ordering: First (1), second (2), etc.

Non-Examples:

- 0 (if excluding it), -1, 1/2, √2.

Set Notation:

Natural numbers are denoted by ℕ or N. ℕ = {1, 2, 3, 4, ...}

Applications:

Natural numbers are used in everyday counting, indexing in computer science (e.g., array positions starting from 1 in some languages), and in number theory for studying primes.

Whole Numbers

Definition: Whole numbers are natural numbers including 0. They extend natural numbers to include the concept of "nothing" or zero. This set is useful when counting can include absence.

Properties:

- Closed under addition and multiplication.
- Not closed under subtraction (e.g., 2 - 3 = -1, not whole) or division.
- Infinite, starting from 0 with no upper bound.
- All non-negative integers without fractions.

Examples:

- 0, 1, 2, 3, 4, ...
- Bank balance: If you have $0, it's a whole number representing no money.
- Temperature in Kelvin: Always non-negative, like 0 K (absolute zero).

Non-Examples:

- -5, 3.14, √3.

Set Notation:

Whole numbers are often denoted by W or $\mathbb{N}_0$ (natural numbers including 0). W = {0, 1, 2, 3, ...}

Relationship to Natural Numbers:

Whole numbers = Natural numbers ∪ {0}. All natural numbers are whole numbers, but 0 is a whole number that is not natural (if natural starts from 1).

Applications:

Used in inventory management (e.g., 0 items in stock), non-negative measurements, and as a foundation for integers.

Integers

Definition: Integers include all whole numbers and their negative counterparts. They represent directed quantities, like debts or temperatures below zero.

Properties:

- Closed under addition, subtraction, and multiplication.
- Not closed under division (e.g., $5 \div 2 = 2.5$, not an integer).
- Infinite in both positive and negative directions.
- No fractional parts; discrete values.

Examples:

- ..., -3, -2, -1, 0, 1, 2, 3, ...
- Elevation: Sea level is 0, Mount Everest is +8,848 meters, Dead Sea is -430 meters.
- Financial transactions: +$100 (credit), -$50 (debit).

Non-Examples:

- $1/2$, π, $\sqrt{5}$.

Set Notation:

Integers are denoted by $\mathbb{Z}$ or Z (from German "Zahlen," meaning numbers). $\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$

Relationship to Previous Sets:

Integers = Whole numbers ∪ Negative whole numbers (excluding 0, which is already included). All whole numbers (and thus natural numbers) are integers.

Applications:

Integers are crucial in programming (e.g., int data type), accounting (profits and losses), and physics (e.g., charge: +1 for proton, -1 for electron).

Rational Numbers

Definition: Rational numbers are numbers that can be expressed as the ratio of two integers (a fraction p/q), where p and q are integers, and q ≠ 0. They include all integers, fractions, and terminating or repeating decimals.

Properties:

- Closed under addition, subtraction, multiplication, and division (except by zero).
- Dense: Between any two rationals, there is another rational.
- Can be positive, negative, or zero.
- Equivalent fractions represent the same rational (e.g., 1/2 = 2/4).

Examples:

- 1/2, -3/4, 5 (which is 5/1), 0.25 (1/4), 0.333... (1/3).
- Dividing a pizza: 3/8 of a pizza remaining.
- Measurements: 2.5 meters (5/2).

Non-Examples:

- √2 (approximately 1.414..., non-repeating), π.

Set Notation:

Rational numbers are denoted by ℚ or Q (from "quotient"). $\mathbb{Q} = \{p/q \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0\}$

Relationship to Previous Sets:

All integers are rational (since any integer n = n/1).

Thus, integers ⊂ rationals.

Applications:

Rationals are used in recipes (e.g., 1/2 cup), probability (e.g., 3/4 chance), and engineering calculations where exact fractions are preferred over decimals.

Irrational Numbers

Definition: Irrational numbers cannot be expressed as a simple fraction p/q where p and q are integers and q ≠ 0. Their decimal expansions are non-terminating and non-repeating.

Properties:

- Not closed under addition or multiplication (e.g., $\sqrt{2} + (-\sqrt{2}) = 0$, which is rational).
- Dense: Between any two irrationals, there is another irrational.
- Can be positive or negative (though negatives are less common in basic contexts).
- Often arise from roots, logarithms, or transcendental functions.

Examples:

- $\sqrt{2}$ (≈1.414213562...), π (≈3.1415926535...), e (≈2.718281828...).
- Diagonal of a square: For a square with side 1, diagonal is $\sqrt{2}$.
- Circumference of a circle: $C = 2\pi r$, where π is irrational.

Non-Examples:

- 1/3 (repeating decimal but rational), $\sqrt{4}$ (which is 2, rational).

Set Notation:

Irrational numbers are often denoted by $\mathbb{Q}'$ or $\mathbb{R} \setminus \mathbb{Q}$ (real numbers minus rationals). No standard single symbol.

Relationship to Previous Sets:

Irrationals are disjoint from rationals; no overlap.

Applications:

Used in geometry (e.g., Pythagorean theorem), physics (e.g., wave functions involving e), and cryptography (e.g., irrational-based algorithms for security).

Real Numbers

Definition: Real numbers include all rational and irrational numbers. They represent all points on the continuous number line, filling in all gaps.

Properties:

- Closed under addition, subtraction, multiplication, and division (except by zero).
- Complete: Every non-empty set of reals with an upper bound has a least upper bound (supremum).
- Infinite and uncountable (unlike rationals, which are countable).

- Include all measurable quantities.

Examples:

- All previous examples: 5, -1/2, √3, π, e.
- Distance: Any length, like 1.732 meters (≈√3).
- Time: Continuous values, not just integers.

Non-Examples:

- Complex numbers like √(-1) = i (imaginary unit).

Set Notation:

Real numbers are denoted by ℝ or R. ℝ = ℚ ∪ (ℝ \ ℚ) = Rationals ∪ Irrationals.

Relationship to Previous Sets:

All natural, whole, integers, rationals, and irrationals are real numbers. Reals are the union of rationals and irrationals.

