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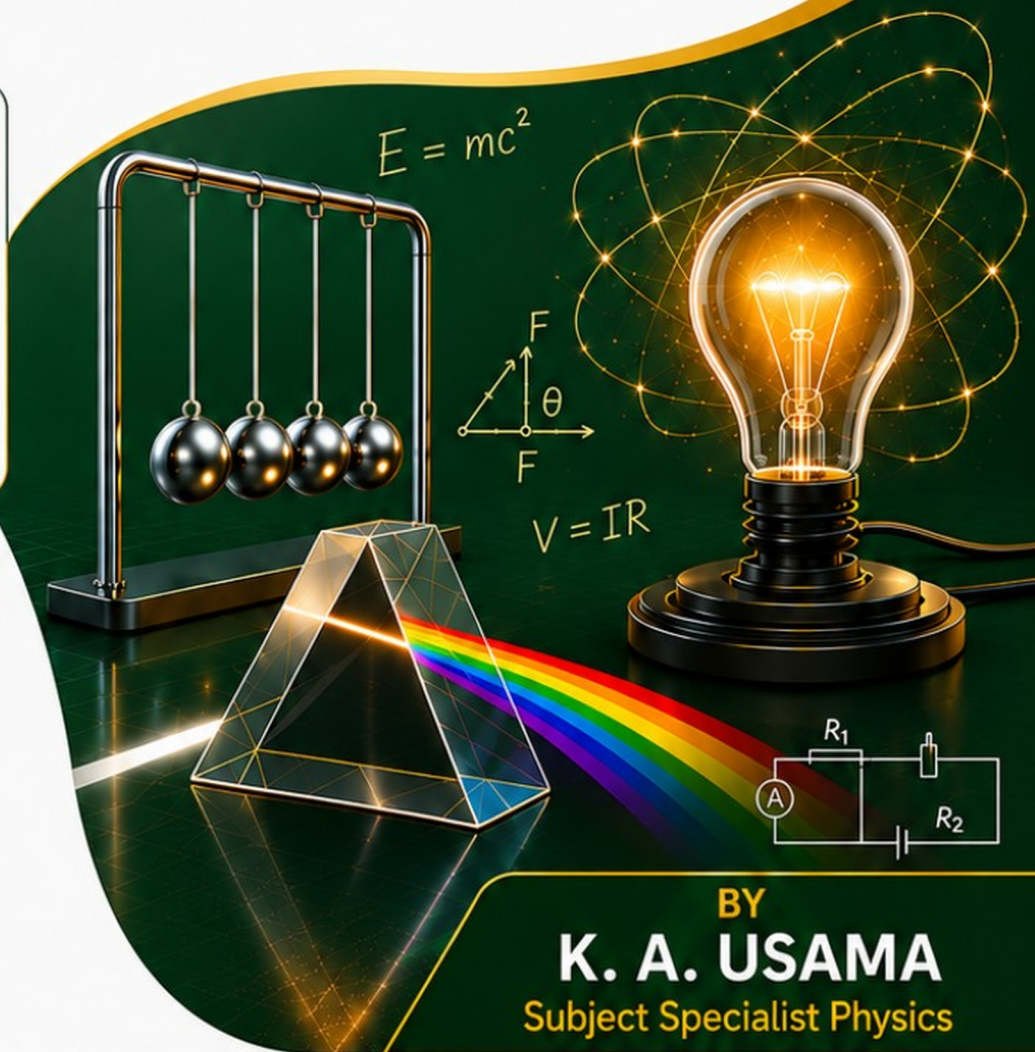
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Part I: Classical Mechanics



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Chapter 1: Classical Mechanics

Introduction

Classical mechanics is the branch of physics that deals with the study of motion and the forces that cause it. It forms the bedrock upon which much of classical physics is built. From the falling of an apple to the orbiting of planets around the Sun, from the spin of a fidget spinner to the trajectory of a cricket ball—all these phenomena are governed by the fundamental principles of mechanics.

Every mechanical engineer who reconstructs a traffic accident uses physics. Every dance trainer who coaches a ballerina on how to leap uses physics. Analyzing complicated motion of any sort requires simplification via an understanding of physics. The complicated motion of a system of objects—such as a car or a ballerina—can be simplified by determining a special point: the **center of mass** of that system. The beauty of mechanics lies in its logical structure: the same physical principles that describe a car moving along a straight road also describe a satellite orbiting the Earth. By understanding these principles deeply, you will develop the analytical tools needed to solve problems ranging from simple projectile trajectories to complex rotational dynamics.

The Center of Mass

Definition and Physical Meaning

The **center of mass (com)** of a system of particles is the point that moves as though:

1. All of the system's mass were concentrated there
2. All external forces were applied there

For Two Particles

Consider two particles of masses m_1 and m_2 separated by distance d . If we place the origin at m_1 :

$$x_{\text{com}} = \frac{m_2}{m_1 + m_2} d$$

In a generalized coordinate system:

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

For Many Particles (One Dimension)

$$x_{\text{com}} = \frac{1}{M} \sum_{i=1}^n m_i x_i \text{ where } M = \sum_{i=1}^n m_i$$

For Many Particles (Three Dimensions)

$$x_{\text{com}} = \frac{1}{M} \sum_{i=1}^n m_i x_i, y_{\text{com}} = \frac{1}{M} \sum_{i=1}^n m_i y_i, z_{\text{com}} = \frac{1}{M} \sum_{i=1}^n m_i z_i$$

Vector Form

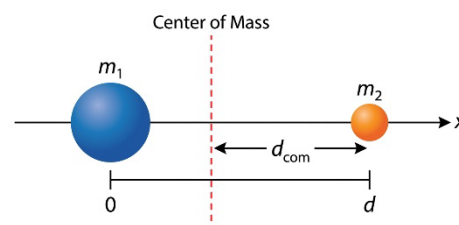
$$\vec{r}_{\text{com}} = \frac{1}{M} \sum_{i=1}^n m_i \vec{r}_i$$

Center of Mass for Continuous Bodies

For a continuous distribution of matter, the sums become integrals:

$$x_{\text{com}} = \frac{1}{M} \int x \, dm, y_{\text{com}} = \frac{1}{M} \int y \, dm, z_{\text{com}} = \frac{1}{M} \int z \, dm$$

For uniform density ($\rho = \text{constant}$):



$$v_f^2 = v_i^2 + 2aS$$

Physical Interpretation: These equations describe how an object's velocity changes under constant acceleration. The first equation gives velocity as a linear function of time; the second gives position as a quadratic function of time; the third relates velocity to displacement without explicit time dependence.

Free Fall Under Gravity

A special case of uniformly accelerated motion is **free fall**, where the only acceleration is due to gravity. Near the Earth's surface, the acceleration due to gravity is approximately constant:

$$g = 9.8 \text{ m/s}^2 \text{ (downward)}$$

For an object dropped from rest ($v_i = 0$), the equations become:

$$v_f = gt \text{ and } h = \frac{1}{2}gt^2$$

For an object thrown vertically upward, g is negative (opposing motion):

$$v_f = v_i - gt \text{ and } h = v_i t - \frac{1}{2}gt^2$$

Important Insight: The time to reach maximum height equals the time to fall back to the starting point, and the speed upon returning equals the initial speed (in the absence of air resistance).

Projectile Motion

Projectile motion is the two-dimensional motion of an object thrown into the air under the action of gravity only. The key insight is that **horizontal and vertical motions are independent**.

Analysis of Projectile Motion

Consider a projectile launched with initial velocity v_i at an angle θ above the horizontal. Resolve the initial velocity into components:

$$v_{ix} = v_i \cos \theta \text{ (horizontal, constant)}$$

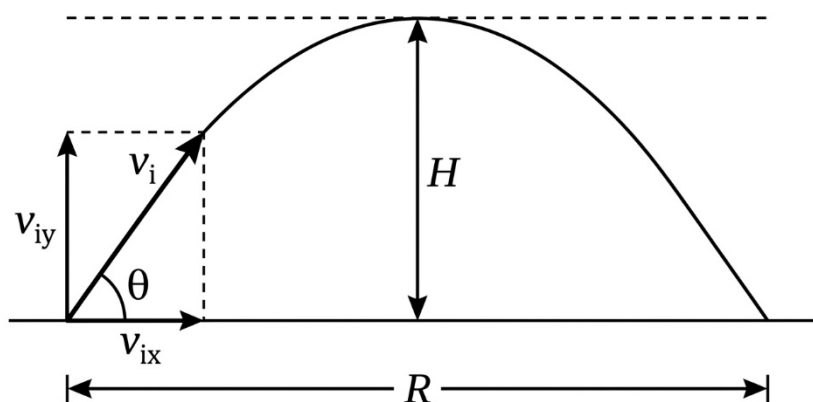
$$v_{iy} = v_i \sin \theta \text{ (vertical, affected by gravity)}$$

The acceleration components are:

$$a_x = 0 \text{ (no horizontal force)}$$

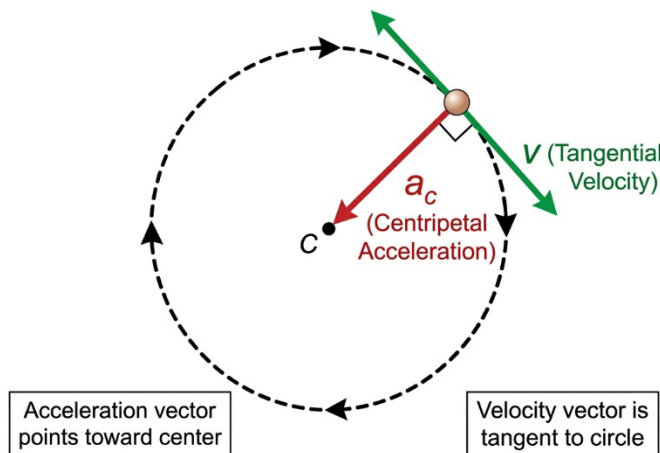
$$a_y = -g \text{ (gravity downward)}$$

Figure: Projectile Trajectory



Centripetal Acceleration

Figure: Direction of Centripetal Acceleration



The magnitude of centripetal acceleration is:

$$a_c = \frac{v^2}{r} = r\omega^2$$

In vector form, pointing inward (negative radial direction):

$$\vec{a}_c = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r}$$

Physical Interpretation: Centripetal acceleration arises solely from the **change in direction** of velocity, not its magnitude. Even at constant speed, a centripetal acceleration exists.

Centripetal Force

By Newton's second law, the net force causing centripetal acceleration is:

$$\vec{F}_c = m\vec{a}_c = -\frac{mv^2}{r}\hat{r} = -mr\omega^2\hat{r}$$

Critical Insight: Centripetal force is **not a new fundamental force**. It is the name given to any net force that causes circular motion. The actual physical force could be tension, gravity, friction, normal force, or any combination thereof.

Examples of Centripetal Force

1. Tension as Centripetal Force (Conical Pendulum):

A ball swung in a circle at the end of a string: the tension in the string provides the centripetal force.

$$T = \frac{mv^2}{r}$$

2. Friction as Centripetal Force (Car on Unbanked Curve):

When a car turns on a flat road, static friction provides the centripetal force.

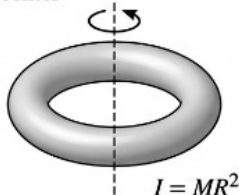
$$f_s = \frac{mv^2}{r}$$

3. Normal Force as Centripetal Force (Banked Curve):

For a frictionless banked curve at angle θ :

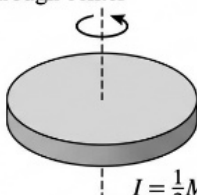
Thin rod	Through center, perpendicular to length	$I = \frac{1}{3}ML^2$
Thin rod	Through end, perpendicular to length	$I = \frac{1}{2}ML^2$

1. Hoop with axis through center



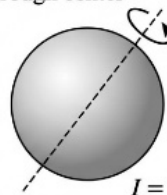
$$I = MR^2$$

2. Solid disk with axis through center



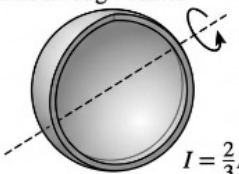
$$I = \frac{1}{2}MR^2$$

3. Solid sphere with axis through center



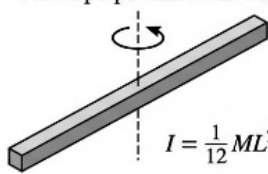
$$I = \frac{2}{5}MR^2$$

4. Thin spherical shell with axis through center



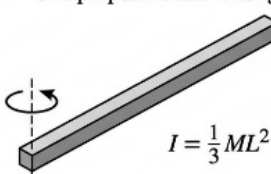
$$I = \frac{2}{3}MR^2$$

5. Thin rod with axis through center perpendicular to length



$$I = \frac{1}{12}ML^2$$

6. Thin rod with axis through end perpendicular to length



$$I = \frac{1}{3}ML^2$$

Parallel-Axis Theorem

$$I = I_{\text{com}} + Mh^2$$

where h is the perpendicular distance between the axis through the center of mass and the parallel axis.

Application: This theorem allows us to find rotational inertia about any axis if we know it about the center of mass.

Torque

Torque (τ) is the rotational analog of force—it causes angular acceleration.

Definition (Scalar Form for Fixed Axis)

$$\tau = rF\sin\phi = rF_{\perp} = r_{\perp}F$$

where:

- $r_{\perp}$ = moment arm (perpendicular distance from axis to line of action)
- $F_{\perp}$ = component of force perpendicular to $\vec{r}$

SI Unit: newton-meter (N·m)

Sign convention: Counterclockwise = positive, Clockwise = negative

Definition (Vector Form)

$$\vec{\tau} = \vec{r} \times \vec{F}$$

The direction is given by the **right-hand rule**.

Newton's Second Law for Rotation

$$\tau_{\text{net}} = I\alpha$$

This is the rotational analog of $F_{\text{net}} = ma$.

Derivation: For a point mass m at distance r from axis, a tangential force F produces torque $\tau = rF$.

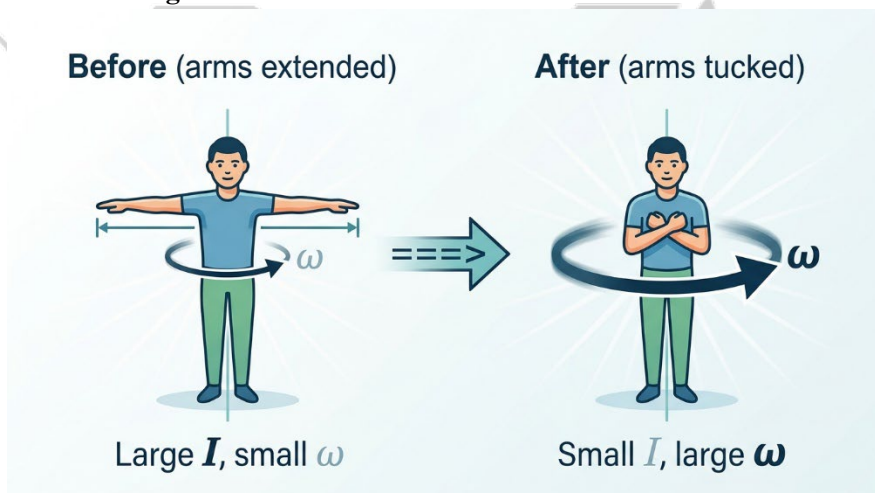
Since $F = ma_t = m(r\alpha)$, we get $\tau = r(mr\alpha) = mr^2\alpha = I\alpha$.

Component Conservation: If the torque component along an axis is zero, the angular momentum

Applications of Angular Momentum Conservation

Example	What Happens
Ice Skater	Pulls arms inward, decreasing I and increasing ω —the skater spins faster.
Springboard Diver	Tucks into a ball (small I) to spin rapidly, then extends limbs (large I) to slow rotation before entering water.
Long Jumper	Windmill arms transfers angular momentum to arms, keeping body upright.
Tour Jeté (Ballet)	Bringing in leg decreases I , increases spin.
Gyroscope	A spinning gyroscope maintains its orientation due to conservation of angular momentum. Used in navigation systems, airplanes, and spacecraft.
Flywheel	A heavy rotating wheel stores rotational energy. When torque is applied, it speeds up, storing kinetic energy. The stored energy can be released when needed, smoothing power delivery in engines.

Figure: Conservation of Angular Momentum in an Ice Skater



RIGID BODY DYNAMICS

A **rigid body** is an idealization in which the distance between any two points remains constant. The dynamics of a rigid body is a central topic in advanced mechanics.

Euler's Equations for Rigid Body Motion

For a rigid body rotating about its center of mass with angular velocity $\omega = (\omega_1, \omega_2, \omega_3)$ along the principal axes, the equations of motion are:

$$I_1 \dot{\omega}_1 + (I_3 - I_2) \omega_2 \omega_3 = \tau_1$$

$$I_2 \dot{\omega}_2 + (I_1 - I_3) \omega_3 \omega_1 = \tau_2$$

$$I_3 \dot{\omega}_3 + (I_2 - I_1) \omega_1 \omega_2 = \tau_3$$

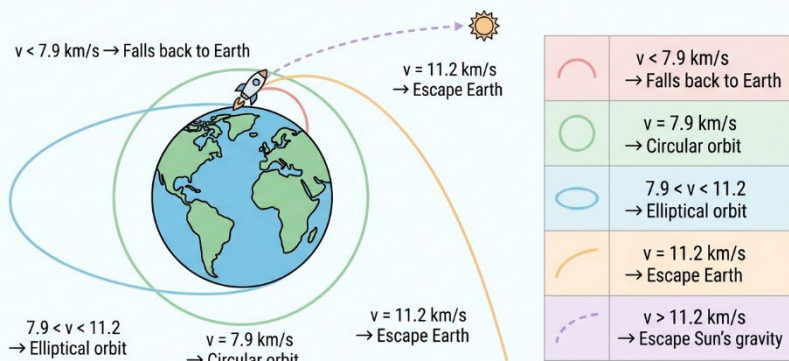
where I_1, I_2, I_3 are the principal moments of inertia.

Torque-Free Motion ($\tau_1 = \tau_2 = \tau_3 = 0$)

For a symmetric top ($I_1 = I_2 \neq I_3$):

$$I_3 \dot{\omega}_3 = 0 \Rightarrow \omega_3 = \text{constant}$$

ORBITAL AND ESCAPE VELOCITIES DIAGRAM



Satellites and Orbits

A satellite is any object that orbits another object due to gravity. Satellites can be natural (moons) or artificial (human-made).

Orbital Velocity

For a satellite in circular orbit at distance r from Earth's center, gravity provides the centripetal force:

$$G \frac{M_E m}{r^2} = \frac{mv_o^2}{r}$$

Solving for orbital velocity:

$$v_o = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{GM_E}{R_E + h}}$$

where h is the altitude above Earth's surface.

For a satellite just above Earth's surface ($r = R_E$):

$$v_o = \sqrt{\frac{GM_E}{R_E}} = \sqrt{gR_E} \approx 7.9 \text{ km/s}$$

This is called **circular orbital velocity** or first cosmic velocity.

Orbital Period

The time for one complete orbit:

$$T = \frac{2\pi r}{v_o} = 2\pi r \sqrt{\frac{r}{GM_E}} = 2\pi \sqrt{\frac{r^3}{GM_E}}$$

Squaring both sides gives **Kepler's Third Law**:

$$T^2 = \frac{4\pi^2}{GM_E} r^3$$

The square of the period is proportional to the cube of the orbital radius.

Orbital Energy

Circular Orbits:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0$$

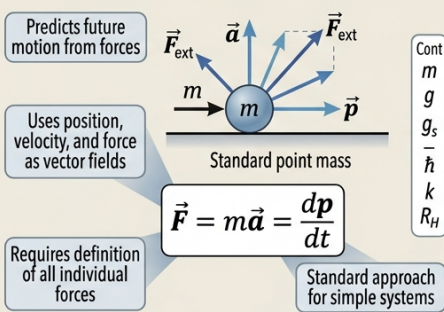
For a system with n degrees of freedom, there are n such equations—one for each generalized coordinate q_i .

Physical Interpretation

Term	Meaning	Units
$\frac{\partial L}{\partial q_i}$	Generalized force	N or N·m
$\frac{\partial L}{\partial \dot{q}_i}$	Generalized momentum	kg·m/s or kg·m ² /s
$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right)$	Rate of change of generalized momentum	N or N·m

NEWTON'S SECOND LAW vs. EULER-LAGRANGE EQUATION: Predicting Classical Trajectories

Panel 1: Newton's Second Law — Predicting via Forces (Vector approach)
Apparatus: Standard Cartesian and other Coordinate systems



Kinematics vs. Dynamics (Force Approach)

Kinematics	Dynamics
Position: $x(t)$	Forces: $\vec{F}(\vec{x}, \dot{\vec{x}})$
Position: $\vec{x}(\vec{x}, \dot{\vec{x}})$	Energy: k
Velocity: $\dot{x}$	Velocity: $\vec{v}$
Coordinate: z	Forms: t
Cartesian: n	Momentum: $\vec{p}$
	Frequency: dt

Derivation from Energy conservation

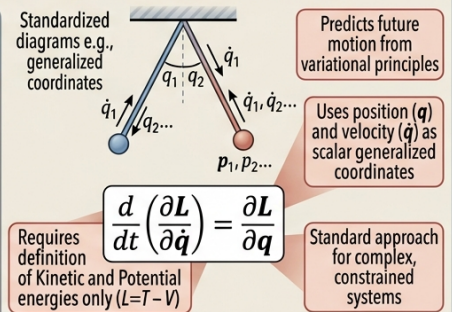
$$E = T + V = \frac{1}{2}m\dot{x}^2 + V(x)$$

$$\frac{dE}{dt} = m\ddot{x}\dot{x} + \dot{x}\left(\frac{\partial V}{\partial x}\right) = 0$$

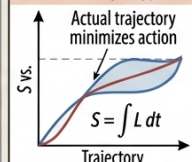
$$m\ddot{x} = -\frac{\partial V}{\partial x} = \vec{F}$$

COMPARISON AND ANALOGIES: Bridging vs. Methods

Feature: Newtonian Method	Lagrangian Method (Scalar approach)
Specific individual force vectors	Single Lagrangian function ($L=T-V$)
Coordinates: Cartesian (x, y, z) or other coordinates	Generalized coordinates (q) (scalar values)
System of (x, y, z) or other coordinates (vector bases)	Unified system of generalized scalar equations
Equation Type: individual vector	Hamilton Principle of least Action
System of individual differential equations	Unified system: scalar equations
Fundamental force laws (Newton's Laws)	Hamilton's Principle of least Action
Force $\vec{F}_z$	Generalized Force $Q_z = \frac{\partial L}{\partial q_z}$
Momentum $\vec{p}_z$	Generalized Momentum $p_z = \frac{\partial L}{\partial \dot{q}_z}$
Primary Analogy: $\frac{d}{dt}(\vec{p}_z) = \vec{F}_z$	$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_z} \right) = \frac{\partial L}{\partial q_z}$ (Equation of motion)



Action Integral Variation (Variation Principle Approach)



Comparison with Newtonian results (Example systems)

Harmonic Oscillator: $m\ddot{x} + kx = 0$
Simple Pendulum: $\ddot{\theta} + \frac{g}{L}\sin\theta = 0$
Simple Pendulum: $\ddot{\theta} + \frac{g}{L}\sin\theta = 0$
Simple Pendulum: $\ddot{\theta} + \frac{g}{L}\sin\theta = 0$

Key Insight: The Euler-Lagrange equation states that the rate of change of generalized momentum equals the generalized force—this is Newton's second law in generalized coordinates!