Applications:

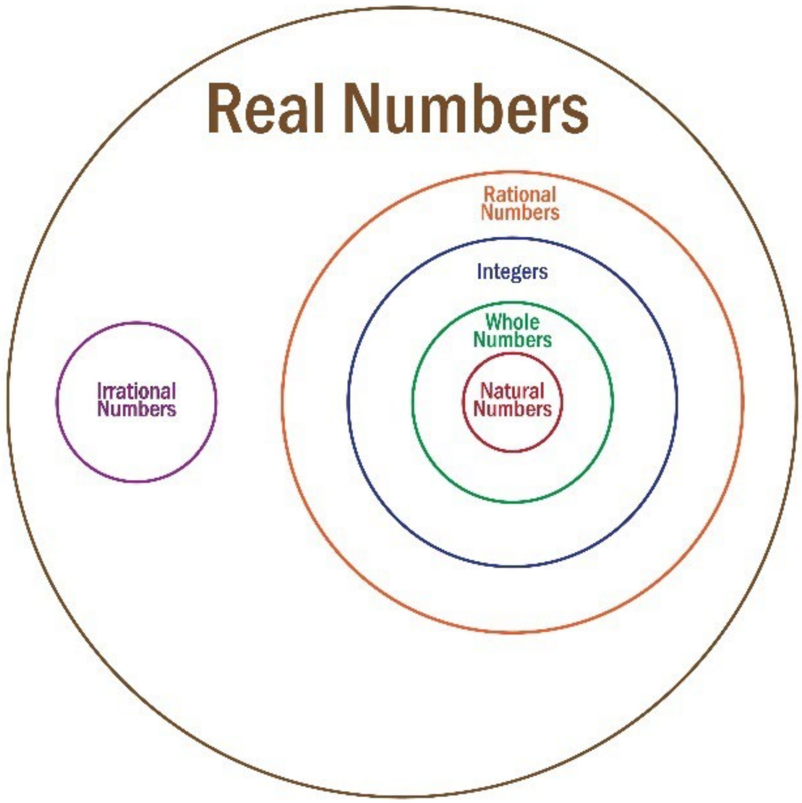
Reals are the foundation for calculus, physics (e.g., velocity as real-valued), economics (e.g., continuous growth models), and computer graphics (e.g., floating-point numbers approximating reals).

Summary of Relationships (Number Line Hierarchy)

- Natural ⊂ Whole ⊂ Integers ⊂ Rational ⊂ Real
- Irrational ⊂ Real
- Real = Rational ∪ Irrational (disjoint union)

This hierarchy shows how each set expands to include more numbers, allowing for broader mathematical operations and representations.

Venn Diagram Representation



Key Differences Table

Type	Includes Negatives?	Includes Fractions?	Decimal Form	Examples
Natural	No	No	Whole positives	1, 2, 3
Whole	No	No	Whole non-negatives	0, 1, 2
Integers	Yes	No	Whole numbers	-2, 0, 3
Rational	Yes	Yes	Terminating/repeating	-1/2, 0.25, 1/3
Irrational	Yes (possible)	No	Non-terminating/non-repeating	√2, π, e
Real	Yes	Yes	All decimals	All above

Practice Problems of NUMBERS

Problem 1
Find the sum of first 20 natural numbers.

Solution
Formula: $S = n(n + 1) / 2$
$S = (20 \times 21) / 2 = 210$

Answer: 210

Problem 2
Find the 10th natural number.

Solution
Natural numbers start from 1.
10th natural number = 10

Answer: 10

Problem 3
How many natural numbers are there between 15 and 30?

Solution
Numbers = 16 to 29
Count = 29 – 16 + 1 = 14

Answer: 14 numbers

Problem 4
Is 0 a natural number?

Solution
No, because natural numbers start from 1.
0 is a whole number, not a natural number.

Problem 5
Find the smallest whole number.

Solution
Whole numbers start from 0.

Answer: 0

Problem 6
Find the value: –7 + 12

Solution
–7 + 12 = 5

Answer: 5

Problem 7
Evaluate: –15 – (–8)

Solution
Subtracting a negative means adding:
–15 + 8 = –7

Answer: –7

Problem 8
Which is greater: –3 or –7?

Solution
On a number line, –3 lies to the right of –7.

Answer: –3 is greater

Problem 9
Is 245 even or odd?

Solution
Last digit = 5, which is odd.

Answer: Odd

Problem 10
Find the sum of the first 10 even numbers.

Solution
First 10 even numbers: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
Sum formula: $S = n(n + 1)$
$S = 10 \times 11 = 110$

Answer: 110

Problem 11
Find the sum of the first 15 odd numbers.

Solution
Formula: $S = n^2$
$S = 15^2 = 225$

Answer: 225

Problem 12
Is –5 a natural number, whole number, or integer?

Solution
Natural? No.
Whole? No.
Integer? Yes.

Answer: Integer only

Problem 13
How many integers are between –10 and 5?

Solution
Numbers: –9 to 4
Count: 4 – (–9) + 1 = 14

Answer: 14 integers

Problem 14
Find the additive inverse of –18.

Solution
Additive inverse is the number that makes the sum zero.
–18 + 18 = 0

Answer: 18

Problem 15
Find: (–4) × (–6)

Solution
Negative × Negative = Positive
Result = 24

Answer: 24

Problem 16
Simplify: 8 – 12 + 5

Solution
8 – 12 = –4
–4 + 5 = 1

Answer: 1

Problem 17
Find the greatest integer less than 3.7.

Solution
The greatest integer less than or equal to 3.7 is 3.

Answer: 3

2. Prime and Composite Numbers

Prime and composite numbers are classifications within natural numbers (starting from 1). These categories are essential in number theory, cryptography, and factorization problems. Prime numbers are the building blocks of all natural numbers greater than 1, as every such number can be uniquely factored into primes (Fundamental Theorem of Arithmetic). Understanding primes and composites helps in divisibility rules, greatest common divisors (GCD), and least common multiples (LCM).

Least Common Multiple (LCM)

1. Introduction to LCM

The **Least Common Multiple (LCM)** of two or more numbers is the smallest multiple that is divisible by each number without a remainder.

Example: LCM of 4 and 5 is 20.

2. Methods to Find LCM

A. Listing Multiples Method

1. List multiples of each number.
2. Identify common multiples.
3. Smallest common multiple is the LCM.

Example 1: Find LCM of 4 and 6.

- Multiples of 4: 4, 8, 12, 16, 20, 24, …
- Multiples of 6: 6, 12, 18, 24, 30, …
- Common multiples: 12, 24, …
- **LCM = 12**

B. Prime Factorization Method

1. Prime factorize each number.
2. Take highest power of each prime factor.
3. Multiply these powers.

Example 2: Find LCM of 12 and 15.

- 12 = 2^2 × 3^1
- 15 = 3^1 × 5^1
- Highest powers: 2^2, 3^1, 5^1
- LCM = 2^2 × 3 × 5 = 4 × 3 × 5 = **60**

C. Division Method (Ladder Method)

1. Write numbers in a row.

$490 = 2 \times 5 \times 7^2$
$560 = 2^4 \times 5 \times 7$
Common factors: $2 \times 5 \times 7 = 70$
Answer = 70

Problem 18
A teacher has 72 pens and 108 pencils. She wants to distribute them equally among students. Find the maximum number of students who can receive equal items.

Solution:
Prime factorization:
$72 = 2^3 \times 3^2$
$108 = 2^2 \times 3^3$
Common factors: $2^2 \times 3^2 = 36$
Maximum students = 36

Problem 19
Find the greatest number that can divide 252 and 420 such that no remainder is left.

Solution:
Prime factorization:
$252 = 2^2 \times 3^2 \times 7$
$420 = 2^2 \times 3 \times 5 \times 7$
Common factors: $2^2 \times 3 \times 7 = 84$
Answer = 84

Problem 20
Three bells ring at intervals of 12 minutes, 18 minutes, and 30 minutes. Find the maximum time interval that divides all these intervals exactly.

Solution:
Prime factorization:
$12 = 2^2 \times 3$
$18 = 2 \times 3^2$
$30 = 2 \times 3 \times 5$
Common factor: $2 \times 3 = 6$
Maximum time interval = 6 minutes

Number Series and Patterns

Introduction

A **number series** is an ordered list of numbers that follow a specific rule or pattern.
A **number pattern** describes the relationship between the numbers in the series.

Understanding number series helps in:
• Logical thinking
• Problem solving
• Algebraic reasoning
• Competitive exams and school mathematics

1. Types of Number Series

Number series can be broadly classified into the following types:

1. Arithmetic Series
2. Geometric Series

Fractions, Decimals & Percentages

1. Fractions

Definition

A **fraction** represents a part of a whole and is written in the form:

a / b

where
a = numerator (top number)
b = denominator (bottom number), $b \neq 0$

Types of Fractions

Proper Fraction:
Numerator < Denominator
Example: 3/5, 7/9

Improper Fraction:
Numerator ≥ Denominator
Example: 7/4, 9/3

Mixed Fraction:
Combination of a whole number and a proper fraction
Example: 1 3/4, 2 5/6

Equivalent Fractions:
Fractions with different forms but same value
Example:
1/2 = 2/4 = 3/6

2. Operations with Fractions

A. Addition of Fractions

Case 1: Same Denominator

Add the numerators, keep the denominator same.

Example:
3/7 + 2/7 = (3 + 2)/7 = 5/7

Case 2: Different Denominators

Steps:

Detailed Example 3 (Successive): 10% inc, then 10% dec on salary $40,000.

Final?

40,000→44,000→39,600.

Net -1%.

Explanation: Loss due to base.

Detailed Example 4 (Mixed): Population 10,000, +5% year1, -3% year2, +2% year3.

Final?

10,000×1.05=10,500; ×0.97≈10,185; ×1.02≈10,388.7.

Net ≈3.89%.

Explanation: Compound growth.

Detailed Example 5 (Reverse): Final price after 25% inc is $125.

Original?

Original ×1.25=125

→ Original=100.

Explanation: Divide by multiplier.

Summary Table of Formulas and Examples

Concept	Formula	Example	Result
% of Number	(x/100) × y	30% of 200	60
% Increase	[(New - Orig)/Orig] ×100	100 to 120	20%
% Decrease	[(Orig - New)/Orig] ×100	100 to 80	20%
New after Inc	Orig × (1 + x/100)	200 +15%	230
New after Dec	Orig × (1 - x/100)	200 -15%	170
Successive Two	x + y + (x y)/100	+10% then +20%	32%
Overall Multiplier	∏(1 + p_i/100)	+5%, -10%	0.945

Ratio and Proportion

1. Introduction to Ratio and Proportion

What is a Ratio?
A ratio is a relationship between two quantities, indicating how many times the first number contains the second. It is expressed in the form:

Ratio = a/b = a : b

Where:

- a and b are two quantities
- The ratio a : b represents how many times a is compared to b

Example of Ratio:
If there are 6 boys and 4 girls in a class, the ratio of boys to girls is:
6/4 = 6 : 4 or 3 : 2
This means that for every 3 boys, there are 2 girls.

2. Types of Ratios

- **Simple Ratios:** Involves only two quantities
- **Compound Ratios:** Involves more than two quantities, e.g., the ratio of a : b and c : d is a : b : c : d

3. Proportion

A proportion is an equation that expresses the equality of two ratios. If two ratios are equal, they are said to be in proportion.

Formula for Proportion:

a/b = c/d

This means that the ratio of a to b is equal to the ratio of c to d. The quantities a, b, c, and d are said to be in proportion.

Example of Proportion:
If the ratio of boys to girls in two classes is the same, such as:

6/4 = 9/6

This is a proportion, as both ratios are equal, i.e., 3 : 2 = 3 : 2.

Problem 24
Two numbers are in the ratio 7 : 11. If their difference is 24, find the numbers.

Solution:
Difference in parts = 11 − 7 = 4
1 part = 24 ÷ 4 = 6
First number = 7 × 6 = 42
Second number = 11 × 6 = 66
Answer: 42 and 66

Problem 25
The ratio of the number of red balls to blue balls is 2 : 5. If there are 140 balls in total, find the number of blue balls.

Solution:
Total parts = 2 + 5 = 7
1 part = 140 ÷ 7 = 20
Blue balls = 5 × 20 = 100
Answer: 100

Problem 26
The ratio of expenditure to savings of a person is 7 : 3. If his savings are Rs. 6,000, find his total income.

Solution:
3 parts = 6,000
1 part = 6,000 ÷ 3 = 2,000
Total parts (expenditure + savings) = 7 + 3 = 10
Total income = 10 × 2,000 = 20,000
Answer: Rs. 20,000

Partnership and Sharing

1. Introduction to Partnership

In mathematics, partnership problems usually refer to the division of profits or losses between partners in a business, based on their investment or effort. The concept can be broken down into:

- Sharing profits and losses based on capital contribution
- Time-based partnerships, where investment is multiplied by time spent

2. Formula for Partnership Problems

The share of profit or loss for each partner is determined by the ratio of their investments or efforts:

Partner's Share = (Investment of Partner / Total Investment) × Total Profit or Loss

Where:

- Investment of Partner: Amount of money or effort contributed
- Total Investment: Sum of all partners' investments
- Total Profit or Loss: Amount to be shared

Answer: Rs. 3,000

Problem 10
A invests Rs. 12,000 for the whole year. B invests Rs. 8,000 for 9 months. If the total profit is Rs. 10,000, find each person's share.

Solution:
A's capital-time = 12,000 × 12 = 144,000
B's capital-time = 8,000 × 9 = 72,000
Ratio = 144,000 : 72,000 = 2 : 1
Total parts = 2 + 1 = 3
A's share = (2/3) × 10,000 ≈ 6,667
B's share = (1/3) × 10,000 ≈ 3,333

Answer: A ≈ Rs. 6,667, B ≈ Rs. 3,333

Allegation/Mixture

1. Introduction to Allegation/Mixture Problems

An allegation or mixture problem involves mixing two or more substances to form a mixture, typically involving finding proportions, cost, strength, or concentration of the mixture.

The Allegation Method solves these problems quickly and efficiently, especially with mixtures of different quantities, prices, or concentrations.

2. Allegation Formula

The allegation method is based on a simple rule applied when two quantities with different values are mixed to obtain a desired result.