Example: Simple Pendulum

For the pendulum:

$$L = \frac{1}{2}mL^2\dot{\theta}^2 - mgL(1 - \cos\theta)$$

Compute:

- $\frac{\partial L}{\partial \theta} = -mgL\sin\theta$
- $\frac{\partial L}{\partial \dot{\theta}} = mL^2\dot{\theta}$
- $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) = mL^2\ddot{\theta}$

Substituting into Euler-Lagrange:

$$mL^2\ddot{\theta} + mgL\sin\theta = 0$$

$$mL^2\ddot{\theta} = -mgL\sin\theta$$

$$L\ddot{\theta} = -g\sin\theta$$

For small angles ($\sin\theta \approx \theta$):

$$\ddot{\theta} + \frac{g}{L}\theta = 0$$

This is exactly the equation of SHM with $\omega = \sqrt{g/L}$ —confirming our earlier result!

Generalized Momenta and Cyclic Coordinates

Generalized Momentum

The generalized momentum conjugate to q_i is defined as:

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

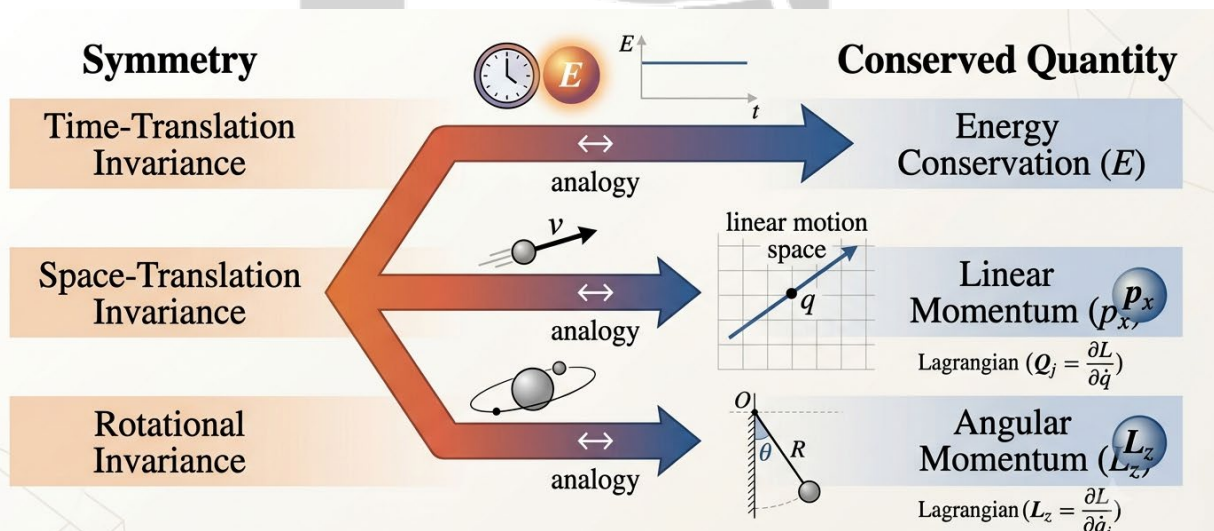
Cyclic (Ignorable) Coordinates

If the Lagrangian does not depend on a particular coordinate q_i , that coordinate is called **cyclic** or **ignorable**. The corresponding generalized momentum is conserved:

If $\frac{\partial L}{\partial q_i} = 0$, then $\frac{dp_i}{dt} = 0$, so $p_i = \text{constant}$.

Physical Significance: Conservation laws in Lagrangian mechanics emerge naturally from symmetries of the Lagrangian.

Symmetry of Lagrangian	Conserved Quantity
Independence of time (t)	Energy (E)
Independence of x (translation)	Linear momentum (p_x)
Independence of θ (rotation)	Angular momentum (L)



Example: Particle in a Central Potential

Consider a particle of mass m moving in a central potential $V(r)$. In spherical coordinates:

$$T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2)$$

- Hamiltonian: $H = p\dot{x} - L = \frac{p^2}{2m} + \frac{1}{2}kx^2$

Hamilton's equations:

$$\dot{x} = \frac{\partial H}{\partial p} = \frac{p}{m}$$

$$\dot{p} = -\frac{\partial H}{\partial x} = -kx$$

Differentiating the first and substituting the second:

$$\ddot{x} = \frac{\dot{p}}{m} = -\frac{k}{m}x$$

Thus:

$$\ddot{x} + \frac{k}{m}x = 0$$

This is the same SHM equation—but expressed as a pair of first-order equations rather than a single second-order equation.

Advantages of Hamiltonian Mechanics

Feature	Advantage
Phase space	Describes the system in $2n$ -dimensional phase space (q_i, p_i) —geometric interpretation
Conservation laws	If H does not depend on q_i , then p_i is conserved
Quantum mechanics	Hamiltonian is the basis of the Schrödinger equation
Statistical mechanics	Phase space volume is fundamental to partition functions
Canonical transformations	Allows transformation to simpler coordinates

Phase Space

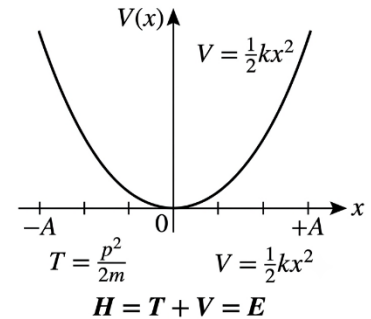
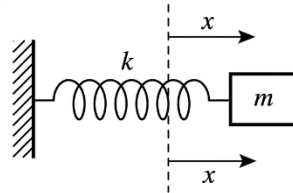
The state of a system is represented by a point in **phase space**—a $2n$ -dimensional space with coordinates $(q_1, \dots, q_n, p_1, \dots, p_n)$. As time evolves, this point traces a path (trajectory) in phase space.

Key properties:

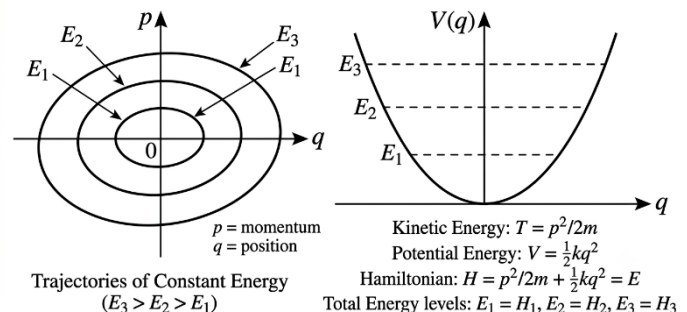
Property	Meaning
Liouville's theorem	Phase space volume is conserved (incompressible flow)
Trajectories	Never cross (deterministic evolution)
Equilibrium	Fixed points where $\dot{q}_i = \dot{p}_i = 0$

CENTRAL FORCE PROBLEM

A **central force** is a force that always points toward or away from a fixed point (the center of force) and whose magnitude depends only on the distance from that center. Mathematically:



Phase Space for Harmonic Oscillator



$$p = \frac{L^2}{mk} \text{ and } e = \sqrt{1 + \frac{2EL^2}{mk^2}}$$

Physical Interpretation:

Eccentricity e	Orbit Type	Energy
$e = 0$	Circle	$E < 0, E = -\frac{mk^2}{2L^2}$
$0 < e < 1$	Ellipse	$E < 0$
$e = 1$	Parabola	$E = 0$
$e > 1$	Hyperbola	$E > 0$

Kepler's Laws from the Central Force Problem

Using the above orbit equation, we can derive Kepler's three laws:

- Law of Orbits:** Planets move in ellipses with the Sun at one focus. This follows immediately from the orbit equation $r(\theta) = p/(1 + e \cos \theta)$.
- Law of Areas:** The radius vector sweeps out equal areas in equal times. From conservation of angular momentum:

Kepler's Laws: Visual Summary

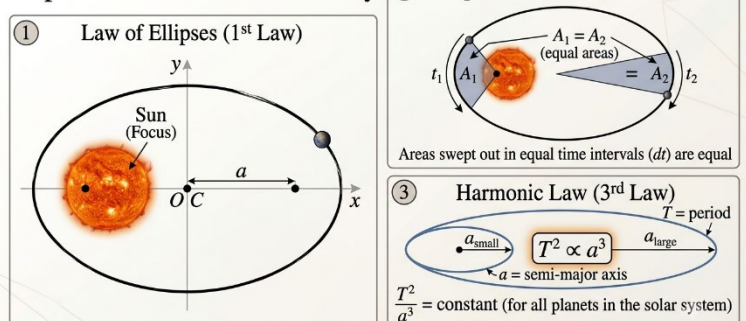


Fig. 1. Integrated visual summary of Kepler's three laws of planetary motion, illustrating the orbital geometries, equal areas principle, and the period-radius relationship.

$$\frac{dA}{dt} = \frac{L}{2m} = \text{constant}$$

Law of Periods: $T^2 \propto a^3$. For an elliptical orbit with semi-major axis a :

$$T^2 = \frac{4\pi^2}{GM} a^3$$

SUMMARY

Concept	Definition	Key Equation
Center of mass	Point that moves as if all mass concentrated there	$\vec{p} = m\vec{v}$
Linear momentum	Product of mass and velocity	$\vec{p}_i = \vec{p}_f$
Conservation of momentum	Total momentum constant for isolated system	$\vec{J} = \int \vec{F} dt$
Impulse	Integral of force over time	$\theta = s/r$
Angular displacement	Angle swept by position vector	$\omega = d\theta/dt$
Angular velocity	Rate of change of angular displacement	$\alpha = d\omega/dt$
Angular acceleration	Rate of change of angular velocity	$a_c = v^2/r = r\omega^2$
Centripetal acceleration	Acceleration toward center	$F_c = mv^2/r = mr\omega^2$
Centripetal force	Net force causing circular motion	$I = \sum m_i r_i^2 = \int r^2 dm$
Moment of inertia	Rotational analog of mass	$\tau = I\alpha$

Torque	Rotational analog of force	$L = I\omega$
Angular momentum	Rotational analog of momentum	$I_i\omega_i = I_f\omega_f$
Conservation of angular momentum	L constant for no external torque	$F = Gm_1m_2/r^2$
Gravitational force	Attraction between masses	$g = GM/r^2$
Gravitational field strength	Force per unit mass	$v_o = \sqrt{GM/r}$
Orbital velocity	Speed for circular orbit	$v_{esc} = \sqrt{2GM/R}$
Escape velocity	Speed to escape gravity	$V = -GM/r$
Gravitational potential	Potential energy per unit mass	$T^2 = (4\pi^2/GM)a^3$
Kepler's 3rd law	Relation between period and orbit size	$E = (F/A)/(\Delta L/L)$
Lagrangian	$L = T - V$	Energy difference; fundamental function of mechanics
Euler-Lagrange equation	$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$	Generalization of Newton's second law
Hamiltonian	$H = \sum p_i \dot{q}_i - L$	Total energy (usually)
Hamilton's equations	$\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$	First-order equations of motion
Central force orbit	$r(\theta) = \frac{p}{1 + e \cos \theta}$	Conic sections: circle, ellipse, parabola, hyperbola
Euler's equations	$I_i\dot{\omega}_i + (I_j - I_k)\omega_j\omega_k = \tau_i$	Rotational dynamics of rigid bodies
Gyroscope precession	$\Omega_p = \frac{Mgr}{I\omega}$	Conservation of angular momentum with torque

COMPREHENSIVE FORMULA SHEET

Center of Mass and Linear Momentum

Quantity	Formula
Center of mass (discrete)	$\vec{r}_{com} = \frac{1}{M} \int \vec{r} dm$
Center of mass (continuous)	$\vec{p} = m\vec{v}$
Linear momentum	$\vec{F}_{net} = d\vec{p}/dt$
Newton's second law (momentum form)	$\vec{J} = \int \vec{F} dt$
Impulse	$\Delta\vec{p} = \vec{J}$
Impulse-momentum theorem	$\vec{P}_i = \vec{P}_f$ (isolated system)
Conservation of momentum	$\vec{r}_{com} = \frac{1}{M} \int \vec{r} dm$

Collisions

Type	Momentum Conserved?	KE Conserved?	Key Equations
Elastic	Yes	Yes	$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$
Inelastic	Yes	No	$V = \frac{m_1v_{1i} + m_2v_{2i}}{m_1 + m_2}$



Completely inelastic	Yes	No (max loss)	$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$
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Kinematics (Constant Acceleration)

Linear	Rotational
$S = v_i t + \frac{1}{2} a t^2$	$\theta = \omega_i t + \frac{1}{2} \alpha t^2$
$v_f^2 = v_i^2 + 2aS$	$\omega_f^2 = \omega_i^2 + 2\alpha\theta$
$S = v_i t + \frac{1}{2} a t^2$	$\theta = \omega_i t + \frac{1}{2} \alpha t^2$

Projectile Motion

Quantity	Formula
Time of flight	$H = \frac{v_i^2 \sin^2 \theta}{2g}$
Maximum height	$R = \frac{v_i^2 \sin 2\theta}{g}$
Range	$R_{\max} = \frac{v_i^2}{g} \text{ (at } \theta = 45^\circ \text{)}$
Maximum range	$H = \frac{v_i^2 \sin^2 \theta}{2g}$

Rotation

Quantity	Formula
Rotational kinetic energy	$\tau_{\text{net}} = I\alpha$
Newton's second law for rotation	$\vec{l} = \vec{r} \times \vec{p}$
Angular momentum (particle)	$L = I\omega$
Angular momentum (rigid body)	$\vec{\tau}_{\text{net}} = d\vec{L}/dt$
Newton's second law (angular)	$\vec{L}_i = \vec{L}_f \text{ (if } \vec{\tau}_{\text{net}} = 0 \text{)}$
Conservation of angular momentum	$W = \int \tau d\theta$
Work done by torque	$P = \tau\omega$
Power (rotational)	$\Omega = Mgr/(I\omega)$
Precession rate	$\tau_{\text{net}} = I\alpha$

Rotational Inertia (Common Shapes)

Object	Axis	SIS
Hoop (thin)	Through center, $\perp$ plane	$\frac{1}{2} MR^2$
Solid cylinder/disk	Through center, $\perp$ plane	$\frac{2}{5} MR^2$
Solid sphere	Through center	$\frac{2}{3} MR^2$
Thin spherical shell	Through center	$\frac{1}{2} ML^2$

1. Classical Mechanics

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4. For two particles, the center of mass from mass m_1 is $x_{\text{com}} = \frac{m_2}{m_1 + m_2} d$.
5. The generalized coordinate for the center of mass of two particles is $x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$.
6. In three dimensions, the center of mass vector is $\vec{r}_{\text{com}} = \frac{1}{M} \sum m_i \vec{r}_i$.
7. For a continuous body, the center of mass is found using integrals of the form $x_{\text{com}} = \frac{1}{M} \int x \, dm$.
8. For bodies with uniform density, volume integrals can be used to find the center of mass.
9. A point of symmetry in a body places the center of mass at that exact point.
10. If a body has a line of symmetry, the center of mass lies on that line.
11. A key insight is that the center of mass need not lie within the physical object itself (e.g., a doughnut).
12. The motion of the center of mass of a system is governed solely by the net external force.
13. Newton's Second Law for a system is $\vec{F}_{\text{net}} = M \vec{a}_{\text{com}}$.
14. In an exploding firework, the fragments' center of mass follows the same parabolic path as the unexploded rocket.
15. In a ballet grand jeté, the dancer's center of mass follows a parabolic path.
16. The center of gravity is the point where the gravitational force effectively acts on a body.
17. If the gravitational field $\vec{g}$ is uniform, the center of gravity coincides with the center of mass.
18. Linear momentum is defined as the product of mass and velocity: $\vec{p} = m \vec{v}$.
19. The SI unit for linear momentum is kilogram-meter per second ($\text{kg} \cdot \text{m/s}$).
20. Momentum is a vector quantity, sharing the direction of the velocity vector.
21. Newton's Second Law in its most general form is $\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$.
22. The net force on an object equals the time rate of change of its momentum.
23. The total linear momentum of a system is the vector sum of the individual momenta.
24. The total momentum of a system can be expressed as $\vec{P} = M \vec{v}_{\text{com}}$.
25. Impulse is the integral of force over time: $\vec{J} = \int_{t_i}^{t_f} \vec{F}(t) dt$.
26. The impulse-linear momentum theorem states $\Delta \vec{p} = \vec{J}$.
27. For a constant force, impulse can be calculated as $J = F_{\text{avg}} \Delta t$.
28. When a projectile rebounds with the same speed, its change in velocity is $\Delta v = -2v$.
29. In a karate chop, the large momentum is reduced to zero in a short time, producing a large force.
30. For a closed and isolated system, the total linear momentum is conserved.
31. The law of conservation of momentum is a consequence of Newton's third law.
32. If the net external force component along an axis is zero, the momentum component along that axis is conserved.
33. Rockets work by expelling mass backward, gaining an equal and opposite forward momentum.
34. Rockets do not require an external medium like air to push against.
35. In all types of collisions, linear momentum is always conserved.
36. Kinetic energy is only conserved in elastic collisions.
37. In elastic collisions, the relative speed of approach equals the relative speed of separation.
38. In a perfectly inelastic collision, the objects stick together and move with a common velocity.

147. Hamilton's equations yield $2n$ first-order differential equations for an n -degree-of-freedom system.
148. The Hamiltonian is the fundamental basis of the Schrödinger equation in quantum mechanics.
149. Liouville's theorem states that the volume of phase space is conserved.
150. A central force is one that points toward or away from a fixed point.
151. In a central force, angular momentum is conserved, and the motion is planar.
152. The effective potential for a central force includes a centrifugal barrier term.
153. For an inverse-square force, the orbit equation is $r(\theta) = \frac{p}{1+e\cos\theta}$.
154. The eccentricity $e = 0$ corresponds to a circular orbit.
155. The eccentricity $0 < e < 1$ corresponds to an elliptical orbit.
156. The eccentricity $e = 1$ corresponds to a parabolic orbit.
157. The eccentricity $e > 1$ corresponds to a hyperbolic orbit.

Practice MCQs

1. The center of mass of a system of particles moves as though:

- A) All internal forces were concentrated there.
- B) All of the system's mass and all external forces were concentrated there.
- C) The system's total kinetic energy were concentrated there.
- D) Half the mass were at two different points.

Correct Answer: B

2. For two particles of masses m_1 and m_2 , the center of mass from m_1 is located at:

- A) $\frac{m_1}{m_1+m_2}d$
- B) $\frac{m_2}{m_1+m_2}d$
- C) $\frac{m_1m_2}{m_1+m_2}d$
- D) $\frac{m_1+m_2}{m_1m_2}d$

Correct Answer: B

3. The vector form for the center of mass of a system is:

- A) $\vec{r}_{com} = \sum m_i \vec{r}_i$
- B) $\vec{r}_{com} = \frac{1}{M} \sum m_i \vec{r}_i$
- C) $\vec{r}_{com} = \frac{\sum \vec{r}_i}{M}$
- D) $\vec{r}_{com} = \frac{1}{M} \sum \frac{\vec{r}_i}{m_i}$

Correct Answer: B

4. Which of the following is true for the center of mass of a doughnut?

- A) It lies at the geometric center of the

doughnut.

- B) It lies exactly at the center of the hole.
- C) It lies on the surface of the doughnut.
- D) It lies outside the physical material of the doughnut.

Correct Answer: D

5. Newton's second law for a system of particles is $\vec{F}_{net} = M\vec{a}_{com}$. Here, $\vec{F}_{net}$ is the:

- A) Sum of all internal and external forces.
- B) Sum of all internal forces only.
- C) Sum of all external forces only.
- D) Net force on the heaviest particle.

Correct Answer: C

6. The center of gravity coincides with the center of mass when:

- A) The body is in free fall.
- B) The body is in a non-inertial frame.
- C) The gravitational field is uniform over the body.
- D) The body is symmetric about an axis.

Correct Answer: C

7. The SI unit of impulse is equivalent to:

- A) $\text{kg} \cdot \text{m/s}$
- B) $\text{J} \cdot \text{s}$
- C) N/s
- D) Both A and B

Correct Answer: D

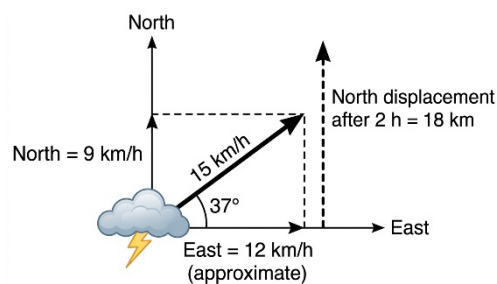
8. A ball of mass 0.5 kg is moving with a velocity of 10 m/s. It hits a wall and rebounds with the same speed. The change in momentum is:

Chapter 2: Vector Analysis

Introduction

Physics deals with a great many quantities that have both size and direction, and it needs a special mathematical language — the language of vectors — to describe those quantities. This language is also used in engineering, the other sciences, and even in common speech. If you have ever given directions such as "Go five blocks down this street and then hang a left," you have used the language of vectors. Navigation of any sort is based on vectors, but physics and engineering also need vectors in special ways to explain phenomena involving rotation and magnetic forces.

Real-World Opening Example: A thunderstorm is moving at 15 km/h in a direction 37° north of east. How far north does the thunderstorm move in 2.0 hours? To answer this, we need vectors. (Answer: 18 km north)



Signboard Example: A signboard provides information about distances and directions of other places relative to the location of the signboard. Distance is a scalar quantity. Knowing the distance alone is not enough to get to the desired location. We must also know the direction from the signboard to the destination. The direction along with the distance is a vector quantity called the displacement.

The Nature of Physical Theories

Physics is an experimental science. Physicists observe the phenomena of nature and try to find patterns that relate these phenomena. These patterns are called physical theories or, when they are very well established and widely used, physical laws or principles.