Rule of Allegation:

If we mix two substances with different prices, concentrations, or qualities, the mixing ratio is:

Required Ratio = $(C_2 - C) / (C - C_1)$

Where:

- C_1 = Cost, concentration, or quality of the first substance
- C_2 = Cost, concentration, or quality of the second substance
- C = Desired cost, concentration, or quality of the mixture

3. Types of Allegation/Mixture Problems

A. Allegation of Prices (Cost Mixture Problems)

Mixing two or more goods with different prices to obtain a mixture with a certain cost.

Example 1: Mixing Two Different Priced Items

A = 5000 × (1 + 10/100)^2 = 5000 × (1.10)^2 = 5000 × 1.21 = Rs. 6,050
CI = A - P = 6050 - 5000 = Rs. 1,050

Answer: Compound Interest = Rs. 1,050

Example 2:
Problem: Rs. 8,000 invested at 12% per annum for 3 years, compounded annually. Find total amount.

Solution:
P = Rs. 8,000, R = 12%, T = 3 years
A = 8000 × (1 + 12/100)^3 = 8000 × (1.12)^3 = 8000 × 1.404928 = Rs. 11,239.42
CI = A - P = 11239.42 - 8000 = Rs. 3,239.42

Answer: Total Amount = Rs. 11,239.42, Compound Interest = Rs. 3,239.42

Different Compounding Periods:
For interest compounded 'n' times per year:
A = P × (1 + R/(100×n))^(n×T)

Where n = number of compounding periods per year (e.g., n=4 for quarterly, n=12 for monthly)

4. Differences Between Simple and Compound Interest

Aspect	Simple Interest	Compound Interest
Interest Calculation	Calculated only on principal	Calculated on principal + accumulated interest
Formula	SI = (P×R×T)/100	A = P(1+R/100)^T, CI = A - P
Growth Pattern	Linear growth	Exponential growth
Interest per Period	Constant	Increases over time
Best For	Short-term loans/investments	Long-term investments/savings

Practice Problems

Problem:
A person invests Rs. 10,000 at 5% per annum. What is the interest earned after 4 years?

Solution:
P = Rs. 10,000, R = 5%, T = 4 years
SI = (P × R × T) / 100
SI = (10,000 × 5 × 4) / 100 = (200,000) / 100 = Rs. 2,000

Binary System

M K P R E P A R A T

1. Introduction to the Binary System

The binary system is a numerical system that operates using only two digits: 0 and 1. It is the foundation of all modern computing systems, which rely on binary data representation. In contrast to the decimal system (which uses ten digits: 0 through 9), the binary system operates on a base of 2.

2. Why Binary System?

The binary system is closely related to the physical structure of digital electronic circuits. Computers and other electronic devices operate using switches (transistors), which can be in one of two states:

- ON (1)
- OFF (0)

Thus, binary values naturally map to these two states, allowing for efficient data storage, processing, and transmission in computers.

3. Binary Digits (Bits)

A binary digit, or bit, is the smallest unit of information in computing, represented by either 0 or 1.

- Bit is short for Binary Digit.
- A group of 8 bits is called a byte.

4. Place Value in the Binary System

The binary system is a positional number system, similar to the decimal system, but with only two base values. The place values in binary are powers of 2.

The rightmost bit is the least significant bit (LSB), and the leftmost bit is the most significant bit (MSB).

The place values for each digit are powers of 2. For example:

- $2^0 = 1$
- $2^1 = 2$
- $2^2 = 4$

ALGEBRA

Algebraic Expressions and Identities

1. Introduction to Algebraic Expressions

An **algebraic expression** is a combination of numbers, variables, and arithmetic operations (addition, subtraction, multiplication, division, etc.). It does not have an equality sign.

Example:
$2x + 3$, $4a^2 - 5a + 6$, and $3y^2 - 7y + 2$ are algebraic expressions.

Components of an Algebraic Expression:

- **Variables:** Letters representing unknown quantities (e.g., x, y, a).
- **Coefficients:** Numbers multiplying the variables (e.g., in 2x, the coefficient is 2).
- **Constants:** Numbers that do not change (e.g., in 2x + 3, the constant is 3).
- **Operators:** Mathematical operations (+, −, ×, ÷).

2. Types of Algebraic Expressions

A. Monomial – One term.
Example: $4x$, $7y^2$, $-3a$.

B. Binomial – Two terms.
Example: $2x + 3$, $5a - 4b$, $x^2 + y^2$.

C. Trinomial – Three terms.
Example: $x^2 + 2x + 1$, $3a^2 + 5a + 7$.

D. Polynomial – More than one term.
Example: $2x^3 + 3x^2 - 5x + 6$.

3. Basic Operations on Algebraic Expressions

A. Addition of Algebraic Expressions
Combine like terms (terms with the same variables raised to the same powers).

Example 1: Add $3x + 5$ and $2x - 4$.
$(3x + 5) + (2x - 4) = (3x + 2x) + (5 - 4) = \mathbf{5x + 1}$

B. Subtraction of Algebraic Expressions
Subtract like terms.

Algebra Word Problems

1. Age Problems

Age problems are a common type of algebra word problem where relationships between the ages of two or more people are given, and you are asked to find their ages at different points in time (present, past, or future). These problems require setting up equations based on the given relationships.

General Steps to Solve Age Problems:

1. **Define variables:** Assign variables for the unknowns (usually ages). For example, if finding the current age of two people, let x represent the age of person A, and y represent the age of person B.
2. **Translate the information into equations:** Use the relationships in the problem to form algebraic equations. This may involve relationships like "in 5 years," "10 years ago," or "three times as old."
3. **Solve the system of equations:** Use algebraic methods such as substitution or elimination.
4. **Check the solution:** Verify that the found ages satisfy all conditions in the problem.

Types of Age Problems:

1.1 Simple Age Difference

These problems involve a simple difference in the ages of two people.

- **Example:** John is 5 years older than Jane. If Jane's current age is x, express John's age in terms of x.
- **Solution:** Let Jane's age be x. John's age is x + 5.

1.2 Age Difference at a Given Time

These problems give the relationship between ages at a specific point in the past or future.

- **Example:** John is currently 5 years older than Jane. In 10 years, John will be twice as old as Jane. How old are they now?
- **Solution:**
  - Let Jane's current age be x.
  - John's current age is x + 5.
  - In 10 years: Jane's age = x + 10, John's age = (x + 5) + 10 = x + 15.

Range:
The function can output any real number except 0, because 1/(x-2) never equals 0.
Range = $\{y \in \mathbb{R} : y \neq 0\} = (-\infty, 0) \cup (0, \infty)$.

Answer:
Domain: $(-\infty, 2) \cup (2, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$

Problem 2: Inverse Function

Problem:
Find the inverse of the function $f(x) = 3x + 4$.