A theory is not just a random thought or an unproven concept. Rather, a theory is an explanation of natural phenomena based on observation and accepted fundamental principles. No theory is ever regarded as the ultimate truth — it is always possible that new observations will require that a theory be revised or discarded.

Idealized Models

In physics, a model is a simplified version of a physical system that would be too complicated to analyze in full detail. For example, to analyze the motion of a thrown baseball, we ignore the size, shape, and rotation of the ball by representing it as a point object or particle. We ignore air resistance and make the weight constant. A useful model simplifies a problem enough to make it manageable, yet keeps its essential features.

SCALARS AND VECTORS

Definitions

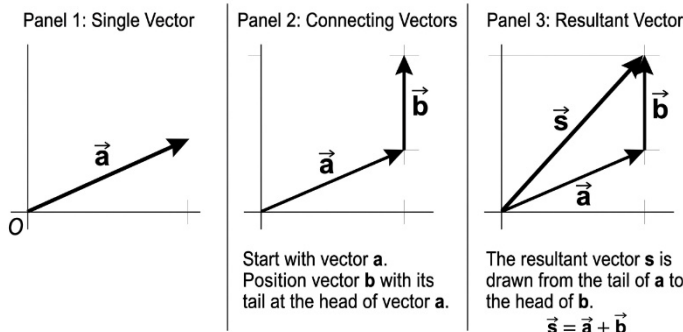
Scalars are physical quantities that have magnitude only. They are specified by a number with a unit (e.g., 10°C , 5 kg, 30 s) and obey the rules of arithmetic and ordinary algebra.

Examples of Scalars: Temperature, pressure, energy, mass, time, speed, work, power, electric potential, density

Vectors are physical quantities that have both magnitude and direction. They obey the rules of vector algebra, which differ from ordinary arithmetic.

3. The vector sum $\vec{s}$ is the vector that extends from the tail of $\vec{a}$ to the head of $\vec{b}$

HEAD-TO-TAIL VECTOR ADDITION



Properties of Vector Addition

Commutative Law: $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (order does not matter)

Associative Law: $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ (grouping does not matter)

Magnitude of the Vector Sum

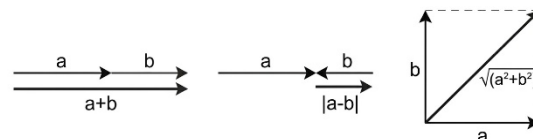
A common misconception: $|\vec{a} + \vec{b}| = a + b$ is generally FALSE.

The magnitude of the sum depends on the angle ϕ between the vectors:

$$|\vec{a} + \vec{b}| = \sqrt{a^2 + b^2 + 2ab\cos\phi}$$

Special Cases:

Condition	Angle	Magnitude of Sum
Parallel	$\phi = 0^\circ$	$a + b$ (maximum)
Antiparallel	$\phi = 180^\circ$	$ a - b $ (minimum)
Perpendicular	$\phi = 90^\circ$	$\sqrt{a^2 + b^2}$



Example: For vectors of magnitudes 3 m and 4 m:

- Maximum sum magnitude = 7 m (when parallel)
- Minimum sum magnitude = 1 m (when antiparallel)
- Sum magnitude = 5 m (when perpendicular)

Vector Subtraction

Vector subtraction is defined using the negative vector:

$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

Procedure:

1. Reverse the direction of $\vec{b}$ to obtain $-\vec{b}$
2. Add $\vec{a}$ and $-\vec{b}$ using the head-to-tail method

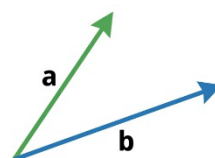
Alternative Method (Parallelogram): Place both vectors tail-to-tail; the difference vector points from the head of the subtracted vector to the head of the other vector.

Sample Problem: Orienteering

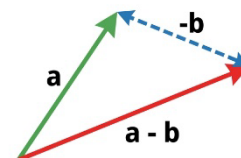
In an orienteering class, you have the goal of moving as far (straight-line distance) from base camp as possible by making three straight-line moves. You may use the following displacements in any order:

Vector Subtraction: $\vec{a} - \vec{b}$

Panel 1: Vectors ' $\vec{a}$ ' and ' $\vec{b}$ ' from Common Tail



Panel 2: Addition of $-\vec{b}$ ($\vec{a} + (-\vec{b})$)



- $\vec{a}$: 2.0 km due east
- $\vec{b}$: 2.0 km at 30° north of east
- $\vec{c}$: 1.0 km due west

Alternatively, you may substitute either $-\vec{b}$ for $\vec{b}$ or $-\vec{c}$ for $\vec{c}$. What is the greatest distance you can be from base camp?

Solution: The distance is greatest for the arrangement $\vec{a}$, $\vec{b}$, and $-\vec{c}$. The magnitude is $d = 4.8$ km.

COMPONENTS OF VECTORS

Concept of Vector Resolution

Resolution is the process of splitting a single vector into two or more vectors (called components) that together have the same effect as the original vector. These components are usually taken along mutually perpendicular directions (rectangular components) for mathematical convenience.

Why Resolve Vectors?

- To simplify addition of multiple vectors
- To analyze forces acting on an object in different directions
- To study the effect of a vector in a specific direction

Rectangular Components

The (scalar) components a_x and a_y of any two-dimensional vector $\vec{a}$ along the coordinate axes are found by dropping perpendicular lines from the ends of $\vec{a}$ onto the coordinate axes.

Mathematical Expressions:

If a vector $\vec{a}$ has magnitude a and makes an angle θ measured from the positive x-axis toward the positive y-axis:

$$a_x = a \cos \theta$$

$$a_y = a \sin \theta$$

The algebraic sign of a component indicates its direction along the associated axis.

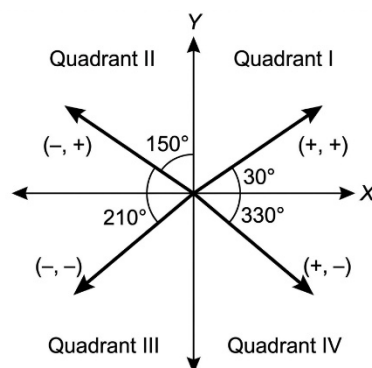
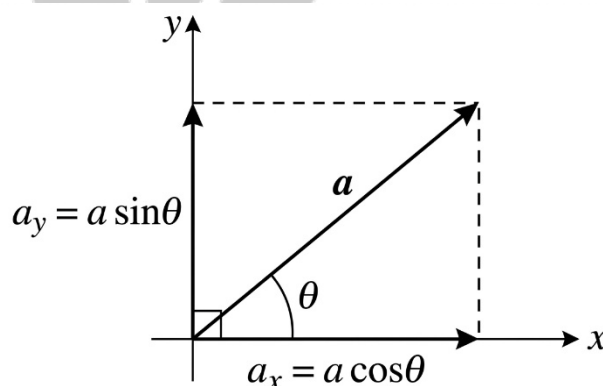
Sign Conventions

Quadrant	Angle Range	$a_x = a \cos \theta$	$a_y = a \sin \theta$
I	0° to 90°	Positive	Positive
II	90° to 180°	Negative	Positive
III	180° to 270°	Negative	Negative
IV	270° to 360°	Positive	Negative

Reconstructing a Vector from Its Components

Given components a_x and a_y , we can find the magnitude and orientation:

$$a = \sqrt{a_x^2 + a_y^2}$$



Sample Problem: Scalar Product

Find the scalar product of $\vec{A} = 4.00$ at 53.0° and $\vec{B} = 5.00$ at 130.0° .

Solution:

- Method 1: $\phi = 130.0^\circ - 53.0^\circ = 77.0^\circ$, $\vec{A} \cdot \vec{B} = (4.00)(5.00)\cos 77.0^\circ = 4.50$
- Method 2 (components):
 - $A_x = 4.00\cos 53.0^\circ = 2.407$, $A_y = 4.00\sin 53.0^\circ = 3.195$
 - $B_x = 5.00\cos 130.0^\circ = -3.214$, $B_y = 5.00\sin 130.0^\circ = 3.830$
 - $\vec{A} \cdot \vec{B} = (2.407)(-3.214) + (3.195)(3.830) = 4.50$

Sample Problem: Power Delivered

Find the power delivered by an engine for attaining velocity $(3\hat{i} + 4\hat{j})$ m/s while it exerts a force $(8\hat{i} - 2\hat{j})$ N.

Solution:

- $P = \vec{F} \cdot \vec{v} = (3\hat{i} + 4\hat{j}) \cdot (8\hat{i} - 2\hat{j}) = 3 \times 8 + 4 \times (-2) = 24 - 8 = 16 \text{ W}$

The Vector (Cross) Product

Definition: The vector product of $\vec{a}$ and $\vec{b}$ produces a third vector $\vec{c}$ whose magnitude is:

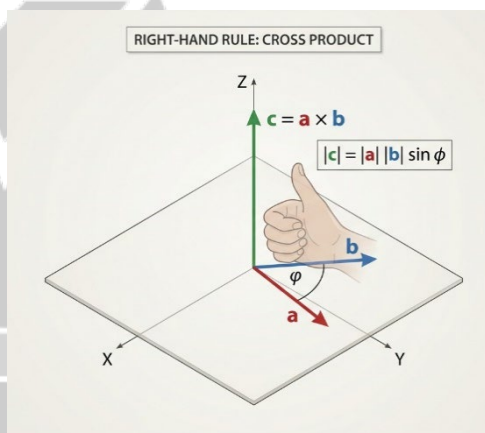
$$c = |\vec{a} \times \vec{b}| = ab \sin \phi$$

where ϕ is the smaller angle between $\vec{a}$ and $\vec{b}$ ($0^\circ \leq \phi \leq 180^\circ$).

Direction of Cross Product:

- $\vec{c}$ is perpendicular to the plane containing $\vec{a}$ and $\vec{b}$
- Direction given by the **right-hand rule**

Properties:



Property	Mathematical Statement
Anticommutative	$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$
Distributive	$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
Parallel vectors	$\vec{a} \times \vec{b} = 0$ ($\sin 0^\circ = 0$)
Perpendicular vectors	(maximum, $\sin 90^\circ = 1$)
Cross product with itself	$\vec{a} \times \vec{a} = 0$

Unit Vector Cross Products (Right-Handed System):

$$\begin{aligned} \hat{i} \times \hat{i} &= 0, \hat{j} \times \hat{j} = 0, \hat{k} \times \hat{k} = 0 \\ \hat{i} \times \hat{j} &= \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j} \\ \hat{j} \times \hat{i} &= -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}, \hat{i} \times \hat{k} = -\hat{j} \end{aligned}$$

Component Form (Determinant Method):

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Expanding:

$$\vec{a} \times \vec{b} = (a_y b_z - a_z b_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$$

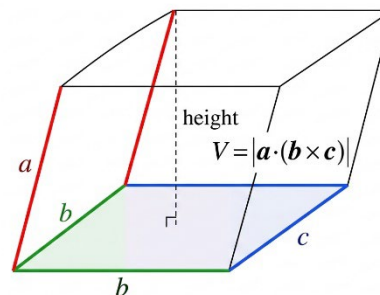
$$[\vec{a}, \vec{b}, \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$$

Geometric Interpretation: The absolute value gives the volume of the parallelepiped formed by the three vectors as adjacent edges.

$$V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$$

Component Form (Determinant):

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$



Properties:

1. **Cyclic Permutation:**

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$$

Anticyclic Permutation (changes sign):

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = -\vec{a} \cdot (\vec{c} \times \vec{b})$$

Zero if vectors are coplanar

2. **Linear in each argument**

Vector Triple Product

Definition: The vector triple product is $\vec{a} \times (\vec{b} \times \vec{c})$.

BAC-CAB Rule:

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$$

Similarly,

$$(\vec{a} \times \vec{b}) \times \vec{c} = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{a}(\vec{b} \cdot \vec{c})$$

Note: The vector triple product is not associative:

$$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$$

Lagrange's Identity

$$|\vec{a} \times \vec{b}|^2 = a^2 b^2 - (\vec{a} \cdot \vec{b})^2$$

Binet-Cauchy Identity

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$$

Reciprocal Vectors

For non-coplanar vectors $\vec{a}, \vec{b}, \vec{c}$:

$$\vec{a}' = \frac{\vec{b} \times \vec{c}}{[\vec{a}, \vec{b}, \vec{c}]}, \vec{b}' = \frac{\vec{c} \times \vec{a}}{[\vec{a}, \vec{b}, \vec{c}]}, \vec{c}' = \frac{\vec{a} \times \vec{b}}{[\vec{a}, \vec{b}, \vec{c}]}$$

Properties: $\vec{a} \cdot \vec{a}' = 1, \vec{a} \cdot \vec{b}' = 0, \vec{a} \cdot \vec{c}' = 0$ (cyclic)

VECTOR DIFFERENTIATION AND INTEGRATION

Differentiation of Vectors

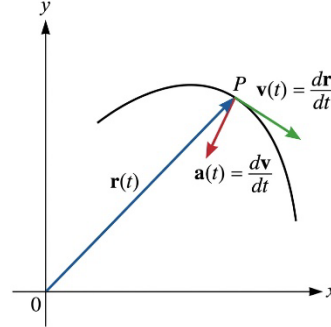
If $\vec{A}(t) = A_x(t)\hat{i} + A_y(t)\hat{j} + A_z(t)\hat{k}$:

$$\frac{d\vec{A}}{dt} = \frac{dA_x}{dt}\hat{i} + \frac{dA_y}{dt}\hat{j} + \frac{dA_z}{dt}\hat{k}$$

Physical Interpretation: For position $\vec{r}(t)$, $d\vec{r}/dt = \vec{v}$ (velocity), $d^2\vec{r}/dt^2 = \vec{a}$ (acceleration).

Differentiation Rules:

1. $\frac{d}{dt}(\vec{A} + \vec{B}) = \frac{d\vec{A}}{dt} + \frac{d\vec{B}}{dt}$
2. $\frac{d}{dt}(f\vec{A}) = \frac{df}{dt}\vec{A} + f\frac{d\vec{A}}{dt}$
3. $\frac{d}{dt}(\vec{A} \cdot \vec{B}) = \frac{d\vec{A}}{dt} \cdot \vec{B} + \vec{A} \cdot \frac{d\vec{B}}{dt}$
4. $\frac{d}{dt}(\vec{A} \times \vec{B}) = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$
5. If $|\vec{A}|$ is constant, $\vec{A} \cdot \frac{d\vec{A}}{dt} = 0$ (derivative is perpendicular to the vector)



Integration of Vectors

Indefinite integral:

$$\int \vec{A}(t) dt = \left(\int A_x(t) dt \right) \hat{i} + \left(\int A_y(t) dt \right) \hat{j} + \left(\int A_z(t) dt \right) \hat{k} + \vec{C}$$

Definite integral:

$$\int_{t_1}^{t_2} \vec{A}(t) dt = \left(\int_{t_1}^{t_2} A_x(t) dt \right) \hat{i} + \left(\int_{t_1}^{t_2} A_y(t) dt \right) \hat{j} + \left(\int_{t_1}^{t_2} A_z(t) dt \right) \hat{k}$$

Sample Problem: Vector Differentiation

A particle moves with position $\vec{r}(t) = t^2\hat{i} + 3t\hat{j} + t^3\hat{k}$. Find velocity and acceleration at $t = 2$ s.

Solution:

- $\vec{v}(t) = 2t\hat{i} + 3\hat{j} + 3t^2\hat{k}$
- $\vec{a}(t) = 2\hat{i} + 0\hat{j} + 6t\hat{k}$
- At $t = 2$ s: $\vec{v} = 4\hat{i} + 3\hat{j} + 12\hat{k}$ m/s, $\vec{a} = 2\hat{i} + 12\hat{k}$ m/s²

VECTOR DIFFERENTIAL OPERATORS

These operators are essential in advanced physics (electromagnetism, fluid dynamics):

Gradient (scalar → vector):

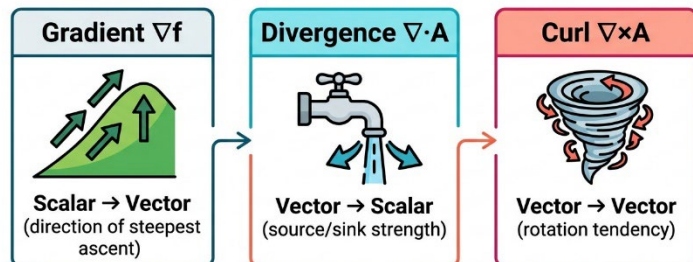
$$\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

Divergence (vector → scalar):

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

sCurl (vector → vector):

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \hat{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \hat{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \hat{k}$$



SUMMARY OF VECTOR OPERATIONS

Operation	Method	Result
Addition	$\vec{a} + \vec{b} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j} + (a_z + b_z)\hat{k}$	Vector
Subtraction	$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$	Vector
Scalar multiplication	$s\vec{a} = (sa_x)\hat{i} + (sa_y)\hat{j} + (sa_z)\hat{k}$	Vector
Resolution	$a_x = a \cos \theta, a_y = a \sin \theta$	Scalars
Dot product	$\vec{a} \cdot \vec{b} = ab \cos \phi = a_x b_x + a_y b_y + a_z b_z$	Scalar
Cross product	$\vec{a} \times \vec{b} = ab \sin \phi \hat{n}$ (right-hand rule)	Vector
Scalar triple product	$\vec{a} \cdot (\vec{b} \times \vec{c}) = \det \begin{pmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{pmatrix}$	Scalar
Vector triple product	$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$	Vector
Derivative	$\frac{d\vec{a}}{dt} = \frac{da_x}{dt}\hat{i} + \frac{da_y}{dt}\hat{j} + \frac{da_z}{dt}\hat{k}$	Vector

Quick Reference: Dot vs. Cross Product

Feature	Dot Product	Cross Product
Result	Scalar	Vector
Formula	$ab \cos \phi$	$ab \sin \phi \hat{n}$
Parallel (0°)	ab	0
Perpendicular (90°)	0	ab
Commutative?	Yes	No

Advanced Formulas

Lagrange: $|\vec{a} \times \vec{b}|^2 = a^2 b^2 - (\vec{a} \cdot \vec{b})^2$

BAC-CAB: $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$

Lami: $\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$

PREPARATIONS
LET'S MAKE IT HAPPEN

2. Vector Analysis

Vector Analysis: One-Liners

Nature of Physical Theories and Models

1. Physics is an experimental science where physicists observe phenomena and try to find patterns called physical theories.
2. A theory is an explanation of natural phenomena based on observation and accepted fundamental principles.
3. No theory is ever regarded as the ultimate truth; new observations may require a theory to be revised or discarded.
4. A model is a simplified version of a physical system that would be too complicated to analyze in full detail.
5. A useful model simplifies a problem enough to make it manageable yet keeps its essential features.

Scalars and Vectors

6. Scalars are physical quantities that have magnitude only and obey the rules of ordinary arithmetic.
7. Examples of scalars include temperature, pressure, energy, mass, time, speed, work, power, electric potential, and density.
8. Vectors are physical quantities that have both magnitude and direction and obey the rules of vector algebra.
9. Examples of vectors include displacement, velocity, acceleration, force, momentum, torque, electric field, and magnetic field.
10. A vector is represented graphically by an arrow whose length represents magnitude and whose direction represents the vector's direction.
11. Two vectors are equal if and only if they have the same magnitude and the same direction.
12. A vector can be shifted without changing its value if its length and direction are not changed, known as the free vector property.
13. The negative of a vector has the same magnitude but opposite direction.
14. The zero vector has magnitude zero and undefined (arbitrary) direction.
15. A position vector specifies the location of a point relative to an origin.

Vector Addition and Subtraction

16. Vectors are added using the head-to-tail (geometrical) method.
17. Vector addition is commutative: $\vec{a} + \vec{b} = \vec{b} + \vec{a}$.
18. Vector addition is associative: $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$.
19. The magnitude of the sum of two vectors depends on the angle between them: $|\vec{a} + \vec{b}| = \sqrt{a^2 + b^2 + 2ab\cos\phi}$.
20. The maximum magnitude of the sum of two vectors occurs when they are parallel ($\phi = 0^\circ$), giving $a + b$.
21. The minimum magnitude of the sum of two vectors occurs when they are antiparallel ($\phi = 180^\circ$), giving $|a - b|$.
22. The magnitude of the sum of two perpendicular vectors is $\sqrt{a^2 + b^2}$.
23. Vector subtraction is defined as $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$.

Practice MCQs

1. Which of the following is a scalar quantity?

- A) Displacement
- B) Velocity
- C) Work
- D) Force

Correct Answer: C

2. Which of the following is a vector quantity?

- A) Temperature
- B) Speed
- C) Momentum
- D) Energy

Correct Answer: C

3. Two vectors are equal if and only if they have:

- A) The same magnitude only.
- B) The same direction only.
- C) The same magnitude and the same direction.
- D) The same magnitude, direction, and point of application.

Correct Answer: C

4. The negative of a vector has:

- A) The same magnitude and same direction.
- B) The same magnitude but opposite direction.
- C) Opposite magnitude but same direction.
- D) Opposite magnitude and opposite direction.

Correct Answer: B

5. The zero vector has:

- A) Zero magnitude and zero direction.
- B) Zero magnitude and undefined direction.
- C) Unit magnitude and zero direction.
- D) Infinite magnitude and undefined direction.

Correct Answer: B

6. Vector addition is:

- A) Commutative only.
- B) Associative only.
- C) Both commutative and associative.
- D) Neither commutative nor associative.

Correct Answer: C

7. The magnitude of the sum of two vectors $\vec{a}$ and $\vec{b}$ is maximum when the angle between them is:

- A) 0°
- B) 90°
- C) 180°
- D) 270°

Correct Answer: A

8. The magnitude of the sum of two vectors $\vec{a}$ and $\vec{b}$ is minimum when the angle between them is:

- A) 0°
- B) 90°
- C) 180°
- D) 270°

Correct Answer: C

9. If two vectors of magnitudes 3 and 4 are perpendicular, the magnitude of their sum is:

- A) 1
- B) 5
- C) 7
- D) 12

Correct Answer: B ($\sqrt{3^2 + 4^2} = 5$)

10. The vector subtraction $\vec{a} - \vec{b}$ is equivalent to:

- A) $\vec{a} + \vec{b}$
- B) $\vec{a} + (-\vec{b})$
- C) $-\vec{a} + \vec{b}$
- D) $-(\vec{a} + \vec{b})$

Correct Answer: B

11. The component of a vector $\vec{a}$ along the x-axis is given by:

- A) $a_x = a \sin \theta$
- B) $a_x = a \cos \theta$
- C) $a_x = a \tan \theta$
- D) $a_x = a \sec \theta$

Correct Answer: B

12. The component of a vector $\vec{a}$ along the y-axis is given by:

- A) $a_y = a \sin \theta$
- B) $a_y = a \cos \theta$
- C) $a_y = a \tan \theta$

2. Vector Analysis

Chapter 3: Motion and Force

What is Physics of Motion?

Physics is fundamentally the study of motion—how objects move, what causes them to move, and how their motion changes over time. Every motion we observe in the universe, from the flight of a cricket ball to the orbit of the Moon around Earth, is governed by forces.

Definition — Force: A force is a push or a pull that one object exerts on another. Forces are the fundamental cause of changes in motion: they start objects moving, stop them, speed them up, slow them down, and change their direction.

The systematic study of the relationship between force and motion is called **dynamics**. This chapter unifies two great pillars of classical mechanics:

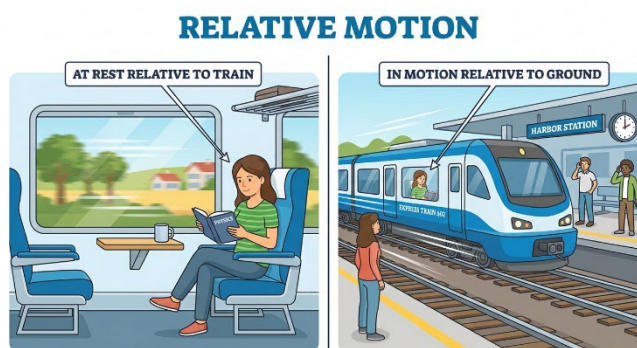
Branch	Focus	Question Answered
Kinematics	Description of motion	<i>How</i> does an object move? (position, velocity, acceleration)
Dynamics	Causes of motion	<i>Why</i> does an object move? (forces, mass, momentum)

These laws were published by Sir Isaac Newton in 1686 in his monumental work *Philosophiae Naturalis Principia Mathematica*. They remain as valid today for describing the motion of everyday objects.

Rest and Motion — A Relative Concept

Definitions:

- An object is said to be **at rest** if its position does not change with respect to a chosen reference point as time passes.
- An object is in **motion** if its position changes with respect to a reference point with the passage of time.



Critical Insight: Rest and motion

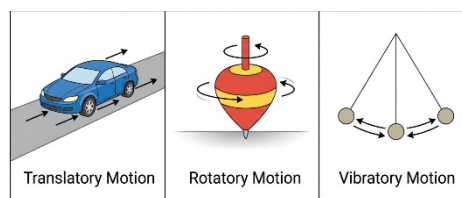
are **relative** terms. A person sitting in a moving train is at rest relative to the train but in motion relative to the ground. There is no absolute state of rest.

Classification of Motion

Type of Motion	Description	Example
Translatory Motion	All points move uniformly in the same direction; orientation unchanged	Car on a straight road
Rotatory Motion	Object rotates about its own axis	Spinning top, Earth's rotation
Vibratory Motion	Back-and-forth motion about a mean position	Pendulum, plucked string

Translatory motion is further divided into:

- Rectilinear motion** (straight-line path)
- Curvilinear motion** (curved path)
- Circular motion** (constant radius curved path)
- Random motion** (no specific path)



Critical Rule: When v and a have the **same sign**, speed increases. When they have **opposite signs**, speed decreases.

Example: Car accelerates from rest to 20 m/s in 5 seconds:

$$a = \frac{20 - 0}{5} = 4 \text{ m/s}^2$$

(Each second, speed increases by 4 m/s)

Constant Acceleration — The Five Kinematic Equations

When acceleration is constant, we can derive five fundamental equations.

Derivation: Starting from $a = \text{constant}$:

$$v = \int a \, dt = at + C$$

At $t = 0$, $v = v_0$, so $C = v_0$:

$$v = v_0 + at \quad (1)$$

$$x = \int v \, dt = \int (v_0 + at) \, dt = v_0 t + \frac{1}{2} at^2 + C'$$

At $t = 0$, $x = x_0$, so $C' = x_0$:

$$x - x_0 = v_0 t + \frac{1}{2} at^2 \quad (2)$$

The Complete Set

$$\begin{aligned} v &= v_0 + at \\ x - x_0 &= v_0 t + \frac{1}{2} at^2 \\ v^2 &= v_0^2 + 2a(x - x_0) \\ x - x_0 &= \frac{1}{2}(v_0 + v)t \\ x - x_0 &= vt - \frac{1}{2} at^2 \end{aligned}$$

Problem-Solving Strategy:

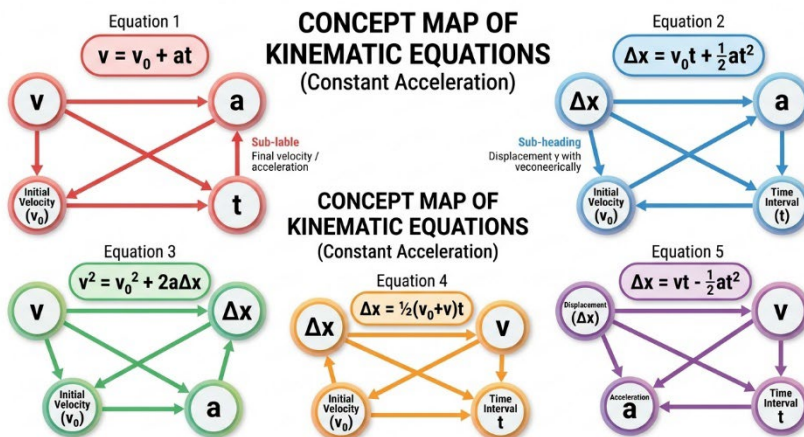
1. Identify known quantities (3 of 5 variables)
2. Identify unknown
3. Choose equation containing knowns and unknown
4. Solve algebraically before substituting numbers
5. Check sign and reasonableness

Free Fall — Motion Under Gravity

Definition: Free fall is motion under **only** gravity, with air resistance neglected.

Key Facts:

- Acceleration is constant and directed **downward**
- Magnitude: $g = 9.8 \text{ m/s}^2$ ($\approx 10 \text{ m/s}^2$ for rough calculations)
- g varies slightly with latitude and altitude
- All objects in free fall have the same acceleration, regardless of mass



Projectile Motion

Definition: Two-dimensional motion of a particle launched with initial velocity $\vec{v}_0$, under **only gravity** (air resistance neglected).

The Fundamental Principle: Independence of Motions (Galileo's Discovery)

- Horizontal motion: $a_x = 0$ (constant velocity)
- Vertical motion: $a_y = -g$ (constant acceleration, same as free fall)

Initial Velocity Components:

$$v_{0x} = v_0 \cos \theta_0$$

$$v_{0y} = v_0 \sin \theta_0$$

Equations of Projectile Motion (setting $x_0 = 0, y_0 = 0$):

Component	Equation
Horizontal position	$x = (v_0 \cos \theta_0)t$
Horizontal velocity	$v_x = v_0 \cos \theta_0$ (constant)
Vertical position	$y = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2$
Vertical velocity	$v_y = v_0 \sin \theta_0 - gt$
Vertical velocity squared	$v_y^2 = (v_0 \sin \theta_0)^2 - 2gy$

The Trajectory Equation (Parabola):

$$y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2}$$

Maximum Height (at $v_y = 0$):

$$H = \frac{v_0^2 \sin^2 \theta_0}{2g}$$

Time of Flight:

$$T = \frac{2v_0 \sin \theta_0}{g}$$

Horizontal Range (launch and land at same height):

$$R = \frac{v_0^2 \sin(2\theta_0)}{g}$$

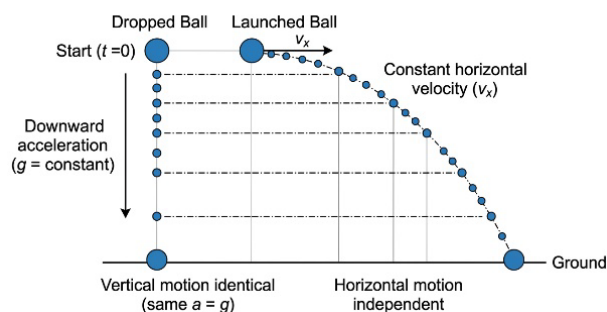
Important Observations:

Condition	Result
$\theta_0 = 45^\circ$	Maximum range
θ and $90^\circ - \theta$	Same range (complementary angles)
Range $\propto v_0^2$	Doubling speed quadruples range

Example: Ball kicked at 20 m/s, 30° .

- $v_{0x} = 20 \cos 30^\circ = 17.32$ m/s

Figure 1: Independence of Horizontal and Vertical Projectile Motion



6. Solve for unknowns

Worked Example 1: Car (1200 kg) accelerates from 12 m/s to 20 m/s in 6.4 s.

$$a = (20 - 12)/6.4 = 1.25 \text{ m/s}^2$$

$$F = ma = 1200 \times 1.25 = 1500 \text{ N}$$

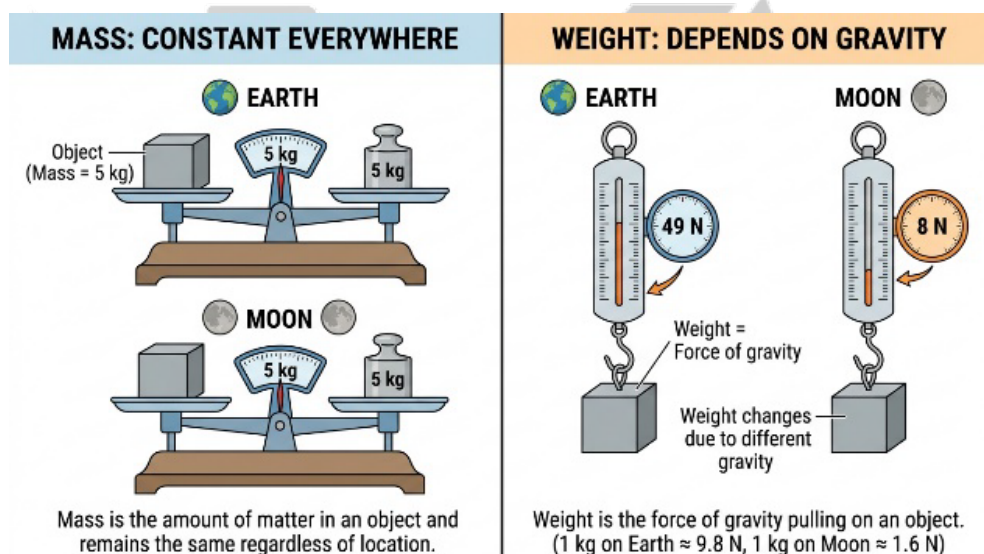
Worked Example 2: Force to give 0.1 kg ball acceleration of 500 m/s².

$$F = 0.1 \times 500 = 50 \text{ N}$$

Mass vs. Weight — Critical Distinction

Quantity	Mass	Weight
Definition	Amount of matter	Force of gravity
Type	Scalar	Vector
SI Unit	kilogram (kg)	newton (N)
Dependence	Constant everywhere	Depends on g
Measurement	Balance	Spring scale

$$W = mg$$



Example: 50 kg person on Earth: $W = 50 \times 9.8 = 490 \text{ N}$

Same person on Moon: $W = 50 \times 1.6 = 80 \text{ N}$

Mass remains 50 kg in both locations.

Gravitational Field Strength:

$$\vec{g} = \frac{\vec{F}_g}{m}$$

Units: N/kg (magnitude equals acceleration due to gravity)

g values in solar system (N/kg):

| Mercury 3.7 | Venus 8.9 | Earth 9.8 | Mars 3.7 |

| Jupiter 24.7 | Saturn 9.0 | Uranus 8.7 | Neptune 11.0 | Sun 274 |

With static friction (verge of slipping):

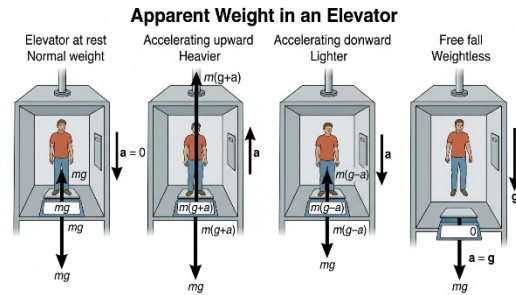
$$\tan \theta_{\max} = \mu_s$$

Apparent Weight in an Elevator

Setup: Person of mass m in elevator accelerating upward with a .

$$F_N - mg = ma$$

$$F_N = m(g + a)$$



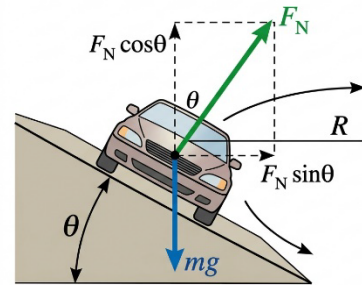
Elevator Motion	Acceleration Direction	F_N vs mg
At rest or constant velocity	0	Equal
Accelerating upward	Upward (+)	Greater (feel heavier)
Accelerating downward	Downward (-)	Less (feel lighter)
Free fall	$a = -g$	$F_N = 0$ (weightless)

Banked Curves (No Friction)

Setup: Car on banked curve of radius R at angle θ , no friction.

- Horizontal (centripetal): $F_N \sin \theta = \frac{mv^2}{R}$
- Vertical: $F_N \cos \theta = mg$

$$\tan \theta = \frac{v^2}{gR}$$



Circular Motion in a Vertical Loop

At the top (minimum speed to maintain contact):

$$F_N + mg = \frac{mv^2}{R}$$

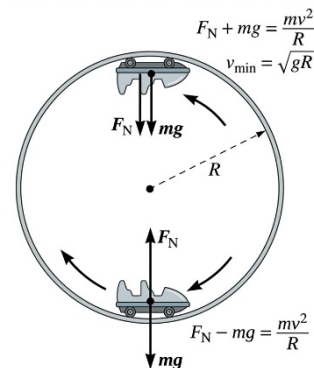
Minimum speed when $F_N = 0$:

$$v_{\min} = \sqrt{gR}$$

At the bottom:

$$F_N - mg = \frac{mv^2}{R}$$

$$F_N = mg + \frac{mv^2}{R}$$



Drag Force and Terminal Speed

Drag Force

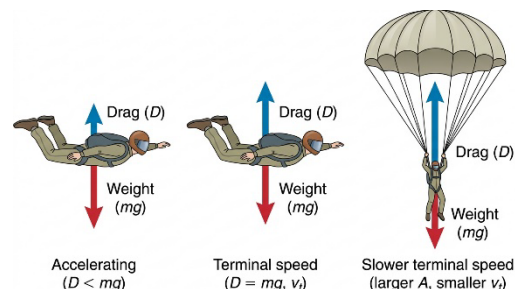
When an object moves through a fluid, it experiences a drag force opposing motion.

Magnitude (turbulent flow, blunt objects):

$$D = \frac{1}{2} C \rho A v^2$$

where:

- C = drag coefficient (0.4–1.0 for typical objects)
- ρ = fluid density (air $\approx 1.2 \text{ kg/m}^3$ at sea level)



- Crowbar used by pulling near the end

Principle of Moments

When an object is in equilibrium (not rotating):

$$\text{Sum of clockwise moments} = \text{Sum of anticlockwise moments}$$

Conditions for Complete Equilibrium

For both translational and rotational equilibrium:

1. **No resultant force:** $\sum \vec{F} = 0$
2. **No resultant torque:** $\sum \tau_{\text{clockwise}} = \sum \tau_{\text{anticlockwise}}$

Centre of Mass and Stability

Centre of Mass

The point at which the entire mass of an object can be considered to be concentrated.

- For symmetrical objects, centre of mass lies on axis of symmetry
- Under gravity, also called **centre of gravity** (point where weight appears to act)

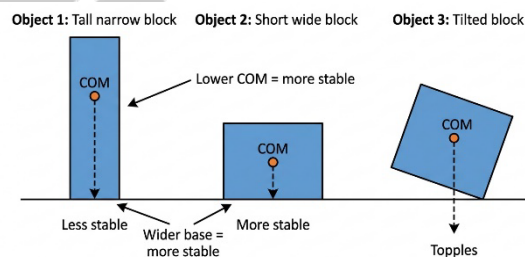
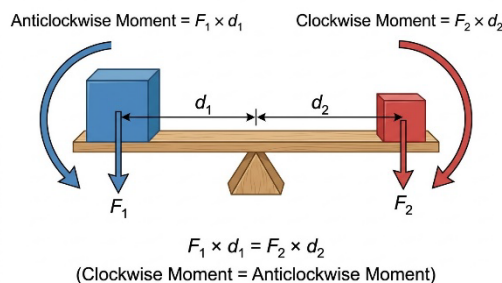
Factors Affecting Stability

Factor	Effect
Lower centre of mass	More stable
Wider base	More stable

Object topples if line of action of weight falls **outside its base**.

Examples:

- Double-decker buses have heavy weights underneath to lower centre of mass
- Racing cars have low centres of mass to prevent rolling
- Pyramids are extremely stable (low centre of mass, wide base)
- Tightrope walkers use long poles to adjust centre of mass position



Comprehensive Summary Tables

Kinematics Summary

Quantity	1D Definition	2D/3D Definition	Vector?
Position	x	$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$	Yes
Displacement	$\Delta x = x_2 - x_1$	$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$	Yes
Average Velocity	$v_{\text{avg}} = \Delta x / \Delta t$	$\vec{v}_{\text{avg}} = \Delta \vec{r} / \Delta t$	Yes
Instantaneous Velocity	$v = dx/dt$	$\vec{v} = d\vec{r}/dt$	Yes
Average Acceleration	$a_{\text{avg}} = \Delta v / \Delta t$	$\vec{a}_{\text{avg}} = \Delta \vec{v} / \Delta t$	Yes
Instantaneous Acceleration	$a = dv/dt$	$\vec{a} = d\vec{v}/dt$	Yes

Dynamics Summary

Quantity	Symbol	Formula	Notes
Newton's 1st Law	—	$\vec{F}_{\text{net}} = 0 \Rightarrow \vec{a} = 0$	Law of inertia
Newton's 2nd Law	—	$\vec{F}_{\text{net}} = m\vec{a}$	$F \propto a$
Newton's 3rd Law	—	$\vec{F}_{AB} = -\vec{F}_{BA}$	Action-reaction
Weight	W	$W = mg$	Depends on g



Motion and Force: ONE-LINERS

Physics of Motion and Fundamental Concepts

1. Physics is fundamentally the study of motion—how objects move, what causes them to move, and how their motion changes over time.
2. A force is a push or a pull that one object exerts on another, serving as the fundamental cause of changes in motion.
3. Dynamics is the systematic study of the relationship between force and motion.
4. Kinematics describes *how* an object moves (position, velocity, acceleration), while dynamics explains *why* it moves (forces, mass, momentum).
5. Newton's laws were published in 1686 in *Philosophiae Naturalis Principia Mathematica*.
6. An object is at rest if its position does not change with respect to a chosen reference point as time passes.
7. An object is in motion if its position changes with respect to a reference point with the passage of time.
8. Rest and motion are relative terms—there is no absolute state of rest.
9. Translatory motion occurs when all points move uniformly in the same direction with orientation unchanged.
10. Rotatory motion occurs when an object rotates about its own axis.
11. Vibratory motion is back-and-forth motion about a mean position.
12. Translatory motion includes rectilinear, curvilinear, circular, and random motion.
13. The particle model treats objects as point-like masses with no internal structure, valid when size is negligible and rotation/deformation are ignored.

Scalars and Vectors

14. Scalars are described completely by magnitude alone (mass, distance, speed, time, temperature, energy).
15. Vectors require both magnitude and direction for complete specification (displacement, velocity, acceleration, force, momentum).
16. Displacement is the change in position: $\Delta x = x_2 - x_1$, with sign indicating direction.
17. Distance is a scalar (total path length, always positive), while displacement is a vector (net change with direction).
18. Average speed is total distance divided by total time.
19. Instantaneous speed is the speed at a specific instant (what a speedometer shows).
20. Average velocity is displacement divided by time: $v_{\text{avg}} = \Delta x / \Delta t$.
21. Instantaneous velocity is $v = dx/dt$, the derivative of position with respect to time.
22. Velocity is a vector; speed is its magnitude (scalar).
23. A satellite moving at constant speed in a circular orbit does NOT have constant velocity because direction changes continuously.
24. Acceleration is the rate of change of velocity and can result from changes in speed, direction, or both.
25. Average acceleration is $a_{\text{avg}} = \Delta v / \Delta t$.
26. Instantaneous acceleration is $a = dv/dt = d^2x/dt^2$.

Practice MCQs

1. Physics is fundamentally the study of:

- A) Forces only
- B) Motion and its causes
- C) Energy only
- D) Matter composition

Correct Answer: B

2. A force is defined as:

- A) A scalar quantity measuring energy
- B) A push or pull that one object exerts on another
- C) The rate of change of position
- D) The amount of matter in an object

Correct Answer: B

3. Dynamics is the branch of mechanics that studies:

- A) How objects move (position, velocity, acceleration)
- B) Why objects move (forces, mass, momentum)
- C) The shape of objects
- D) The composition of matter

Correct Answer: B

4. Kinematics is the branch of mechanics that studies:

- A) Forces and their effects
- B) How objects move without considering causes
- C) The nature of energy
- D) The structure of matter

Correct Answer: B

5. Rest and motion are:

- A) Absolute concepts
- B) Relative concepts
- C) Independent of reference frame
- D) Always the same for all observers

Correct Answer: B

6. A person sitting in a moving train is:

- A) At rest relative to the ground
- B) In motion relative to the train
- C) At rest relative to the train and in motion relative to the ground
- D) Neither at rest nor in motion

Correct Answer: C

7. Which type of motion involves all points moving uniformly in the same direction with orientation unchanged?

- A) Rotatory motion
- B) Vibratory motion
- C) Translatory motion
- D) Random motion

Correct Answer: C

8. The particle model is valid when:

- A) The object's rotation is significant
- B) All parts move at different rates
- C) The object's size is negligible compared to distances involved
- D) The object is deformable

Correct Answer: C

9. Which of the following is a scalar quantity?

- A) Displacement
- B) Velocity
- C) Speed
- D) Acceleration

Correct Answer: C

10. Which of the following is a vector quantity?

- A) Mass
- B) Temperature
- C) Displacement
- D) Energy

Correct Answer: C

11. Displacement is defined as:

- A) Total path length travelled
- B) Change in position with direction
- C) Distance divided by time
- D) Rate of change of speed

Correct Answer: B

12. If a person walks 3 km east then 4 km west, the distance travelled is:

- A) 1 km
- B) 3 km
- C) 4 km

Chapter 4: Work and Energy

What Is Energy?