Solution:
Let $y = f(x) = 3x + 4$.
Solve for x:
$y - 4 = 3x$
$x = (y - 4)/3$

Swap x and y to obtain the inverse:
$f^{-1}(x) = (x - 4)/3$

Answer:
$f^{-1}(x) = (x - 4)/3$

Problem 3: Function Composition

Problem:
Given $f(x) = x^2 + 1$ and $g(x) = 2x - 3$, find the composite function $(f \circ g)(x)$.

Solution:
$(f \circ g)(x) = f(g(x))$
Substitute g(x) into f(x):
$f(g(x)) = (2x - 3)^2 + 1$
Expand: $(2x - 3)^2 = 4x^2 - 12x + 9$
So, $f(g(x)) = 4x^2 - 12x + 9 + 1 = 4x^2 - 12x + 10$

Answer:
$(f \circ g)(x) = 4x^2 - 12x + 10$

Problem 4: Function with Specific Domain and Range

Problem:
Let $f(x) = \sqrt{x - 2}$. Find the domain and range of the function.

GEOMETRY

A. Plane Geometry

• Lines and Angles (Properties, Theorems)

1. Basic Definitions

- **Point**: A location in space with no dimensions (no length, width, or height).
- **Line**: A straight one-dimensional figure that extends infinitely in both directions. It has no width or thickness but has length.
- **Line Segment**: A part of a line that has two distinct endpoints.
- **Ray**: A part of a line that starts at a point (called the origin) and extends infinitely in one direction.
- **Angle**: The figure formed by two rays (called the sides of the angle) sharing a common endpoint (called the vertex).
- **Collinear Points**: Points that lie on the same straight line.
- **Coplanar Points**: Points that lie on the same plane.

2. Types of Lines

- **Parallel Lines**: Two lines that never meet and are always the same distance apart. These lines lie on the same plane.
- **Perpendicular Lines**: Two lines that intersect at a 90° angle.
- **Intersecting Lines**: Lines that meet at exactly one point.
- **Oblique Lines**: Lines that are neither parallel nor perpendicular to each other.

3. Types of Angles

- **Acute Angle**: An angle less than 90°.
- **Right Angle**: An angle equal to 90°.
- **Obtuse Angle**: An angle greater than 90° but less than 180°.
- **Straight Angle**: An angle of 180° (i.e., a straight line).
- **Reflex Angle**: An angle greater than 180° but less than 360°.

3. **Equilateral Triangle Theorem**: All angles of an equilateral triangle are 60° and all sides are equal.
4. **Angle Bisector Theorem**: An angle bisector divides the opposite side into segments that are proportional to the adjacent sides of the triangle.

9. Circle Geometry (Related to Lines and Angles)

- **Tangent to a Circle**: A line that touches the circle at exactly one point.
- **Secant to a Circle**: A line that intersects the circle at two points.
- **Central Angle**: An angle whose vertex is at the center of the circle and whose sides pass through two points on the circle.
- **Inscribed Angle**: An angle formed by two chords in a circle that share a common endpoint.

10. Applications of Lines and Angles

- **Angle of Elevation**: The angle formed between the line of sight and the horizontal when an observer looks at an object above the horizontal.
- **Angle of Depression**: The angle formed between the line of sight and the horizontal when an observer looks at an object below the horizontal.
- **Real-Life Examples**: Calculating heights of buildings using angles of elevation or depression, navigation using bearing angles, etc.

Practice Problems

Problem 1: Complementary Angles

Problem:
Two angles are complementary. One angle is 10° less than the other. Find the measures of both angles.

Solution:
Let the measure of one angle be x.
Then the other angle is x - 10°.
Since complementary angles sum to 90°, we have:
$x + (x - 10°) = 90°$
$2x - 10° = 90°$
$2x = 100°$

Trigonometry

A. Basic Concepts

Trigonometry is the branch of mathematics that deals with the relationships between the sides and angles of triangles, especially right-angled triangles. It also studies trigonometric functions, which are ratios that express these relationships.

1. Trigonometric Ratios (Sine, Cosine, Tangent, etc.)

In a right-angled triangle, for a given angle θ (theta):

- **Hypotenuse (H):** Side opposite the right angle
- **Adjacent side (A):** Side next to angle θ (excluding hypotenuse)
- **Opposite side (O):** Side opposite angle θ

The six trigonometric ratios are defined as:

Ratio	Definition	Formula
Sine (sin θ)	Opposite / Hypotenuse	$\sin\theta = O/H$
Cosine (cos θ)	Adjacent / Hypotenuse	$\cos\theta = A/H$
Tangent (tan θ)	Opposite / Adjacent	$\tan\theta = O/A$
Cotangent (cot θ)	Adjacent / Opposite	$\cot\theta = A/O = 1/\tan\theta$
Secant (sec θ)	Hypotenuse / Adjacent	$\sec\theta = H/A = 1/\cos\theta$
Cosecant (csc θ)	Hypotenuse / Opposite	$\csc\theta = H/O = 1/\sin\theta$

Trigonometric Identities

Trigonometric identities are equations involving trigonometric functions that hold true for all values of the involved variables (where the functions are defined). These identities are essential tools in simplifying trigonometric expressions, solving equations, and proving other mathematical statements.

1. Pythagorean Identities

Derived from the Pythagorean theorem, these are the most fundamental identities:

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

Height = 50 × tan 30°

= 50 × (1/√3)

≈ 50 × 0.577

≈ 28.85 m

Answer: The height of the tower is approximately **28.85 m**.

2. Simple Trigonometric Equations

Trigonometric equations are used to find unknown angles or sides in geometric problems. Solving them involves algebraic techniques and trigonometric identities.

2.1 Solving for Unknown Angles

Given a trigonometric function and its value, use the inverse trigonometric functions:

Equation	Solution
$\sin\theta = x$	$\theta = \sin^{-1}(x)$
$\cos\theta = x$	$\theta = \cos^{-1}(x)$
$\tan\theta = x$	$\theta = \tan^{-1}(x)$

Example 1:

Solve for θ if sin θ = 0.5.

$\theta = \sin^{-1}(0.5) = 30°$

2.2 Solving for Unknown Sides Using Trigonometric Ratios

Given an angle and one side in a right-angled triangle:

Ratio	Formula for Unknown Side
sin θ = Opposite / Hypotenuse	Opposite = sin θ × Hypotenuse
cos θ = Adjacent / Hypotenuse	Adjacent = cos θ × Hypotenuse
tan θ = Opposite / Adjacent	Opposite = tan θ × Adjacent

Example 2:

In a right-angled triangle, θ = 45° and the hypotenuse is 10 units. Find the opposite side.

sin 45° = Opposite / 10

Opposite = 10 × sin 45°

= 10 × (√2 / 2)

≈ 10 × 0.7071

≈ 7.071 units

2.3 Solving Trigonometric Equations Involving Identities

Sometimes equations involve more than one function. Use identities to simplify.

Example 3: Solve $\sin\theta + \cos\theta = 1$.