One of the fundamental goals of physics is to understand energy—a concept that permeates every aspect of our lives. From the flight of an airplane to the beating of our hearts, from the generation of electricity to the motion of planets, energy is the universal currency of change.

Energy is a scalar quantity associated with the state or condition of one or more objects. While this definition may seem abstract, the power of energy lies in a remarkable property of our universe: **Energy can be transformed from one type to another and transferred from one object to another, but the total amount always remains constant.** This principle, known as the **conservation of energy**, is one of the most fundamental and well-tested laws in all of physics.

Think of energy as money in various bank accounts. You can transfer money from one account to another—from your checking account to your savings account, or from your account to a friend's. The numbers change, but the total amount of money in the system can always be accounted for. Similarly, energy can be transferred between objects or converted between different forms, but it is never created or destroyed.

Why Study Work and Energy?

In previous chapters, we analyzed motion using Newton's laws of motion—relating forces to accelerations, and accelerations to changes in velocity. While this approach is powerful, it often requires detailed knowledge of the path taken by an object and the forces acting at every instant.

The **work-energy approach** offers an alternative that is often simpler and more powerful. Instead of tracking accelerations and forces at every moment, we focus on the **initial and final states** of a system. This allows us to:

- Solve problems without knowing the detailed path of motion
- Handle forces that vary with position (like springs and gravity)
- Understand energy transfers between different forms
- Analyze systems where Newton's laws become mathematically difficult

This chapter develops the concepts of work, kinetic energy, potential energy, and power, and shows how they are connected through the work-energy theorem and the law of conservation of energy.

WORK: TRANSFERRING ENERGY BY FORCE

The Physical Meaning of Work

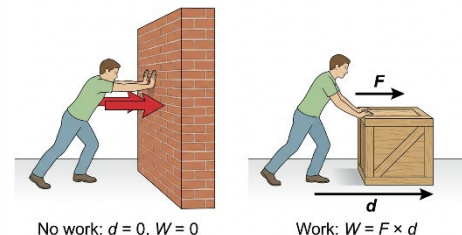
In everyday language, "work" suggests any effortful activity—studying for an exam, lifting weights, or even just thinking hard. In physics, however, **work** has a precise and narrow definition:

Work is energy transferred to or from an object by means of a force acting on the object.

For work to be done, two conditions must be satisfied:

1. A **force** must act on the object
2. The object must **move** while the force acts

If you push against a wall and the wall does not move, you may feel tired, but in the physics sense, **no work is done on the wall** because there is no displacement.



Work Done by Multiple Forces

When several forces act on an object, the **net work** is the sum of the work done by each individual force:

$$W_{\text{net}} = W_1 + W_2 + W_3 + \dots$$

Alternatively, we can find the net force $\vec{F}_{\text{net}}$ and then calculate:

$$W_{\text{net}} = \vec{F}_{\text{net}} \cdot \vec{d} = F_{\text{net}} d \cos \phi_{\text{net}}$$

KINETIC ENERGY AND THE WORK-ENERGY THEOREM

Kinetic energy is the energy associated with the motion of an object. The faster an object moves, the greater its kinetic energy. When an object is stationary, its kinetic energy is zero.

For an object of mass m moving with speed v (where v is well below the speed of light), the kinetic energy K is:

$$K = \frac{1}{2}mv^2$$

Derivation of the Kinetic Energy Formula:

Consider an object of mass m initially at rest ($v_i = 0$). A constant net force F acts on it, causing it to accelerate over a distance d to a final speed $v_f = v$.

From Newton's second law: $F = ma$

From kinematics (equation $v_f^2 = v_i^2 + 2ad$): $v^2 = 0 + 2ad$, so $a = v^2/(2d)$

The work done is: $W = Fd = (ma)d = m\left(\frac{v^2}{2d}\right)d = \frac{1}{2}mv^2$

Since this work appears as energy of motion, we identify $\frac{1}{2}mv^2$ as the kinetic energy.

The Work-Energy Theorem

The **work-energy theorem** states a profound relationship:

The net work done on an object equals the change in its kinetic energy.

$$W_{\text{net}} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Equivalently:

$$K_f = K_i + W_{\text{net}}$$

Physical Interpretation:

- If net work is **positive**, kinetic energy **increases** (the object speeds up)
- If net work is **negative**, kinetic energy **decreases** (the object slows down)
- If net work is **zero**, kinetic energy remains **constant** (speed unchanged)

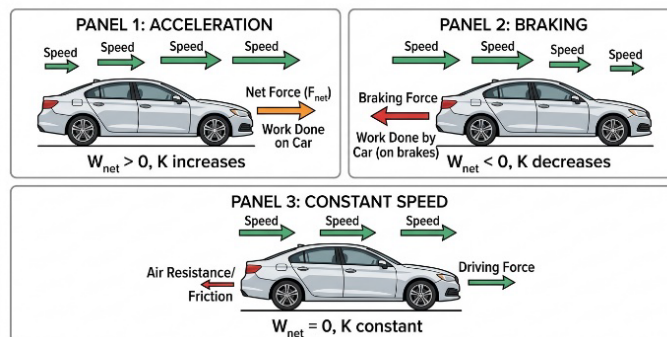
Power of the Theorem: The work-energy theorem allows us to relate speeds at different positions without knowing the details of the motion or the forces as functions of time. We only need the net work done, which often depends only on the initial and final positions for certain forces.

Example: Applying the Work-Energy Theorem

Problem: A 1500 kg car initially traveling at 20 m/s comes to a stop after skidding 50 m. What is the average friction force?

Solution:

WORK-ENERGY THEOREM EXAMPLES



WORK DONE BY A SPRING FORCE

Hooke's Law and the Spring Force

Springs are everywhere—in car suspensions, pens, mattresses, and countless other devices. When we stretch or compress a spring, it exerts a **restoring force** that tries to return the spring to its relaxed (equilibrium) length.

For an ideal spring, this force obeys **Hooke's law**:

$$\vec{F}_s = -k\vec{x}$$

where:

- $\vec{F}_s$ is the force exerted by the spring
- k is the **spring constant** (a measure of stiffness, in N/m)
- $\vec{x}$ is the displacement of the spring's free end from its relaxed position
- The minus sign indicates the force is opposite to the displacement (restoring)

In one dimension, with the origin at the relaxed position:

$$F_s = -kx$$

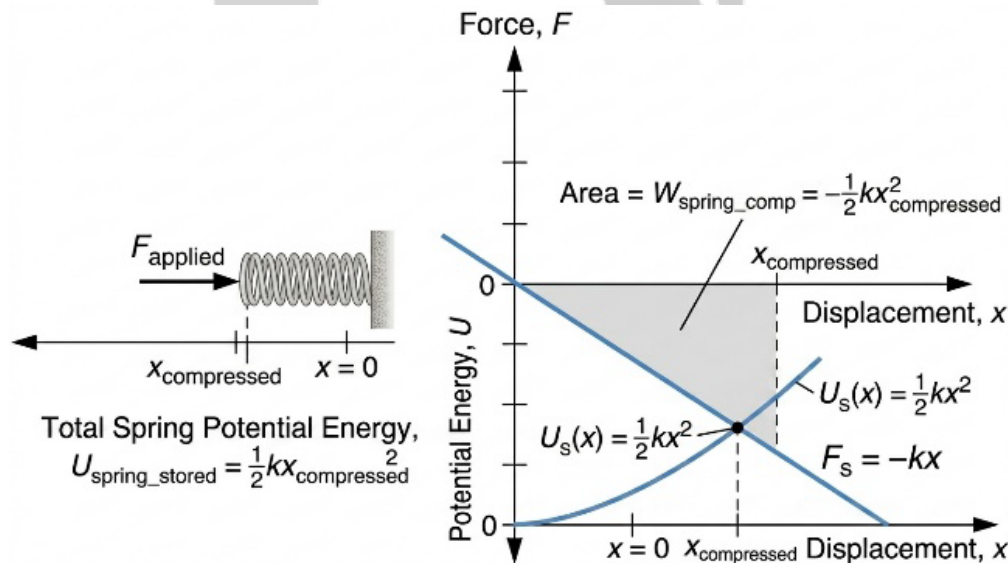
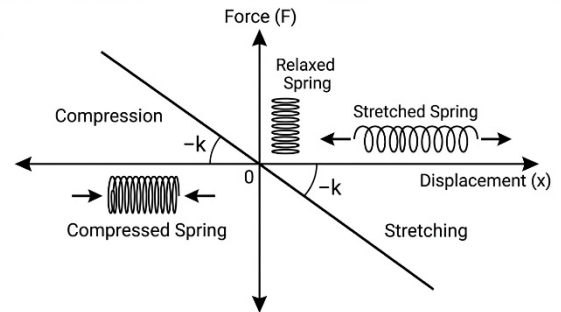
Key properties:

- If $x > 0$ (spring stretched), $F_s < 0$ (force pulls backward, toward equilibrium)
- If $x < 0$ (spring compressed), $F_s > 0$ (force pushes forward, toward equilibrium)
- The spring force is **variable**—it changes with position

Work Done by a Spring Force

Because the spring force varies with position, we cannot simply multiply force by distance. Instead, we must integrate:

$$W_s = \int_{x_i}^{x_f} F_s \, dx = \int_{x_i}^{x_f} (-kx) \, dx = -\frac{1}{2}k[x^2]_{x_i}^{x_f}$$



Evaluating this gives:

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Conservation Principle

For an **isolated system** (no external forces doing work) in which **only conservative forces** do work internally:

The total mechanical energy remains constant.

$$K_f + U_f = K_i + U_i$$

or equivalently:

$$\Delta K + \Delta U = 0 \text{ or } \Delta E_{\text{mec}} = 0$$

This means energy can be transferred between kinetic and potential forms, but their sum never changes.

Applications of Energy Conservation

Example 1: Pendulum motion

A pendulum bob swings back and forth. At the highest points, all energy is gravitational potential; at the lowest point, all energy is kinetic. At intermediate points, energy is shared:

$$mgh_{\text{max}} = \frac{1}{2}mv^2 + mgh$$

We can find the speed at any height without knowing the tension in the string or solving equations of motion.

Example 2: Roller coaster

A roller coaster car starting from rest at height h will have speed at ground level:

$$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$$

Notice this is independent of mass and path (as long as friction is negligible)!

Example 3: Block on a spring

A block attached to a spring, pulled to $x = A$ and released, will oscillate. At any position x :

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

The maximum speed occurs at $x = 0$ (equilibrium): $v_{\text{max}} = A\sqrt{k/m}$

WORK DONE ON A SYSTEM BY EXTERNAL FORCES

Systems and External Work

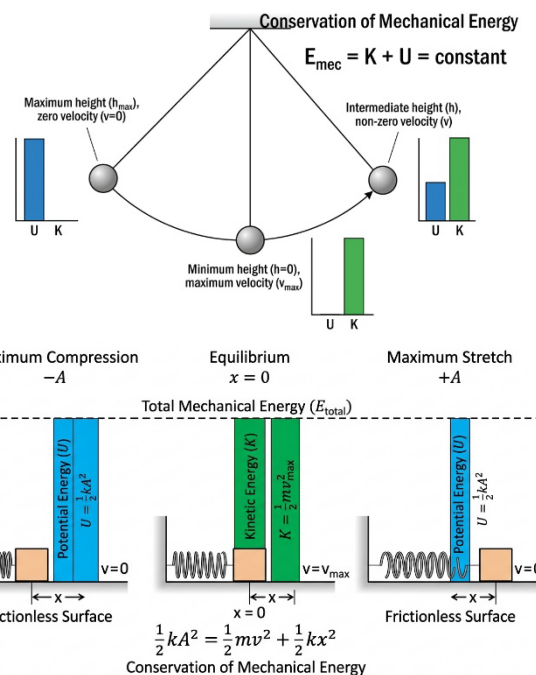
So far, we have focused on work done **by** forces within a system. Often, however, we need to consider work done **on** a system by an external force.

Work is energy transferred to or from a system by means of an external force acting on that system.

No Friction Involved

When no friction acts within the system, the work done by external forces equals the change in mechanical energy:

$$W_{\text{ext}} = \Delta E_{\text{mec}} = \Delta K + \Delta U$$

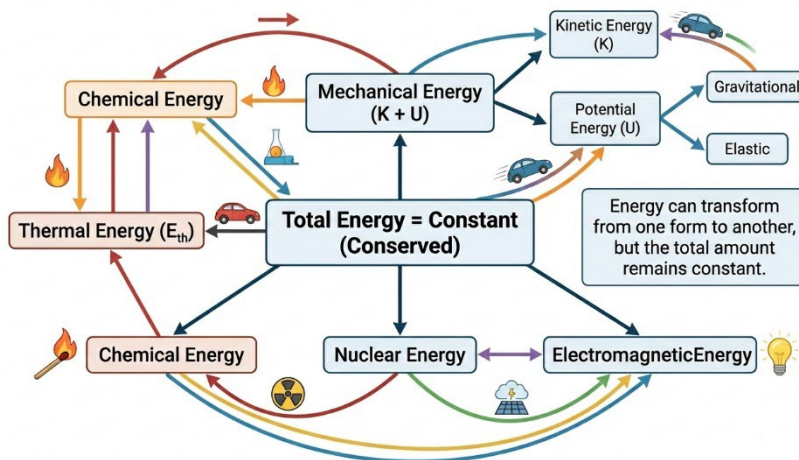


The Principle of Energy Conservation in Context

The law of conservation of energy applies to all these conversions. The total energy input equals the sum of:

- Useful energy output
- Waste heat (often the largest fraction)
- Other non-useful forms (sound, friction, etc.)

This is why improving efficiency—reducing the waste fraction—is such an important technological goal.



CONCEPTUAL SUMMARY

Term	Definition
Work	Energy transferred by a force; $W = \vec{F} \cdot \vec{d} = Fd \cos \phi$
Kinetic energy	Energy of motion; $K = \frac{1}{2}mv^2$
Potential energy	Energy of configuration; associated with conservative forces
Conservative force	Force for which work is path-independent (e.g., gravity, spring)
Nonconservative force	Force for which work depends on path (e.g., friction)
Mechanical energy	$E_{\text{mec}} = K + U$
Power	Rate of energy transfer; $P = dW/dt = \vec{F} \cdot \vec{v}$
Efficiency	$\eta = \text{useful output/input}$

Fundamental Principles

1. **Work-Energy Theorem:** $W_{\text{net}} = \Delta K$
2. **Potential Energy Change:** $\Delta U = -W_c$
3. **Force from Potential:** $F(x) = -dU/dx$
4. **Conservation of Mechanical Energy:** $\Delta K + \Delta U = 0$ (isolated system, only conservative forces)
5. **Work-External Energy:** $W_{\text{ext}} = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}$
6. **Conservation of Total Energy:** E_{total} is constant for an isolated system

Work and Energy: One-Liners

Introduction to Energy and Work

1. Energy is a scalar quantity associated with the state or condition of one or more objects.
2. Energy can be transformed from one type to another and transferred from one object to another, but the total amount always remains constant.
3. The conservation of energy is one of the most fundamental and well-tested laws in all of physics.
4. The work-energy approach focuses on initial and final states of a system rather than detailed path analysis.
5. Work is energy transferred to or from an object by means of a force acting on the object.
6. For work to be done, a force must act on the object and the object must move while the force acts.
7. Pushing against a wall that does not move results in no work done on the wall in the physics sense.
8. The SI unit of work is the joule (J), where $1 \text{ J} = 1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$.
9. One joule is approximately the work required to lift a small apple (about 100 g) vertically by one meter.

Work Done by a Constant Force

10. Work done by a constant force: $W = Fd \cos \phi = \vec{F} \cdot \vec{d}$, where ϕ is the angle between force and displacement.
11. Only the component of force parallel to the displacement does work; the perpendicular component contributes nothing to energy transfer.
12. Work is positive when $0^\circ \leq \phi < 90^\circ$ — energy is transferred **to** the object.
13. Work is zero when $\phi = 90^\circ$ — force is perpendicular to motion, no energy transfer.
14. Work is negative when $90^\circ < \phi \leq 180^\circ$ — energy is transferred **from** the object.
15. A satellite orbiting Earth experiences zero work done by gravity because gravity is perpendicular to velocity at every instant.
16. When several forces act on an object, the net work is the sum of the work done by each individual force.
17. Net work can also be calculated as $W_{\text{net}} = \vec{F}_{\text{net}} \cdot \vec{d} = F_{\text{net}} d \cos \phi_{\text{net}}$.

Kinetic Energy and the Work-Energy Theorem

18. Kinetic energy is the energy associated with the motion of an object: $K = \frac{1}{2}mv^2$.
19. Kinetic energy is zero when an object is stationary.
20. The work-energy theorem states: $W_{\text{net}} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$.
21. If net work is positive, kinetic energy increases and the object speeds up.
22. If net work is negative, kinetic energy decreases and the object slows down.
23. If net work is zero, kinetic energy remains constant and speed is unchanged.
24. The work-energy theorem allows relating speeds at different positions without knowing details of motion or forces as functions of time.

Work Done by Gravitational Force

25. Near Earth's surface, gravitational force is constant: $\vec{F}_g = m\vec{g}$, directed downward.

3. Work is defined as:

- A) The force applied to an object
- B) Energy transferred by a force acting on an object
- C) The rate of energy transfer
- D) The product of mass and acceleration

Correct Answer: B

4. For work to be done in the physics sense:

- A) A force must act and the object must move
- B) The object must move a large distance
- C) A large force must be applied
- D) The object must be at rest

Correct Answer: A

5. Pushing against a wall that does not move results in:

- A) Positive work on the wall
- B) Negative work on the wall
- C) Zero work on the wall
- D) Maximum work on the wall

Correct Answer: C

6. The SI unit of work is the:

- A) Newton
- B) Watt
- C) Joule
- D) Kilogram

Correct Answer: C

7. One joule is equal to:

- A) $1 \text{ N} \cdot \text{m}$
- B) 1 N/s
- C) $1 \text{ kg} \cdot \text{m/s}^2$
- D) $1 \text{ W} \cdot \text{s}$

Correct Answer: A

8. Work done by a constant force is given by:

- A) $W = Fd$
- B) $W = Fd \cos \phi$
- C) $W = Fd \sin \phi$
- D) $W = F/d$

Correct Answer: B

9. Only the component of force _____ to displacement does work.

- A) Perpendicular
- B) Parallel

C) Opposite

D) Equal

Correct Answer: B

10. Work is positive when the angle between force and displacement is:

- A) $0^\circ \leq \phi < 90^\circ$
- B) $90^\circ < \phi \leq 180^\circ$
- C) $\phi = 90^\circ$
- D) $\phi = 180^\circ$

Correct Answer: A

11. Work is negative when the angle between force and displacement is:

- A) $0^\circ \leq \phi < 90^\circ$
- B) $90^\circ < \phi \leq 180^\circ$
- C) $\phi = 90^\circ$
- D) $\phi = 0^\circ$

Correct Answer: B

12. Work is zero when the angle between force and displacement is:

- A) 0°
- B) 90°
- C) 45°
- D) 180°

Correct Answer: B

13. A satellite in circular orbit experiences zero work done by gravity because:

- A) Gravity is zero in orbit
- B) Gravity is perpendicular to velocity at every instant
- C) Gravity is parallel to velocity
- D) There is no displacement

Correct Answer: B

14. Kinetic energy is defined as:

- A) $K = mv$
- B) $K = \frac{1}{2}mv^2$
- C) $K = mgh$
- D) $K = \frac{1}{2}kx^2$

Correct Answer: B

15. The kinetic energy of a stationary object is:

- A) Maximum

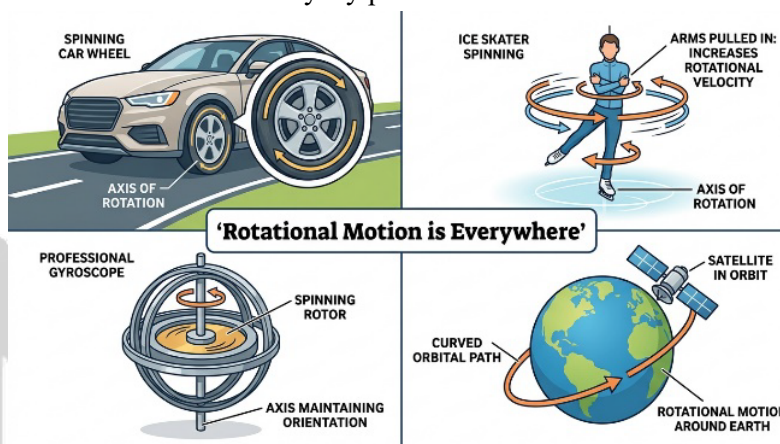
Chapter 5: Rotational and Circular Motion

Introduction

In previous chapters, we focused on **translational motion** — the motion of objects moving along straight or curved paths without any change in their orientation. However, the physical world around us exhibits another fundamental type of motion: **rotation**. From the spinning wheels of a car to the orbiting satellites above Earth, from the blades of a ceiling fan to the rotation of planets around the Sun — rotational motion is everywhere.

Why is the study of rotation so important? Consider these everyday phenomena:

- A **flywheel** in an engine stores energy and smooths out power delivery.
- An **ice skater** spins faster by pulling her arms inward.
- A **gyroscope** maintains its orientation and helps navigate airplanes and ships.
- **Artificial satellites** stay in orbit because gravity provides the necessary centripetal force.



This chapter develops a complete framework for describing rotational motion. You will discover that rotation is mathematically **analogous** to translation — angular displacement corresponds to linear displacement, angular velocity to linear velocity, angular acceleration to linear acceleration, torque to force, and moment of inertia to mass. This powerful analogy allows us to translate our understanding of linear motion directly into the language of rotation.

Rotational Kinematics: Describing Angular Motion

Just as we described translational motion using position, displacement, velocity, and acceleration, we now introduce their **angular counterparts**.

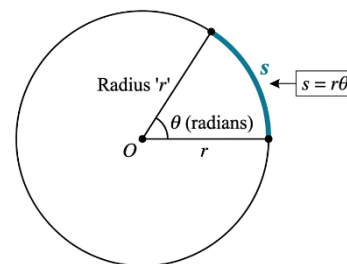
Angular Position (θ)

Consider a rigid body rotating about a fixed axis (called the **axis of rotation**). A rigid body is one that can rotate with all its parts locked together without any change in shape. For such a body, every particle moves in a circle whose center lies on the axis of rotation.

To describe the body's orientation, we fix a **reference line** in the body, perpendicular to the rotation axis. The **angular position** θ of this reference line is the angle it makes with a fixed external reference direction (usually the positive x -axis).

For a point on the body at a perpendicular distance r from the axis, the arc length s traveled along its circular path relates to the angular position by:

$$\theta = \frac{s}{r} \text{ (radian measure)} \quad (5.1)$$



From Newton's second law, any acceleration requires a net force. For circular motion, the radial (centripetal) acceleration requires a **centripetal force** directed toward the center:

$$F_c = ma_c = m \frac{v^2}{r} = mr\omega^2 \quad (5.17)$$

Critical Understanding: Centripetal force is **not a new type of force**. It is the name given to any force (or component of a force) that causes circular motion. Depending on the situation, the centripetal force may be provided by:

- **Tension** (a ball swung on a string)
- **Gravity** (planets orbiting the Sun)
- **Friction** (a car turning on a flat road)
- **Normal force** (a car on a banked curve)

Common Applications of Centripetal Force

Case 1: Horizontal Circular Motion (Ball on a String)

A ball of mass m attached to a string of length r is swung in a horizontal circle. The tension T provides the centripetal force:

$$T = mr\omega^2 = \frac{mv^2}{r} \quad (5.18)$$

If the string breaks, the ball flies off **tangentially** — not radially outward — because no force acts to change its direction.

Case 2: Car on a Level (Unbanked) Curve

For a car of mass m turning on a flat curve of radius r , the centripetal force comes from **static friction** between the tires and road:

$$f_s = \frac{mv^2}{r} \quad (5.19)$$

The maximum safe speed occurs when f_s reaches its maximum value $\mu_s mg$:

$$\mu_s mg = \frac{mv_{\max}^2}{r} \Rightarrow v_{\max} = \sqrt{\mu_s gr} \quad (5.20)$$

Case 3: Car on a Banked Curve (Frictionless)

To eliminate reliance on friction, roads are banked at an angle θ . The horizontal component of the normal force provides the centripetal force.

From the free-body diagram:

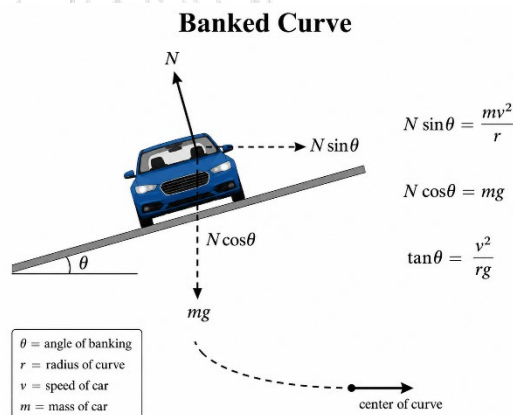
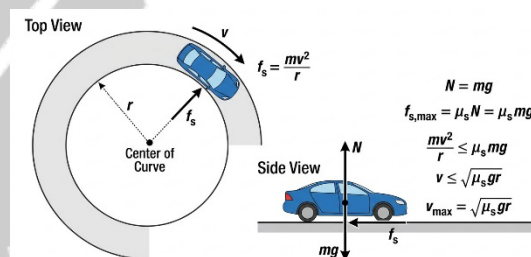
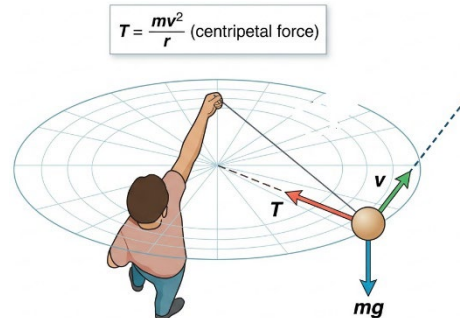
Vertical equilibrium: $N \cos \theta = mg$

Horizontal (centripetal): $N \sin \theta = \frac{mv^2}{r}$

Dividing these equations:

$$\tan \theta = \frac{v^2}{rg} \quad (5.21)$$

For a given speed v and radius r , this equation gives the ideal banking angle at which no friction is required.



Case 4: Conical Pendulum

A mass m moves in a horizontal circle at the end of a string of length L making angle θ with the vertical. The vertical component of tension balances weight, and the horizontal component provides centripetal force:

$$T \cos \theta = mg, T \sin \theta = m\omega^2(L \sin \theta)$$

Solving: $\omega = \sqrt{\frac{g}{L \cos \theta}}$

Rotational Kinetic Energy and Moment of Inertia

Rotational Kinetic Energy

A rotating rigid body consists of many particles, each with its own linear kinetic energy. The total kinetic energy is the sum over all particles:

$$K = \sum \frac{1}{2} m_i v_i^2 = \sum \frac{1}{2} m_i (r_i \omega)^2 = \frac{1}{2} \left(\sum m_i r_i^2 \right) \omega^2$$

The quantity in parentheses is called the **moment of inertia** (or **rotational inertia**) I :

$$I = \sum m_i r_i^2 \text{ (discrete particles)} \quad (5.23)$$

Thus, the rotational kinetic energy is:

$$K_{\text{rot}} = \frac{1}{2} I \omega^2 \quad (5.24)$$

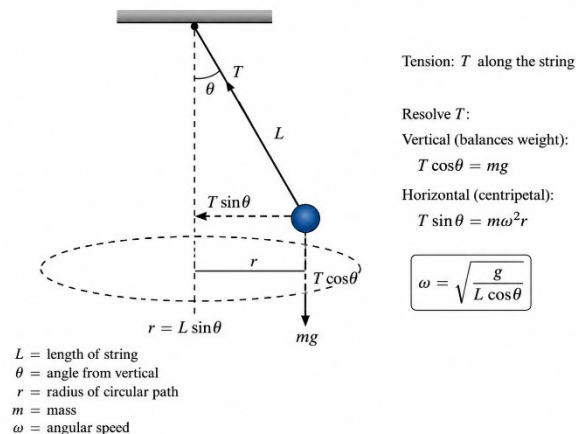
This is the rotational analog of $K_{\text{trans}} = \frac{1}{2} m v^2$ — mass m is replaced by moment of inertia I , and linear speed v by angular speed ω .

Physical Meaning of Moment of Inertia

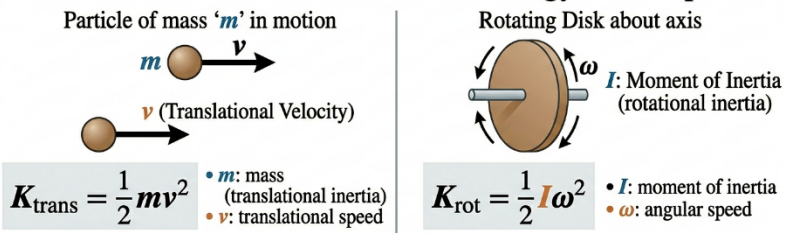
The moment of inertia depends not only on the **mass** of the body but also on how that mass is **distributed** relative to the rotation axis. Mass farther from the axis contributes more to I (because r^2 factor).

Why this matters: A figure skater spins faster by pulling her arms inward. Why? Because pulling arms inward reduces the distance of mass from the rotation axis,

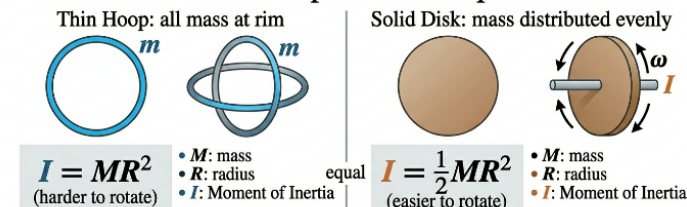
Conical Pendulum



Translational vs. Rotational Kinetic Energy: A Comparison



Mass Distribution Comparison: Hoop vs. Solid Disk



MOMENT OF INERTIA (I) COMPARISON AND CONSEQUENCE

$I = MR^2$ (Hoop) $\leftarrow$ Increased Inertia $\rightarrow$ Hoop mass at far distance $\Rightarrow$ harder to start/stop rotation

$I = \frac{1}{2} MR^2$ (Disk) $\rightarrow$ Decreased Inertia $\leftarrow$ Disk mass distributed closer $\Rightarrow$ easier to start/stop rotation

$\Rightarrow$ Greater ' I ' means greater rotational difficulty. This explains why objects with mass far from the axis are harder to spin.

Condition for Rolling Without Slipping

When a wheel rolls without slipping, the point of contact with the ground is instantaneously at rest. The translational speed of the center of mass and the rotational speed are related by:

$$v_{cm} = R\omega \quad (5.41)$$

where R is the wheel's radius.

Kinetic Energy of Rolling

A rolling object has both translational and rotational kinetic energy:

$$K_{total} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 \quad (5.42)$$

Using $v_{cm} = R\omega$:

$$K_{total} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\left(\frac{v_{cm}}{R}\right)^2 = \frac{1}{2}\left(M + \frac{I_{cm}}{R^2}\right)v_{cm}^2 \quad (5.43)$$

The effective inertia for rolling is larger than the mass M alone, so accelerating a rolling object requires more force than sliding the same object without friction.

Rolling Down an Inclined Plane

Consider a uniform solid cylinder (disk) and a thin hoop, both of mass M and radius R , released from rest at height h on an incline.

Using conservation of energy: $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

With $I = \frac{1}{2}MR^2$ for the solid cylinder and $\omega = v/R$:

$$\begin{aligned} Mgh &= \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 \\ &= \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{3}{4}Mv^2 \end{aligned}$$

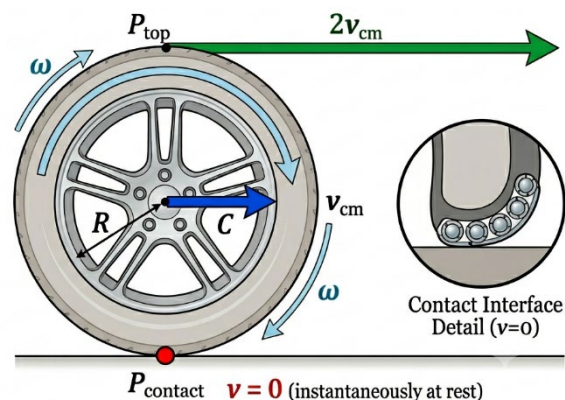
$$v = \sqrt{\frac{4}{3}gh} \quad (5.44)$$

For the hoop ($I = MR^2$):

$$\begin{aligned} Mgh &= \frac{1}{2}Mv^2 + \frac{1}{2}(MR^2)\left(\frac{v}{R}\right)^2 = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 \\ &= Mv^2 \end{aligned}$$

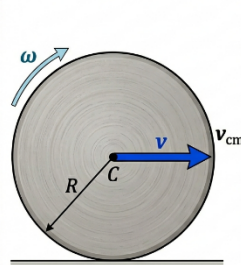
$$v = \sqrt{gh} \quad (5.45)$$

Weightlessness and Artificial Gravity

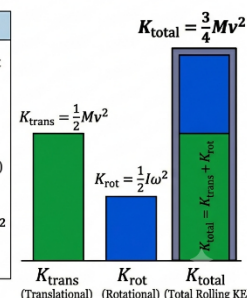


$$\text{Formula: } v_{cm} = R\omega$$

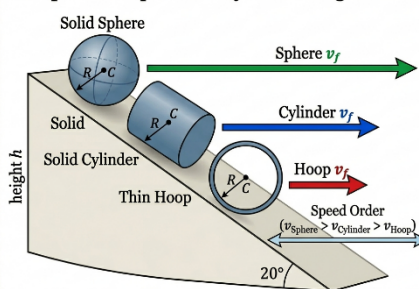
Rolling Kinetic Energy Analysis for a Solid Disk



Formulas
$K_{total} = K_{trans} + K_{rot}$
$K_{trans} = \frac{1}{2}Mv^2$
$K_{rot} = \frac{1}{2}I\omega^2$
$I = \frac{1}{2}MR^2$ (for solid disk)
Combining, K_{total}
$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{4}MR^2\omega^2$
$= \frac{3}{4}Mv^2$



Comparative Speed of Objects Rolling Down an Incline ($v_f = \sqrt{\frac{2gh}{1+\beta}}$)



Formulas
Object (Solid Sphere)
$v_f = \sqrt{\frac{10}{7}gh}$ $I = \frac{2}{5}MR^2$ ($\beta = 0.4$)
Speed (Fastest)
Object (Solid Cylinder)
$v_f = \sqrt{\frac{4}{3}gh}$ $I = \frac{1}{2}MR^2$ ($\beta = 0.5$)
Speed (Middle)
Object (Thin Hoop)
$v_f = \sqrt{gh}$ $I = MR^2$ ($\beta = 1.0$)
Speed (Slowest)
General Form: $v_f = \sqrt{\frac{2gh}{1+\beta}}$ where $I = \beta MR^2$

Diagram sequence after TextbookCL.com style

Weightlessness in Free Fall

Weightlessness does not mean the absence of gravity. Astronauts in orbit are still within Earth's gravitational field — at the International Space Station's altitude (~400 km), gravity is about 90% as strong as on Earth's surface. Weightlessness occurs when an object is in **free fall** — the only force acting is gravity, and there is no contact (normal) force. The sensation of weight comes from contact forces (e.g., the floor pushing up on your feet). In free fall, these contact forces vanish.

Why don't orbiting satellites fall to Earth?

They **are** falling! But their forward (tangential) velocity is so large that Earth's surface curves away beneath them at the same rate they fall. This balance produces a circular (or elliptical) orbit. The orbital speed for a circular orbit at radius r from Earth's center is derived from $F_g = F_c$:

$$\frac{GM_E m}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM_E}{r}} \quad (5.46)$$

For low Earth orbit ($r \approx R_E + 200$ km), $v \approx 7.8$ km/s (about 28,000 km/h).

Artificial Gravity

Weightlessness causes health problems: muscle atrophy, bone density loss, fluid shifts. To counteract this, future long-duration space stations may generate **artificial gravity** by rotation.

Consider a rotating space station shaped like a large wheel. The floor is at the rim. The station rotates so that the floor pushes inward on

astronauts' feet, providing the centripetal force needed for circular motion. By Newton's third law, astronauts' feet push outward on the floor — this contact force feels like weight.

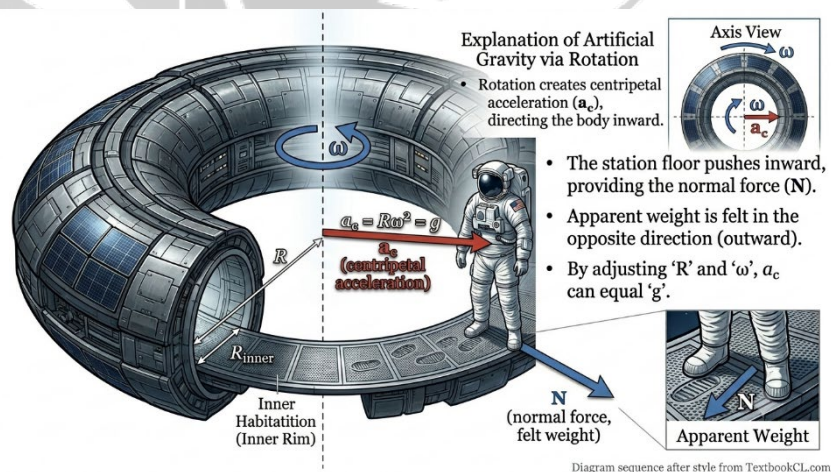
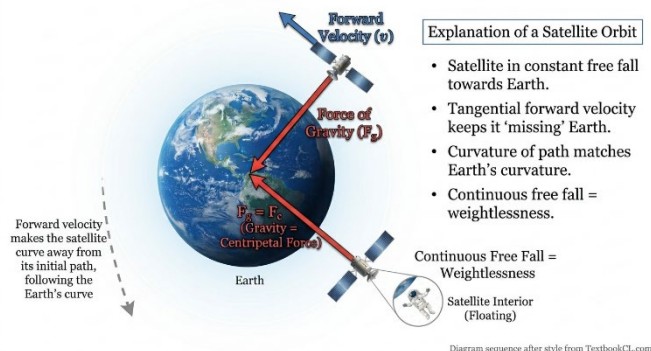
To simulate Earth's gravity ($g = 9.8$ m/s²):

$$a_c = \frac{v^2}{r} = r\omega^2 = g \quad (5.47)$$

The required angular speed is:

$$\omega = \sqrt{\frac{g}{r}} \quad (5.48)$$

The period of rotation:



Applications of geostationary satellites:

- Communication (TV broadcasting, telephone, internet)
- Weather monitoring
- Military surveillance

Three geostationary satellites spaced 120° apart can provide coverage of all inhabited regions of Earth.

Summary of Key Equations

Concept	Equation
Angular displacement	$\Delta\theta = \theta_2 - \theta_1$
Angular velocity (average)	$\omega_{\text{avg}} = \Delta\theta/\Delta t$
Angular velocity (instantaneous)	$\omega = d\theta/dt$
Angular acceleration (average)	$\alpha_{\text{avg}} = \Delta\omega/\Delta t$
Angular acceleration (instantaneous)	$\alpha = d\omega/dt = d^2\theta/dt^2$
Radian relation	$s = r\theta$
Tangential speed	$v = r\omega$
Tangential acceleration	$a_t = r\alpha$
Centripetal (radial) acceleration	$a_r = v^2/r = r\omega^2$
Period	$T = 2\pi/\omega = 2\pi r/v$
Centripetal force	$F_c = mv^2/r = mr\omega^2$
Banking angle (frictionless)	$\tan\theta = v^2/(rg)$
Moment of inertia (point mass)	$I = mr^2$
Moment of inertia (continuous)	$I = \int r^2 dm$
Parallel-axis theorem	$I = I_{\text{com}} + Mh^2$
Rotational kinetic energy	$K_{\text{rot}} = \frac{1}{2}I\omega^2$
Torque (definition)	$\tau = rF\sin\phi = rF_\perp = r_\perp F$
Newton's second law (rotation)	$\tau_{\text{net}} = I\alpha$
Rotational work	$W = \int \tau d\theta$
Rotational power	$P = \tau\omega$
Angular momentum (particle)	$\vec{L} = \vec{r} \times \vec{p}$
Angular momentum (rigid body)	$L = I\omega$
Torque-angular momentum relation	$\tau_{\text{net}} = dL/dt$
Conservation of angular momentum	$I_i\omega_i = I_f\omega_f$ (if $\tau_{\text{ext}} = 0$)
Rolling without slipping	$v_{\text{cm}} = R\omega$
Orbital speed	$v = \sqrt{GM_E/r}$
Artificial gravity	$\omega = \sqrt{g/r}$

5. Rotational and Circular Motion

Rotational and Circular Motion: One-Liners

Introduction to Rotational Motion

1. Rotational motion is a fundamental type of motion where an object rotates about an axis, with examples including spinning wheels, ceiling fans, and planetary rotation.
2. A rigid body is one that can rotate with all its parts locked together without any change in shape.
3. Every particle in a rotating rigid body moves in a circle whose center lies on the axis of rotation.
4. The angular position θ is the angle a reference line in the body makes with a fixed external reference direction.
5. Arc length relates to angular position by $s = r\theta$ where θ is in radians.
6. One complete revolution equals $360^\circ = 2\pi$ radians.
7. One radian equals approximately 57.3° .
8. Angular displacement is $\Delta\theta = \theta_2 - \theta_1$.
9. Counterclockwise rotation is positive; clockwise rotation is negative.
10. Average angular velocity is $\omega_{\text{avg}} = \Delta\theta/\Delta t$.
11. Instantaneous angular velocity is $\omega = d\theta/dt$.
12. Angular velocity has SI units of rad/s; direction is given by the right-hand rule.
13. Average angular acceleration is $\alpha_{\text{avg}} = \Delta\omega/\Delta t$.
14. Instantaneous angular acceleration is $\alpha = d\omega/dt = d^2\theta/dt^2$.
15. Angular acceleration has SI units of rad/s^2 .
16. When ω and α have the same sign, rotation speeds up.
17. When ω and α have opposite signs, rotation slows down.

Kinematic Equations for Constant Angular Acceleration

18. First rotational kinematic equation: $\omega = \omega_0 + \alpha t$.
19. Second rotational kinematic equation: $\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$.
20. Third rotational kinematic equation: $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$.
21. Fourth rotational kinematic equation: $\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$.
22. Fifth rotational kinematic equation: $\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$.

Relating Linear and Angular Variables

23. For a point at distance r from the axis, arc length is $s = r\theta$.
24. Tangential (linear) speed is $v = r\omega$, tangent to the circular path.
25. Tangential acceleration is $a_t = r\alpha$, caused by changes in speed magnitude.
26. Radial (centripetal) acceleration is $a_r = v^2/r = r\omega^2$, directed toward the center.
27. Total linear acceleration is $\vec{a} = \vec{a}_t + \vec{a}_r$.
28. Magnitude of total acceleration is $a = \sqrt{a_t^2 + a_r^2}$.
29. Period of uniform circular motion is $T = 2\pi r/v = 2\pi/\omega$.
30. Frequency is $f = 1/T = \omega/2\pi$, with SI unit hertz (Hz).

Centripetal Force

31. Centripetal force is $F_c = ma_c = mv^2/r = mr\omega^2$, directed toward the center.
32. Centripetal force is not a new fundamental force but the name given to any force causing circular motion.

D) A parabola

Correct Answer: C

3. Angular position θ is measured in:

A) Degrees only

B) Radians

C) Revolutions

D) Both B and C are valid units

Correct Answer: B (Radians are the SI unit)

4. One complete revolution equals:

A) $180^\circ = \pi$ rad

B) $360^\circ = 2\pi$ rad

C) $90^\circ = \pi/2$ rad

D) $270^\circ = 3\pi/2$ rad

Correct Answer: B

5. One radian is approximately:

A) 57.3°

B) 45°

C) 60°

D) 90°

Correct Answer: A

6. Angular displacement is:

A) $\Delta\theta = \theta_2 + \theta_1$

B) $\Delta\theta = \theta_2 - \theta_1$

C) $\Delta\theta = \theta_2/\theta_1$

D) $\Delta\theta = \theta_1 - \theta_2$

Correct Answer: B

7. Counterclockwise rotation has:

A) Positive angular displacement

B) Negative angular displacement

C) Zero angular displacement

D) Undefined angular displacement

Correct Answer: A

8. Clockwise rotation has:

A) Positive angular displacement

B) Negative angular displacement

C) Zero angular displacement

D) Undefined angular displacement

Correct Answer: B

9. Average angular velocity is:

A) $\omega_{\text{avg}} = \Delta\theta \times \Delta t$

B) $\omega_{\text{avg}} = \Delta\theta/\Delta t$

C) $\omega_{\text{avg}} = \Delta t/\Delta\theta$

D) $\omega_{\text{avg}} = \Delta\theta + \Delta t$

Correct Answer: B

10. Instantaneous angular velocity is:

A) $\omega = \Delta\theta/\Delta t$

B) $\omega = d\theta/dt$

C) $\omega = \int \theta dt$

D) $\omega = \theta/t$

Correct Answer: B

11. The SI unit of angular velocity is:

A) m/s

B) rad/s

C) rev/s

D) m/s^2

Correct Answer: B

12. Average angular acceleration is:

A) $\alpha_{\text{avg}} = \Delta\omega/\Delta t$

B) $\alpha_{\text{avg}} = \Delta\theta/\Delta t$

C) $\alpha_{\text{avg}} = \Delta t/\Delta\omega$

D) $\alpha_{\text{avg}} = \omega t$

Correct Answer: A

13. Instantaneous angular acceleration is:

A) $\alpha = d\omega/dt$

B) $\alpha = d\theta/dt$

C) $\alpha = \Delta\omega/\Delta t$

D) $\alpha = \omega t$

Correct Answer: A

14. Angular acceleration has SI units of:

A) rad/s

B) rad/s^2

C) m/s^2

D) $\text{rad}\cdot\text{s}$

Correct Answer: B

15. When ω and α have the same sign:

A) Rotation slows down

B) Rotation speeds up

C) Rotation stops

D) Direction reverses

Correct Answer: B

16. When ω and α have opposite signs:

A) Rotation speeds up

B) Rotation slows down

C) Rotation stops

Chapter 6: Fluid Mechanics

Introduction

Fluids—substances that flow and conform to the boundaries of their containers—surround us in every aspect of daily life. The air we breathe, the water we drink, the blood coursing through our veins, and the fuel powering our vehicles are all fluids. Understanding their behavior is fundamental not only to physics but also to engineering, medicine, meteorology, oceanography, and countless other fields.