Solution:

1. Rearrange: $\sin\theta = 1 - \cos\theta$
2. Square both sides: $\sin^2\theta = (1 - \cos\theta)^2$
3. Use $\sin^2\theta = 1 - \cos^2\theta$:
   $1 - \cos^2\theta = 1 - 2\cos\theta + \cos^2\theta$
4. Simplify: $2\cos^2\theta - 2\cos\theta = 0$
   $2\cos\theta(\cos\theta - 1) = 0$
5. Solve:
   - $\cos\theta = 0 \rightarrow \theta = 90°, 270°$
   - $\cos\theta = 1 \rightarrow \theta = 0°$

Solutions: $\theta = 0°, 90°, 270°$.

Conclusion

Trigonometric applications, particularly in solving height and distance problems and simple trigonometric equations, are fundamental in physics, engineering, and architecture. Mastering these techniques allows efficient solution of many practical and theoretical problems.

Practice Problems

Problem 1

A man is standing 30 meters away from a building. The angle of elevation from his eyes to the top of the building is 45°. Find the height of the building.

Solution:
Given:

- Distance (base) = 30 m
- Angle of elevation $\theta = 45°$

Using formula:
Height = Distance × $\tan\theta$

Statistics and Probability

Statistics is a branch of mathematics that deals with **collecting, analyzing, interpreting, presenting, and organizing data**. It helps us understand **trends and patterns** in data. Three important concepts in statistics are the **Mean, Median, and Mode**. Let's break these down in more detail to understand them clearly.

1. Mean (Arithmetic Mean)

Definition:
The **mean**, often called the **average**, is one of the most commonly used **measures of central tendency**. It gives us a summary of the data by telling us where the **center** of the data is.

To calculate the mean, we **add up all the numbers** in the data set and **divide the sum** by the **total number of numbers** in the set.

Formula:
Mean = $(\Sigma x_i) / n$
Where:
x_i = individual data points
n = total number of data points in the set

How to Calculate:

1. **Add** all the numbers in the data set.
2. **Divide** the sum by the **number of data points**.

Example 1:
For the data set **{10, 12, 14, 16, 18}**:
• Add the numbers: 10 + 12 + 14 + 16 + 18 = **70**
• There are **5** numbers, so divide the sum by 5: 70 / 5 = **14**
Thus, the **mean is 14**.

Properties of Mean:
• **Sensitive to outliers**: The mean can be heavily affected by very large or very small numbers (**outliers**) in the data.
• **Useful for normal (bell-shaped) distributions**: The mean is a good measure for data that is evenly spread out.

2. Median

Definition:
The **median** is the **middle number** in a data set when the numbers are arranged in **order from**

Problem 7: Probability of Drawing a Red Card or a King from a Deck of 52 Cards

Question: What is the probability of drawing a red card or a King?

Solution:

- Red cards: 26 (13 hearts + 13 diamonds).
- Kings: 4.
- Overlap: The **King of Hearts** and **King of Diamonds** are both red cards. Avoid double-counting.
- Favorable outcomes = Red cards + Kings that are not red = 26 + 2 = 28.
- Total outcomes: 52.
- Probability = 28/52 = 7/13.

Answer: The probability is **7/13**.

Problem 8: Probability of Rolling a Number Less Than 3 on a Fair Die

Question: What is the probability of rolling a number less than 3?

Solution:

- Numbers less than 3: {1, 2} → 2 favorable outcomes.
- Total outcomes: 6.
- Probability = 2/6 = 1/3.

Answer: The probability is **1/3**.

Problem 9: Probability of Drawing a Card That is Not a Heart

Question: What is the probability of drawing a card that is not a heart?

Solution:

- Hearts: 13 cards.
- Non-hearts: 52 – 13 = 39 favorable outcomes.
- Total outcomes: 52.
- Probability = 39/52 = 3/4.

Answer: The probability is **3/4**.

ADVANCED QUANTITAVES

A. Sequence and Series

In mathematics, **sequences** and **series** are important topics that deal with ordered lists of numbers and the sums of those lists, respectively. **Arithmetic Progression (AP)** and **Geometric Progression (GP)** are two fundamental types of sequences.

1. Arithmetic Progression (AP)

Definition:

An **Arithmetic Progression (AP)** is a sequence of numbers in which the difference between any two consecutive terms is **constant**. This difference is called the **common difference** and is denoted by **d**.

General Form:

a, a+d, a+2d, a+3d, …

Where:

- **a** = first term
- **d** = common difference
- **n** = number of terms

n-th Term of an AP:

The **n-th term** (denoted T_n) of an AP is given by:

$$T_n = a + (n - 1) \cdot d$$

Where:

- T_n = n-th term
- **a** = first term
- **d** = common difference
- **n** = term number

Sum of the First n Terms of an AP:

The sum of the first **n** terms (denoted S_n) is:

$$S_n = (n/2) \cdot [2a + (n - 1) \cdot d]$$

Seating Arrangements (Permutations):
Assigning seats to guests (order matters) is a permutation problem.

Choosing Teams (Combinations):
Selecting players from a pool (order does not matter) is a combination problem.

Password Creation (Permutations):
Creating passwords from a set of characters (order matters) is a permutation problem.

Summary of Key Differences

Property	Permutations	Combinations
Order matters?	Yes	No
Formula	P(n, r) = n! / (n−r)!	C(n, r) = n! / [r! × (n−r)!]
Example	Arranging people in seats	Selecting members for a team
Use case	Seating arrangements, password formation	Lottery, team selection, committee formation

Permutations and Combinations Practice Problems

Problem 1: Permutation Problem

Problem: How many different ways can 4 books be arranged on a shelf from a collection of 8 books?

Solution:
Since the order matters, use the permutation formula:
$P(n,r) = n! / (n-r)!$
Here $n = 8$, $r = 4$:
$P(8,4) = 8! / (8-4)! = 8! / 4! = 8 \times 7 \times 6 \times 5 = 1680$

Answer: There are **1680** ways.

Problem 2: Combination Problem

Problem: In a class of 20 students, how many ways can a teacher choose 5 students to attend a meeting?

Solution:
Since the order does not matter, use the combination formula:

Solution: $x < -7$ or $x > 3$

Problem 4: Solve $2x^2 - 3x - 5 \geq 0$

Step 1: Factor the quadratic expression.
$2x^2 - 3x - 5 = (2x + 5)(x - 1)$

Step 2: Find the roots.
$(2x + 5)(x - 1) = 0$
$x = -5/2$ or $x = 1$

Step 3: Test intervals on the number line.