Fluid mechanics is traditionally divided into two broad branches: **fluid statics** (the study of fluids at rest) and **fluid dynamics** (the study of fluids in motion). Although these branches address different physical situations, they are united by common principles: the conservation of mass, the conservation of energy, and Newton's laws of motion.

In this chapter, we shall develop a complete understanding of fluid behavior, beginning with the fundamental concepts of density and pressure,

proceeding through the principles of buoyancy and Archimedes' discovery, and culminating in the elegant relationships that govern fluid flow—the continuity equation and Bernoulli's equation. Throughout, we shall emphasize the physical interpretation of every equation and illustrate each concept with real-world applications.

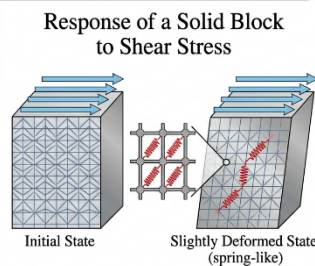
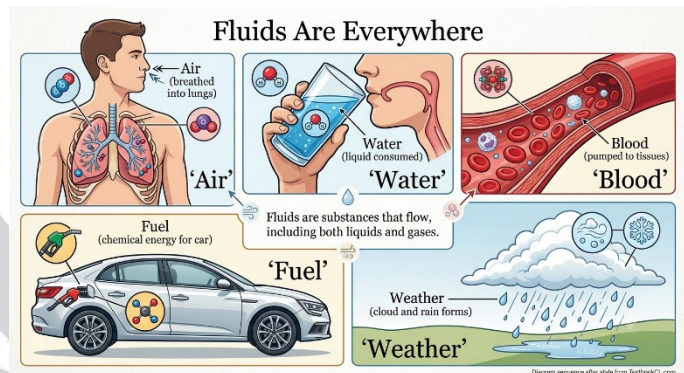
What Is a Fluid?

Before we can analyze fluid behavior, we must understand what distinguishes a fluid from a solid. A **fluid** is a substance that can flow; it conforms to the boundaries of any container in which we place it. This behavior arises because a fluid cannot sustain a shearing stress—a force tangential to its surface. In more formal language, a fluid cannot withstand a shearing stress; it can, however, exert a force perpendicular to its surface.

Both liquids and gases are fluids, but they differ in an important respect. A **liquid** has cohesion—the molecules are close together and exert attractive forces on one another. Consequently, a liquid maintains a constant volume as it flows. A **gas**, by contrast, has negligible cohesion; its molecules are widely separated, and a gas expands to fill any container. Despite this difference, both liquids and gases obey the same fundamental laws of fluid mechanics under appropriate conditions.

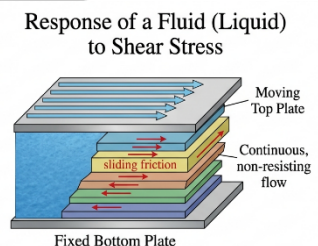
Density

When analyzing rigid bodies, we are concerned with particular lumps of matter—a block of wood, a baseball, a metal rod. With fluids, we are more interested in the extended substance and in properties that can vary from point to point. It is more useful to speak of **density** and **pressure** than of mass and force.



Key Characteristic: Internal structures stretch like springs. Resists deformation.

Solid — resists shear stress



Key Characteristic: Internal layers slide past each other continuously. Continuous flow.

Fluid — flows under shear stress

Diagram sequence after style from TextbookCL.com

This equation tells us that as y increases (moving upward), pressure decreases—exactly as we expect. For a fluid of uniform density ρ and constant g , we can integrate Eq. (8.5) between two points:

$$\int_{p_1}^{p_2} dp = -\rho g \int_{y_1}^{y_2} dy$$

$$p_2 - p_1 = -\rho g(y_2 - y_1) \quad (8.6)$$

or equivalently:

$$p_2 = p_1 + \rho g(y_1 - y_2) \quad (8.7)$$

Pressure at a Depth

It is often more convenient to express pressure in terms of depth below the surface. Let point 1 be at the surface of the liquid, where the pressure is p_0 (usually atmospheric pressure), and let point 2 be at a depth h below the surface. Taking $y_1 = 0$ and $y_2 = -h$, Eq. (8.7) becomes:

$$p = p_0 + \rho gh \quad (8.8)$$

This fundamental result states that **the pressure at a depth h below the surface of a liquid is greater than the pressure at the surface by an amount ρgh .**

Several important points deserve emphasis:

- The pressure depends only on depth, not on the shape of the container or the horizontal dimensions.
- The pressure is the same at all points at the same depth, regardless of the container's shape
- Equation (8.8) gives the **absolute pressure**. The **gauge pressure** is ρgh .

For a gas, the density is not uniform over large vertical distances. However, over short distances (like the height of a room), the variation is negligible. For example, the pressure difference between floor and ceiling in a 3.0 m tall room filled with air of density 1.2 kg/m^3 is only about 35 Pa—roughly 0.00035 atm.

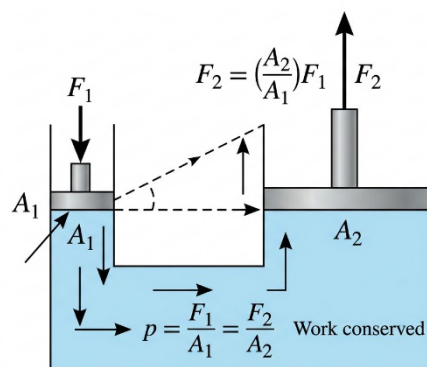
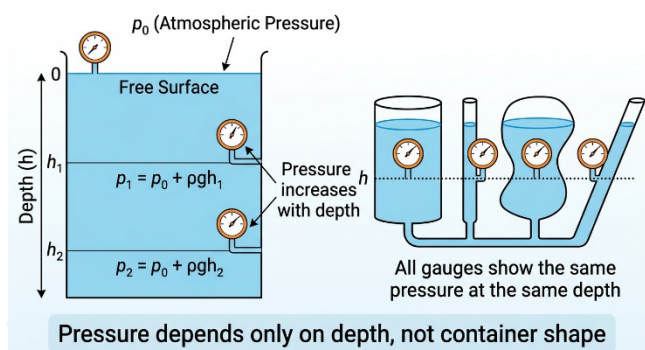
Pascal's Principle

Equation (8.8) reveals an important consequence: if we increase the pressure p_0 at the surface by some amount Δp , the pressure at every depth increases by exactly the same amount. This observation is known as **Pascal's principle** (named for Blaise Pascal, 1623–1662):

A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel.

Pascal's principle is the basis for hydraulic systems. Consider a hydraulic lift, consisting of two pistons of different cross-sectional areas A_1 and A_2 connected by a fluid-filled chamber. An applied force F_1 on the smaller piston creates a pressure $p = F_1/A_1$. This pressure is transmitted undiminished to the larger piston, producing an output force:

$$F_2 = pA_2 = \frac{A_2}{A_1} F_1 \quad (8.9)$$



A wooden block of density 800 kg/m^3 and height 6.0 cm floats in a fluid of density 1200 kg/m^3 . Find the submerged depth.

Solution: From Eq. (8.14):

$$\frac{h_{\text{submerged}}}{H} = \frac{\rho_{\text{block}}}{\rho_f} = \frac{800}{1200} = \frac{2}{3}$$

$$h_{\text{submerged}} = \frac{2}{3} \times 6.0 \text{ cm} = 4.0 \text{ cm}$$

Applications of Buoyancy

Ships: A ship's hull is shaped to displace a volume of water equal to the ship's weight. The hull is designed to maximize displacement while minimizing weight, using strong but lightweight materials.

Submarines: A submarine can control its buoyancy by taking water into or expelling water from ballast tanks. When tanks are filled, the submarine's weight increases and it sinks; when tanks are emptied, it rises.

Hydrometer: This instrument measures the density of liquids. It floats vertically; the depth to which it sinks indicates the liquid's density—the higher the density, the less it sinks.

Hot-air balloons: The balloon rises because the heated air inside has lower density than the surrounding cool air. The buoyant force equals the weight of displaced cool air; when this exceeds the total weight of the balloon (including the hot air and payload), the balloon rises.

Fluid Dynamics: Ideal Fluids

The motion of real fluids is extraordinarily complex and not yet fully understood. However, we can analyze many important situations by considering an **ideal fluid**—a simplified model with four key properties:

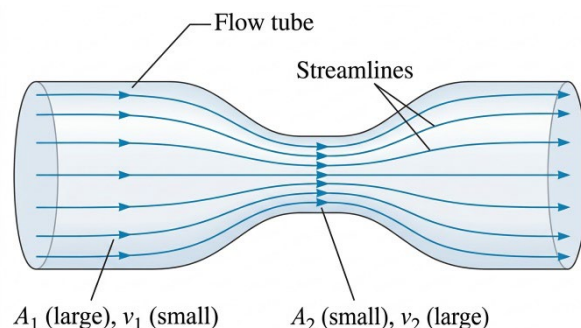
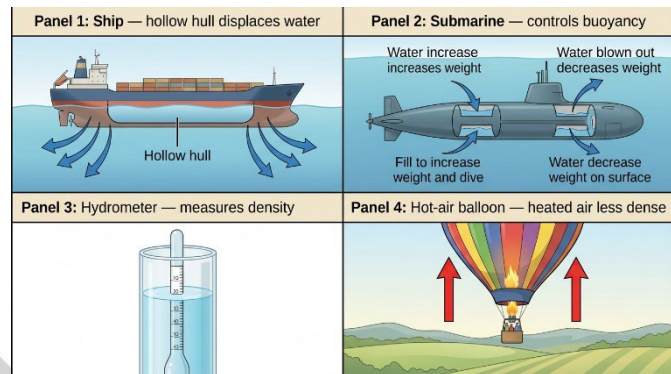
1. **Steady flow** (laminar flow): The velocity at any fixed point does not change with time.
2. **Incompressible flow:** The density is constant and uniform.
3. **Nonviscous flow:** The fluid has no internal friction (no viscosity).
4. **Irrotational flow:** A tiny fluid element does not rotate about its own center of mass.

While no real fluid is perfectly ideal, many fluids (like water in many situations) approximate ideal behavior well enough for practical calculations.

Streamlines and Flow Tubes

A **streamline** is the path followed by an individual fluid particle. At any point, the velocity vector is tangent to the streamline. Streamlines never cross—if they did, a particle at the intersection would have two velocities simultaneously.

A **flow tube** (or tube of flow) is a bundle of streamlines. Because streamlines cannot cross, no fluid can flow across the walls of a flow tube.



$$\Delta K = \frac{1}{2}\Delta mv_2^2 - \frac{1}{2}\Delta mv_1^2 = \frac{1}{2}\rho\Delta V(v_2^2 - v_1^2)$$

The change in gravitational potential energy is:

$$\Delta U = \Delta mgy_2 - \Delta mgy_1 = \rho\Delta Vg(y_2 - y_1)$$

The work–energy theorem ($W = \Delta K + \Delta U$) gives:

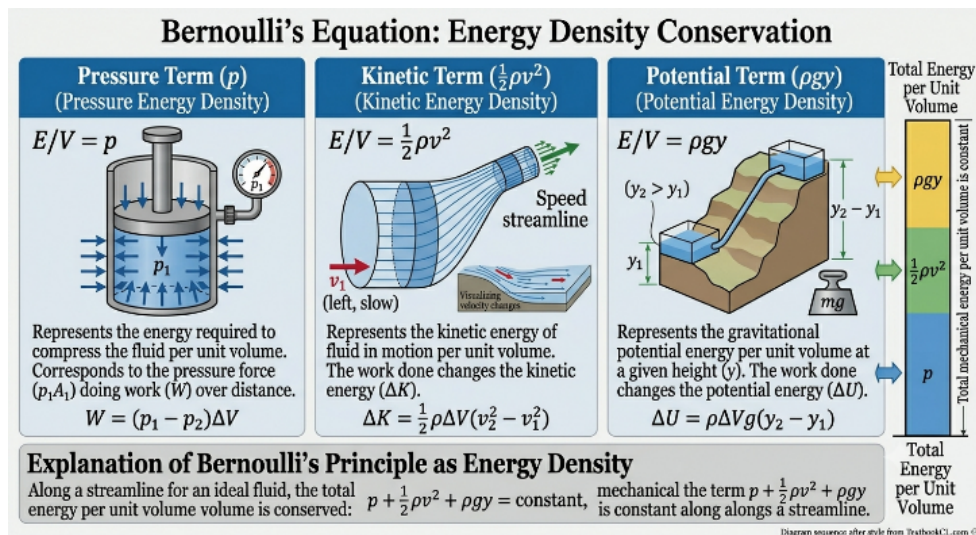
$$(p_1 - p_2)\Delta V = \frac{1}{2}\rho\Delta V(v_2^2 - v_1^2) + \rho\Delta Vg(y_2 - y_1)$$

Dividing by ΔV and rearranging:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho gy_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho gy_2 \quad (8.21)$$

This is **Bernoulli's equation**. Since points 1 and 2 are arbitrary, we can write:

$$p + \frac{1}{2}\rho v^2 + \rho gy = \text{constant} \quad (8.22)$$



Physical Interpretation of Bernoulli's Equation

Each term in Bernoulli's equation has units of pressure (Pa), but we can also interpret them as **energy densities** (energy per unit volume):

- p : Pressure energy density (work done by pressure forces)
- $\frac{1}{2}\rho v^2$: Kinetic energy density
- ρgy : Gravitational potential energy density

Thus, Bernoulli's equation states that the **total mechanical energy per unit volume** is constant along a streamline for an ideal fluid.

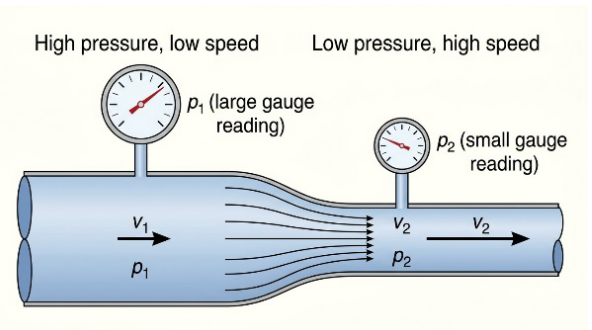
Special Cases

Case 1: Fluid at rest ($v_1 = v_2 = 0$)

Bernoulli's equation reduces to:

$$p_1 + \rho gy_1 = p_2 + \rho gy_2 \Rightarrow p_2 = p_1 + \rho g(y_1 - y_2)$$

which matches our hydrostatic pressure formula, Eq. (8.7).



For laminar flow of a viscous fluid through a cylindrical pipe of radius R and length L , the volume flow rate is given by **Poiseuille's law**:

$$R_V = \frac{\pi R^4 (p_1 - p_2)}{8\eta L} \quad (8.28)$$

The crucial feature is the R^4 dependence: reducing the pipe radius by half reduces the flow rate by a factor of 16 unless the pressure difference increases dramatically. This explains why cholesterol buildup in arteries (narrowing them) can dangerously elevate blood pressure—even a 10% reduction in radius requires a 52% increase in pressure to maintain the same flow.

Superfluidity

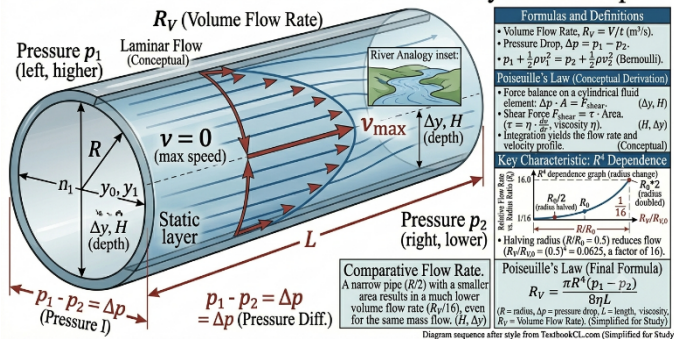
At temperatures near absolute zero, certain liquids (notably liquid helium-4 below 2.17 K) enter a state of **superfluidity** in which they have **zero viscosity**. Superfluids exhibit remarkable behavior:

- They flow without any resistance, even through microscopic channels.
- They creep up and over the walls of their containers, seemingly defying gravity.
- When stirred, they form quantized vortices that spin indefinitely.
- They can be used in precision instruments like gyroscopes to detect gravitational effects.

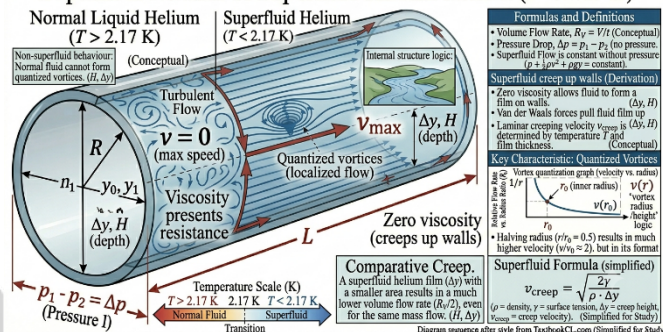
Equations Summary

Equation	Name	When to Use
$\rho = m/V$	Density definition	Always
$p = F/A$	Pressure definition	Uniform pressure on flat surface
$p = p_0 + \rho gh$	Hydrostatic pressure	Fluid at rest, uniform density
$F_2 = (A_2/A_1)F_1$	Hydraulic lift	Pascal's principle, enclosed fluid
$F_b = \rho_f V_{\text{sub}} g$	Archimedes' principle	Object partially/completely submerged
$A_1 v_1 = A_2 v_2$	Continuity equation	Incompressible steady flow
$p + \frac{1}{2} \rho v^2 + \rho gy = \text{constant}$	Bernoulli's equation	Ideal fluid, steady flow
$v = \sqrt{2gh}$	Torricelli's theorem	Efflux from hole in tank
$F_d = 6\pi\eta r v$	Stokes' law	Small sphere in viscous fluid
$v_t = \frac{2r^2(\rho - \rho_f)g}{9\eta}$	Terminal velocity	Sphere falling in viscous fluid

Poiseuille's Law and Laminar Flow in a Cylindrical Pipe



Superfluid Letatuior of Superfluid Helium Helium ($T < 2.17$ K)





Fluid Mechanics: ONE-LINERS

Introduction to Fluids

1. A fluid is a substance that can flow and conforms to the boundaries of any container in which it is placed.
2. A fluid cannot sustain a shearing stress (a force tangential to its surface); it can only exert a force perpendicular to its surface.
3. Both liquids and gases are fluids, but liquids maintain constant volume while gases expand to fill any container.
4. Fluid mechanics is divided into fluid statics (fluids at rest) and fluid dynamics (fluids in motion).
5. The behavior of fluids is governed by the conservation of mass, conservation of energy, and Newton's laws of motion.

Density

6. Density is defined as mass per unit volume: $\rho = m/V$.
7. The SI unit of density is kilogram per cubic meter (kg/m^3).
8. One gram per cubic centimeter equals 1000 kg/m^3 : $1 \text{ g/cm}^3 = 1000 \text{ kg/m}^3$.
9. Specific gravity (relative density) is the ratio of a material's density to the density of water at 4.0°C (1000 kg/m^3).
10. Specific gravity is a pure number without units.
11. Water at 20°C has density approximately 998 kg/m^3 ; seawater has density approximately 1030 kg/m^3 .
12. Mercury has density $13.6 \times 10^3 \text{ kg/m}^3$, about 13.6 times denser than water.
13. Air at 20°C and 1 atm has density approximately 1.20 kg/m^3 .

Pressure

14. Pressure is defined as force per unit area: $p = F/A$.
15. At a given point in a fluid at rest, pressure has the same value regardless of the orientation of the surface—pressure is a scalar quantity.
16. The SI unit of pressure is the pascal (Pa): $1 \text{ Pa} = 1 \text{ N/m}^2$.
17. One atmosphere (atm) equals $1.013 \times 10^5 \text{ Pa} = 760 \text{ torr} = 14.7 \text{ lb/in}^2$.
18. Gauge pressure is the difference between absolute pressure and atmospheric pressure.
19. Absolute pressure = gauge pressure + atmospheric pressure.

Hydrostatic Pressure

20. In a fluid at rest, pressure increases with depth: $dp/dy = -\rho g$, where y is measured upward.
21. Pressure at depth h below the surface: $p = p_0 + \rho gh$, where p_0 is surface pressure.
22. Pressure depends only on depth, not on the shape of the container or horizontal dimensions.
23. Pressure is the same at all points at the same depth, regardless of container shape.
24. For a gas over short vertical distances, pressure variation is negligible.
25. Pascal's principle: A change in pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of the container.
26. Hydraulic lift: $F_2 = (A_2/A_1)F_1$; force is multiplied by the ratio of areas.
27. Work is conserved in hydraulic systems: $F_1 d_1 = F_2 d_2$.
28. A mercury barometer measures atmospheric pressure: $p_{\text{atm}} = \rho gh$.