- For $x < -5/2$ (e.g., $x = -3$): $(2(-3)+5)(-3-1) = (-1)(-4) = 4 \geq 0$
- **For $-5/2 \leq x \leq 1$ (e.g., $x = 0$): $(2(0)+5)(0-1) = (5)(-1) = -5 < 0$**
- For $x > 1$ (e.g., $x = 2$): $(2(2)+5)(2-1) = (9)(1) = 9 \geq 0$

Solution: $x \leq -5/2$ or $x \geq 1$

Problem 5: Solve $|x - 1| \leq 3$

Step 1: Break into a compound inequality.
$-3 \leq x - 1 \leq 3$

Step 2: Solve by adding 1 to all parts.
$-3 + 1 \leq x \leq 3 + 1$
$-2 \leq x \leq 4$

Solution: $-2 \leq x \leq 4$

Problem 6: Solve $4x - 7 < 9$

Step 1: Add 7 to both sides.
$4x - 7 + 7 < 9 + 7$
$4x < 16$

Step 2: Divide both sides by 4.
$4x/4 < 16/4$
$x < 4$

Solution: $x < 4$

Problem 7: Solve $x^2 - 6x + 8 > 0$

Step 1: Factor the quadratic expression.
$x^2 - 6x + 8 = (x - 4)(x - 2)$

Quantitative Reasoning

A. Analytical Problems

1. Clock Problems (Angles, Time Calculations)

Clock problems involve determining the positions or angles of the hands of a clock at a given time, or calculating the time between two events based on the positions of the clock hands.

A.1. Angle Between the Hour and Minute Hands

Basic Concept:

- The **minute hand** moves 360° in 60 minutes → **6° per minute**.
- The **hour hand** moves 360° in 12 hours → **30° per hour** or **0.5° per minute**.

Formula:
Angle = $|30H - 5.5M|$
Where:

- **H** = number of hours
- **M** = number of minutes

If the result is greater than 180°, subtract from 360° to get the smaller angle.

Example:
Find the angle at 3:15.
H = 3, M = 15
Angle = $|30 \times 3 - 5.5 \times 15| = |90 - 82.5| =$ **7.5°**

A.2. Time Calculation Between Two Events

Relative Speed:
Minute hand speed = 6° per minute
Hour hand speed = 0.5° per minute
Relative speed = 6° − 0.5° = **5.5° per minute**

Formula:
Time (in minutes) = (Initial Angle) / (Relative Speed)

Example:
At 4:00, the angle between hands = 30°×4 = 120°.

Question:
Ravi walks 20 meters north, then turns left and walks 10 meters. He turns left again and walks 20 meters. Finally, he turns left again and walks 10 meters. How far is Ravi from his starting point?

Solution:

1. Start at (0,0).
2. Walk 20 m North → (0,20).
3. Turn left (West) → walk 10 m → (-10,20).
4. Turn left (South) → walk 20 m → (-10,0).
5. Turn left (East) → walk 10 m → (0,0).

Answer: Ravi is **0 meters** away from his starting point.

Problem 2: Multiple Turns

Question:
A person walks 15 meters east, turns right and walks 10 meters, turns right again and walks 5 meters, and then turns right again and walks 10 meters. How far is the person from the starting point?

Solution:

1. Start at (0,0).
2. Walk 15 m East → (15,0).
3. Turn right (South) → walk 10 m → (15,-10).
4. Turn right (West) → walk 5 m → (10,-10).
5. Turn right (North) → walk 10 m → (10,0).

Distance from (0,0) to (10,0) = **10 meters**.

Answer: The person is **10 meters** away from the starting point.

Problem 3: Complex Movement

Question:
A man walks 10 meters north, turns right and walks 10 meters, then turns left and walks 10 meters. After that, he turns left again and walks 5 meters. Finally, he turns left and walks 5 meters. How far is the man from his starting point?

Practical Mathematics

1. Quick Calculation Methods

These methods are designed to help you perform calculations faster by simplifying the process.

1.1. Multiplying by 5

To multiply a number by 5, you can **divide the number by 2 and then multiply the result by 10**.

Example:
$25 \times 5 = (25 \div 2) \times 10 = 12.5 \times 10 = \mathbf{125}$
Alternatively, multiply the number by 10 and then divide by 2:
$25 \times 5 = 250 \div 2 = \mathbf{125}$

1.2. Multiplying by 9

To multiply a number by 9, **multiply it by 10 and subtract the original number**.

Example:
$36 \times 9 = (36 \times 10) - 36 = 360 - 36 = \mathbf{324}$

1.3. Multiplying by 11

To multiply a number by 11, **add the number to itself shifted one place to the left** (i.e., multiply by 10 and add the number).

Example:
$23 \times 11 = 23 + (23 \times 10) = 23 + 230 = \mathbf{253}$

1.4. Squaring Numbers Close to 10

When squaring numbers close to 10, use the formula:
$\mathbf{(10 + x)^2 = 100 + 20x + x^2}$

Example:
$12^2 = (10 + 2)^2 = 100 + (20 \times 2) + 4 = 100 + 40 + 4 = \mathbf{144}$

1.5. Multiplying by 12.5

To multiply by 12.5, **multiply by 10 and then divide by 8**.

Example:
$32 \times 12.5 = (32 \times 10) \div 8 = 320 \div 8 = \mathbf{40}$

2. Estimation Skills

Equation: **1/20 + 1/x = 1/12**.
Solve for *x*:
1/x = 1/12 - 1/20 = (5 - 3)/60 = 2/60 = **1/30**.
Therefore, *x* = **30 days**.

Answer: It will take **30 days** for B to complete the job alone.

Problem 4: Time and Work with Different Rates

Question: If 3 people complete a task in 15 days, how long will 5 people take (same rate)?

Solution:
The total work is **3 people × 15 days = 45 person-days**.
For 5 people: Time = Total Work / Number of People = 45 / 5 = **9 days**.

Answer: It will take **9 days** for 5 people to complete the task.

3. Measurement Word Problems

Problem 5: Speed, Distance, and Time

Question: A cyclist travels at 20 km/h for 3 hours, then at 30 km/h for 2 hours. What is the total distance?

Solution:
Use **Distance = Speed × Time**.

- Leg 1: 20 km/h × 3 h = **60 km**.
- Leg 2: 30 km/h × 2 h = **60 km**.
  Total Distance = 60 km + 60 km = 120 km.

Answer: The total distance covered is **120 km**.

Problem 6: Volume of a Rectangular Box

Question: A box has length 4 cm, width 5 cm, height 6 cm. What is its volume?

Solution:
Formula: **Volume = Length × Width × Height**.
Volume = 4 cm × 5 cm × 6 cm = **120 cm³**.

Answer: The volume is **120 cubic centimeters**.

Problem 7: Distance Between Two Points (Coordinate Geometry)

Question: Find the distance between points A(3, 4) and B(7, 1).

Quick Revision Formula Sheets

Geometric Formulas

Here's a concise list of key geometric formulas that are commonly used in mathematics:

1. Area of Basic Shapes

1.1. Square

Area:
$A = s^2$
where s is the side length.

Perimeter:
$P = 4s$
where s is the side length.

1.2. Rectangle

Area:
$A = l \times w$
where l is the length and w is the width.

Perimeter:
$P = 2(l + w)$

1.3. Triangle

Area:
$A = \frac{1}{2} \times b \times h$
where b is the base and h is the height.

Perimeter:
$P = a + b + c$
where a, b, and c are the sides of the triangle.

1.4. Parallelogram

Area:
$A = b \times h$
where b is the base and h is the height.

Perimeter:
$P = 2(a + b)$
where a and b are the adjacent sides.

1.5. Rhombus

Area:
$A = \frac{1}{2} \times d_1 \times d_2$
where d_1 and d_2 are the diagonals.

Perimeter:
$P = 4s$
where s is the side length.

1.6. Trapezoid (Trapezium)

Area:
$A = \frac{1}{2} \times (b_1 + b_2) \times h$
where b_1 and b_2 are the lengths of the parallel sides and h is the height.

Perimeter:
$P = a + b_1 + b_2 + c$
where a, b_1, b_2, and c are the sides.

2. Circle

2.1. Circle Formulas

Area:
$A = \pi r^2$
where r is the radius.

Circumference (Perimeter):
$C = 2\pi r$
where r is the radius.

Diameter:
$D = 2r$
where r is the radius.

3. Polygonal Formulas

3.1. Regular Polygon (n sides)

Area (using side length s and apothem a):
$A = \frac{1}{2} \times n \times s \times a$
where n is the number of sides, s is the side length, and a is the apothem.

Perimeter:
$P = n \times s$
where n is the number of sides and s is the side length.

4. 3D Geometric Formulas

4.1. Cube

Volume:
$V = s^3$
where s is the side length.

Surface Area:
$A = 6s^2$
where s is the side length.

4.2. Rectangular Prism (Cuboid)

Volume:
$V = l \times w \times h$
where l is the length, w is the width, and h is the height.

Surface Area:
$A = 2lw + 2lh + 2wh$
where l is the length, w is the width, and h is the height.

4.3. Sphere

Volume:
$V = (4/3)\pi r^3$
where r is the radius.

Surface Area:
$A = 4\pi r^2$
where r is the radius.

4.4. Cylinder

Volume:
$V = \pi r^2 h$
where r is the radius and h is the height.

Surface Area:
$A = 2\pi r^2 + 2\pi rh$
where r is the radius and h is the height.

4.5. Cone

Volume:
$V = (\frac{1}{3})\pi r^2 h$
where r is the radius and h is the height.

Surface Area:
$A = \pi r^2 + \pi rl$
where r is the radius and l is the slant height.

4.6. Pyramid

Volume:
$V = \frac{1}{3} \times A\_base \times h$
where A_base is the area of the base and h is the height.

Surface Area:
$A = A\_base + \frac{1}{2} \times (\text{perimeter of base}) \times (\text{slant height})$

5. Pythagorean Theorem

For a right-angled triangle:
$a^2 + b^2 = c^2$
where a and b are the legs and c is the hypotenuse.

6. Trigonometry Formulas

6.1. Right-Angled Triangle

Sine:
$\sin \theta$ = opposite / hypotenuse

Cosine:
$\cos \theta$ = adjacent / hypotenuse

Tangent:
$\tan \theta$ = opposite / adjacent

6.2. Area of a Right-Angled Triangle:

A = ½ × base × height

7. Special Triangle Formulas

7.1. Equilateral Triangle

Area:
$A = (s^2\sqrt{3}) / 4$
where s is the side length.

Perimeter:
$P = 3s$
where s is the side length.

7.2. Isosceles Triangle

Area (using base b and height h):
$A = \frac{1}{2} \times b \times h$
where b is the base and h is the height.

8. Coordinate Geometry Formulas

8.1. Distance Between Two Points

Given points (x_1, y_1) and (x_2, y_2), the distance is:
$D = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

8.2. Midpoint of a Line Segment

The midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) is:
$((x_1 + x_2)/2 , (y_1 + y_2)/2)$

Algebraic Identities

Algebraic identities are equations involving polynomials that are true for all values of the variables. These are extremely useful for simplifying expressions and solving problems. Below is a compilation of the key algebraic identities:

1. Basic Algebraic Identities

1.1. Square of a Binomial

Identity:
$(a + b)^2 = a^2 + 2ab + b^2$

This identity helps in expanding the square of a binomial expression.

Example:
$(x + 3)^2 = x^2 + 6x + 9$

1.2. Difference of Squares

Identity:
$a^2 - b^2 = (a + b)(a - b)$

This identity is useful for factoring and simplifying expressions involving the difference of squares.

Example:
$x^2 - 9 = (x + 3)(x - 3)$

1.3. Square of a Binomial (Negative)

Past Paper MCQs

Numbers

1. Which of the following is a prime number?

A. 1
B. 4
C. 9
D. 17

Correct Answer: 17

Solution: A prime number is a natural number greater than 1 with no positive divisors other than 1 and itself.

- 1 is not prime (only one divisor).
- 4 is composite (divisible by 2).
- 9 is composite (divisible by 3).
- 17 is divisible only by 1 and 17, so it is prime.

2. The sum of the first 10 natural numbers is:

A. 45
B. 50
C. 55
D. 60

Correct Answer: 55

Solution: The first 10 natural numbers are 1, 2, 3, …, 10.
Sum = n(n+1)/2 = 10×11/2 = 110/2 = 55.

3. The smallest composite number is:

A. 1
B. 2
C. 3
D. 4

Correct Answer: 4

Solution: A composite number has more than two distinct positive divisors.

- 1 has only one divisor (not composite).
- 2 and 3 are prime (exactly two divisors).
- 4 has divisors 1, 2, and 4, making it the smallest composite number.

4. What is the value of $2^3 \times 3^2$?

A. 36
B. 54
C. 72

D. 108
Correct Answer: 72
Solution:
$2^3 = 8$
$3^2 = 9$
$8 \times 9 = 72$.

5. The HCF of 24 and 36 is:
A. 6
B. 12
C. 18
D. 24
Correct Answer: 12
Solution: List the factors:
Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24.
Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36.
Common factors: 1, 2, 3, 4, 6, 12.
Highest common factor = 12.

6. Which of the following is an irrational number?
A. 2/3
B. √16
C. √2
D. 0.5
Correct Answer: √2
Solution: An irrational number cannot be written as a simple fraction and has a non-repeating, non-terminating decimal expansion.
• 2/3 is a fraction → rational.
• √16 = 4 → integer → rational.
• √2 ≈ 1.414213… (non-repeating, non-terminating) → irrational.
• 0.5 = ½ → fraction → rational.

7. The number $0.\overline{3}$ (repeating) is equivalent to:
A. 1/3
B. 3/10
C. 1/4
D. 2/5
Correct Answer: 1/3
Solution: Let x = 0.3333…