5. The SI unit of density is:

- A) kg/m
- B) kg/m³
- C) g/cm³
- D) kg·m³

Correct Answer: B

6. 1 g/cm³ equals:

- A) 1 kg/m³
- B) 10 kg/m³
- C) 100 kg/m³
- D) 1000 kg/m³

Correct Answer: D

7. Specific gravity is the ratio of a material's density to:

- A) The density of air at STP
- B) The density of water at 4°C
- C) The density of mercury
- D) The density of oil

Correct Answer: B

8. Specific gravity is:

- A) A unitless number
- B) Measured in kg/m³
- C) Measured in N/m³
- D) Always greater than 1

Correct Answer: A

9. Pressure is defined as:

- A) Force per unit volume
- B) Force per unit area
- C) Mass per unit volume
- D) Force times area

Correct Answer: B

10. Pressure at a point in a fluid at rest is:

- A) Different in different directions
- B) The same in all directions
- C) Zero
- D) Maximum horizontally

Correct Answer: B

11. Pressure is a:

- A) Vector quantity
- B) Scalar quantity
- C) Tensor quantity

D) Complex quantity

Correct Answer: B

12. The SI unit of pressure is the:

- A) Newton
- B) Joule
- C) Pascal
- D) Watt

Correct Answer: C

13. One pascal equals:

- A) 1 N·m
- B) 1 N/m
- C) 1 N/m²
- D) 1 kg/m³

Correct Answer: C

14. One atmosphere equals approximately:

- A) 1.013×10^3 Pa
- B) 1.013×10^5 Pa
- C) 1.013×10^7 Pa
- D) 1.013×10^9 Pa

Correct Answer: B

15. One atmosphere equals:

- A) 76 torr
- B) 760 torr
- C) 7.6 torr
- D) 0.76 torr

Correct Answer: B

16. Gauge pressure is:

- A) Absolute pressure
- B) Absolute pressure minus atmospheric pressure
- C) Atmospheric pressure minus absolute pressure
- D) Pressure at depth

Correct Answer: B

17. Hydrostatic pressure at depth h is given by:

- A) $p = p_0 + \rho gh$
- B) $p = p_0 - \rho gh$
- C) $p = \rho gh$
- D) $p = p_0 + \rho gh^2$

Correct Answer: A



Part II: Waves and Optics



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Chapter 7: Oscillations And Waves

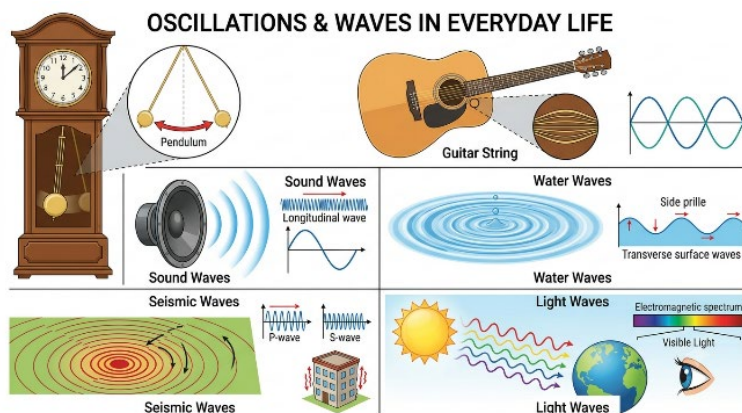
Why Study Oscillations and Waves?

The physical world is filled with motion that repeats itself—from the swinging of a pendulum and the vibration of a guitar string to the propagation of sound from a speaker and the seismic waves that travel through Earth's interior.

Understanding these phenomena requires mastering two fundamental concepts: **oscillations** (motion that repeats in time)

and **waves** (disturbances that travel through space carrying energy).

The importance of waves and oscillations extends far beyond the physics classroom:



Field	Application
Medicine	Ultrasound imaging uses high-frequency sound waves to create images of internal organs
Engineering	Buildings are designed with natural frequencies that avoid destructive resonance during earthquakes
Music	String and wind instruments produce sound through standing wave resonances
Geology	Seismic waves reveal Earth's internal structure and help locate earthquakes
Acoustics	Concert hall design optimizes sound wave reflection and absorption
Astronomy	Redshift of light from distant galaxies reveals the expansion of the universe

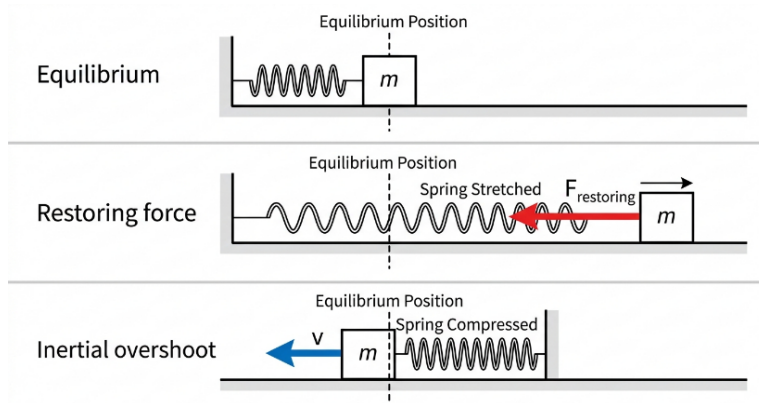
SIMPLE HARMONIC MOTION (SHM)

Fundamental Concepts of Oscillatory Motion

Oscillatory motion (also called **vibratory motion**) is the repeated back-and-forth movement of a body about a fixed central position called the **equilibrium position** or **mean position**.

Key Characteristics of Oscillatory Motion

- Periodicity:** The motion repeats itself after a fixed interval of time.
- Restoring tendency:** There exists a force that always pushes the object back toward the equilibrium position.
- Inertial overshoot:** Due to inertia, the object passes through the equilibrium position and continues to the opposite side.



The Simple Pendulum

The simple pendulum is another classic example of SHM, but with an important approximation: the motion is *approximately* SHM only for small angles (typically less than 10° or about 0.17 radians).

Physical Setup

An ideal simple pendulum consists of:

- A **point mass** m (the "bob")
- Suspended by a **massless, inextensible** string of length L
- From a **fixed frictionless** support

Force Analysis

When the pendulum bob is displaced to an angle θ , the restoring force is the tangential component:

$$F_{\text{rest}} = -mg \sin \theta$$

The Small-Angle Approximation

For small angles (in radians), $\sin \theta \approx \theta$. Also, for small angles, the arc length x (displacement) is related to θ by:

$$\theta = \frac{x}{L}$$

Therefore:

$$F_{\text{rest}} = -mg \frac{x}{L} = -\left(\frac{mg}{L}\right)x$$

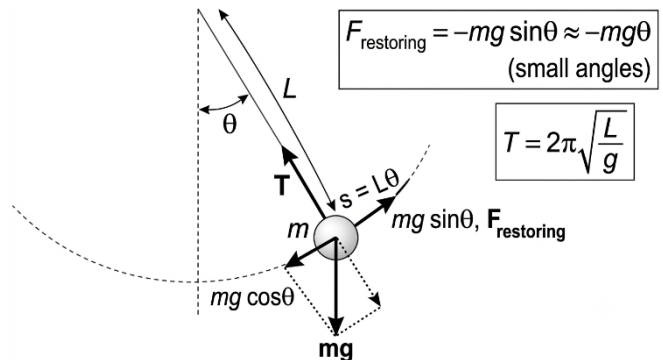
This is of the form $F = -kx$ with $k = mg/L$, confirming SHM for small angles.

Time Period of Simple Pendulum

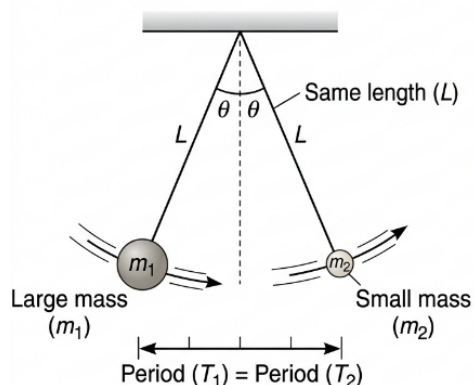
From Newton's second law: $ma = -mg \frac{x}{L} \Rightarrow a = -\frac{g}{L}x$

Comparing with $a = -\omega^2 x$, we get $\omega = \sqrt{g/L}$, therefore:

$$T = 2\pi \sqrt{\frac{L}{g}} \text{ and } f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

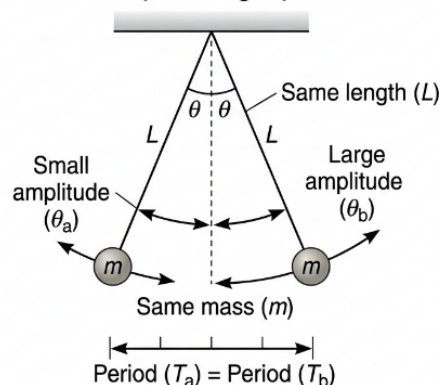


1 Period independent of mass



Two pendulums same length, different bob masses, both swinging with same period.

2 Period independent of amplitude (small angles)



Two pendulums same length, different amplitudes, both swinging with same period.

When an external periodic force is applied to an oscillatory system, the system oscillates at the **driving frequency** rather than its natural frequency. These are called **forced oscillations**.

Examples: A child being pushed on a swing, the pendulum of a clock driven by a motor, a guitar string vibrated by a speaker.

Resonance

Resonance is the phenomenon in which the amplitude of oscillation of a driven system becomes maximum when the driving frequency equals the natural frequency of the system.

Conditions for Resonance

$$\omega_d = \omega_0 \text{ (Resonance condition)}$$

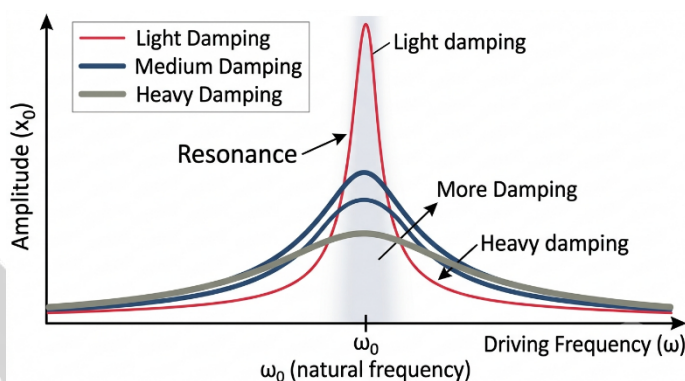
At resonance:

- Energy transfer from the driver to the system is maximized
- Amplitude reaches its maximum possible value
- The system absorbs maximum energy

The Resonance Curve

The sharpness of the resonance peak depends on the amount of damping:

- **Light damping:** Sharp, high peak (narrow bandwidth)
- **Heavy damping:** Broad, low peak (wide bandwidth)



Applications of Resonance

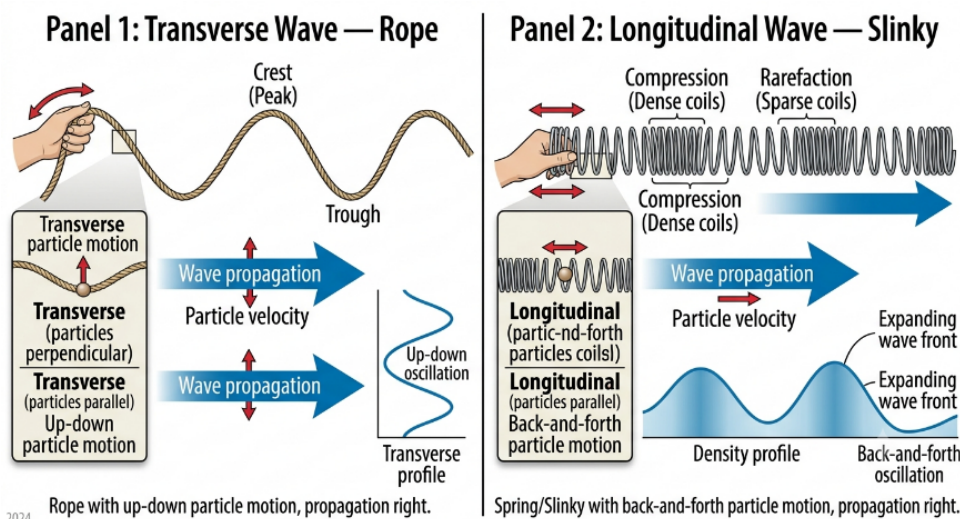
Application	How Resonance is Used
Radio Tuning	Changing the natural frequency of the receiver's electrical circuit to match a station's transmission frequency
Microwave Ovens	Microwaves are tuned to the natural frequency of water molecules, causing them to vibrate and heat food
Guitar Amplification	The hollow wooden box vibrates at the string's frequency, amplifying the sound
MRI (Magnetic Resonance Imaging)	Radio waves cause atomic nuclei to oscillate at their resonant frequency, producing diagnostic images
Ultrasound Imaging	High-frequency sound waves create images of internal organs

Resonance to be Avoided

Situation	Why Resonance is Dangerous
Soldiers Marching on Bridges	Rhythmic steps could match the bridge's natural frequency, causing catastrophic oscillations
Aircraft Wings	Wind gusts at the wing's natural frequency could cause destructive vibrations
Earthquake Engineering	Buildings must be designed with natural frequencies that avoid earthquake frequencies
Rotating Machinery	Imbalanced rotors can excite resonant vibrations in supports

Historical Note: The collapse of the Tacoma Narrows Bridge in 1940 is a famous example of resonance. Strong winds drove oscillations of the bridge deck at its natural frequency, increasing amplitude until the bridge broke apart.

- Described by quantum mechanics (de Broglie waves)



Based on Particle Vibration Direction

Transverse Waves

In transverse waves, the particles of the medium vibrate **perpendicular (at right angles)** to the direction of wave propagation.

Feature	Description
Vibration direction	Up-down or side-to-side
Propagation direction	Horizontal (typically)
Medium requirement	Requires shear strength (solids, surfaces of liquids)
Examples	Waves on a string, electromagnetic waves, water surface waves, seismic S-waves

Longitudinal Waves

In longitudinal waves, the particles of the medium vibrate **parallel (along)** to the direction of wave propagation.

Feature	Description
Vibration direction	Back-and-forth along propagation direction
Regions formed	Compressions (high density/pressure) and rarefactions (low density/pressure)
Medium requirement	Can travel through solids, liquids, gases
Examples	Sound waves, seismic P-waves

Comparison: Transverse vs. Longitudinal Waves

Property	Transverse Waves	Longitudinal Waves
Vibration direction	Perpendicular to propagation	Parallel to propagation
Regions	Crests (peaks) and troughs	Compressions and rarefactions
Can travel through gases?	No (gases lack shear strength)	Yes
Can travel through liquids?	Surface waves only	Yes
Can travel through solids?	Yes	Yes
Examples	Light, radio, waves on string	Sound, seismic P-waves

Seismic Waves: P-waves and S-waves

Earthquakes generate two main types of seismic waves that provide crucial information about Earth's interior:

Wave Type	Type	Speed	Can Travel Through	Particle Motion
P-waves (Primary)	Longitudinal	Fastest (~5-8 km/s)	Solids and liquids	Push-pull (parallel)
S-waves (Secondary)	Transverse	Slower (~3-5 km/s)	Solids only	Side-to-side (perpendicular)

Scientific Importance: The fact that S-waves do not reach certain areas of Earth (the "S-wave shadow zone") provided evidence that Earth's outer core is liquid. This is how seismologists determined Earth's internal structure without ever drilling to the center.

The Wave Function

A traveling wave is described by a function of the form:

$$y(x, t) = h(kx \pm \omega t)$$

For a sinusoidal wave traveling in the **positive x-direction**:

$$y(x, t) = y_m \sin(kx - \omega t + \phi)$$

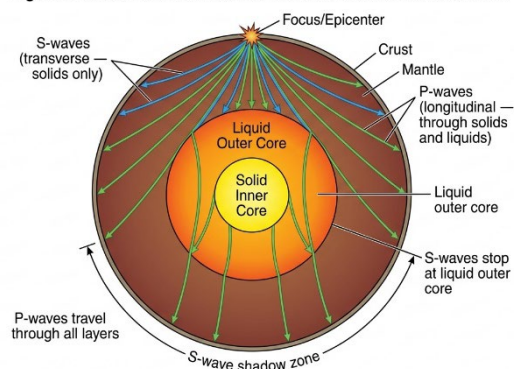
where:

- y_m = amplitude (maximum displacement)
- k = angular wave number (rad/m)
- ω = angular frequency (rad/s)
- ϕ = phase constant

Key Wave Parameters

Parameter	Symbol	Definition	SI Unit	Relationship
Wavelength	λ	Shortest distance between two points where the wave pattern repeats itself	meter (m)	—
Amplitude	y_m or A	Maximum displacement of a particle from its equilibrium position	meter (m)	$E \propto A^2$
Time Period	T	Time required for one complete wave cycle to pass a point	second (s)	$T = 1/f$
Frequency	f	Number of complete wave cycles passing a point per unit time	hertz (Hz)	$f = 1/T$
Wave Speed	v	Distance traveled by the wave per unit time	m/s	$v = f\lambda$
Angular Wave Number	k	$2\pi/\lambda$	rad/m	$k = 2\pi/\lambda$
Angular Frequency	ω	$2\pi f = 2\pi/T$	rad/s	$\omega = 2\pi f$

Figure 1: Cross-Section of Earth's Interior and Seismic Wave Paths



For a sinusoidal sound wave:

$$I = \frac{1}{2} \rho v \omega^2 s_m^2 = \frac{(\Delta p_m)^2}{2 \rho v}$$

Variation with Distance from a Point Source

For an isotropic point source (emitting equally in all directions):

$$I = \frac{P_s}{4\pi r^2}$$

where P_s is the source power (W) and r is the distance from the source.

Inverse Square Law: Doubling the distance reduces intensity by a factor of 4.

The Decibel Scale

Because the human ear can detect sounds over a range of 10^{12} in intensity, we use a logarithmic scale:

$$\beta = (10 \text{ dB}) \log_{10} \left(\frac{I}{I_0} \right)$$

where:

- β = sound level in **decibels (dB)**
- $I_0 = 10^{-12} \text{ W/m}^2$ (threshold of hearing at 1000 Hz)

Important properties:

- Each factor of 10 in intensity adds 10 dB
- A 3 dB increase corresponds to doubling the intensity
- Normal conversation: ~60 dB
- Threshold of pain: ~120 dB

Finding Intensity Ratio from dB Change

If sound level changes by $\Delta\beta$ dB:

$$\frac{I_2}{I_1} = 10^{\Delta\beta/10}$$

WAVE INTERACTIONS

The Principle of Superposition

When two or more waves overlap in the same region of space:

$$y_{\text{net}}(x, t) = y_1(x, t) + y_2(x, t) + \dots$$

Key insight: Waves pass through each other without being altered. The net displacement is simply the algebraic sum of individual displacements.

Interference of Waves Traveling in the Same Direction

Consider two waves with the same amplitude, wavelength, and direction, but with a phase difference ϕ :

$I = P/(4\pi r^2)$
Intensity (I) is Power (P) per unit surface area of the sphere ($A = 4\pi r^2$).

$\text{Area} \propto r^2$

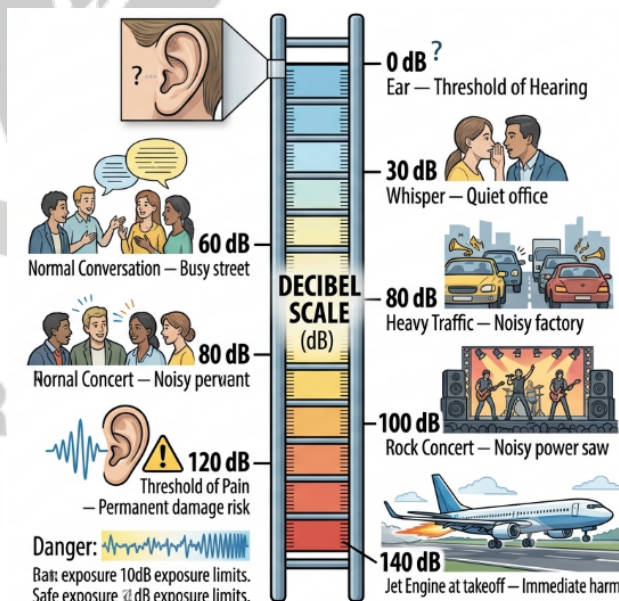
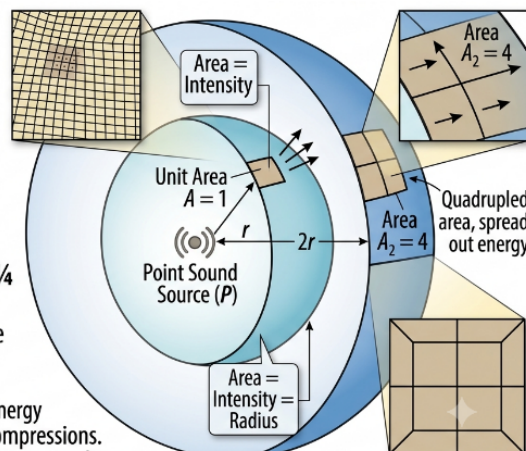
Surface area increases with the square of the radius.



Doubling distance $\rightarrow$ intensity $\times 1/4$

At radius $2r$, the area quadruples ($4A$), so the intensity (I_2) is one-quarter of the intensity at radius r (I_1): $I_2 = I_1 / 4$.

- Sound wave power is energy and mass energy
- Pressure variations transport alternating compressions.
- Pressure variations drive acoustic flow, creating the waves.



Comparison of Pipe Types

Pipe Type	Harmonics Present	Frequency Formula
Open at both ends	All ($n = 1, 2, 3, \dots$)	$f_n = nv/2L$
Closed at one end	Odd only ($n = 1, 3, 5, \dots$)	$f_n = nv/4L$

Beats

Beats are periodic variations in loudness (intensity) that occur when two sound waves of slightly different frequencies interfere.

The Beat Phenomenon

When two sound waves of slightly different frequencies f_1 and f_2 are heard together, the intensity fluctuates at the **beat frequency**:

$$f_{\text{beat}} = |f_1 - f_2|$$

Mathematical Derivation

For two waves: $s_1 =$

$$s_m \cos(2\pi f_1 t) \text{ and } s_2 = s_m \cos(2\pi f_2 t)$$

The superposition gives:

$$s(t) = \left[2s_m \cos\left(2\pi \frac{f_1 - f_2}{2} t\right) \right] \cos\left(2\pi \frac{f_1 + f_2}{2} t\right)$$

The amplitude varies at frequency $f_{\text{beat}}/2$, giving two loudness maxima per cycle, so the perceived beat frequency is $|f_1 - f_2|$.

Applications of Beats

- Tuning musical instruments:** Adjust until beats disappear (instrument in tune with reference)
- Detecting unknown frequencies:** By producing beats with a known frequency, an unknown frequency can be determined
- Generating low frequencies:** Beats can produce very low frequency sounds from higher frequency sources

The Doppler Effect

The **Doppler effect** is the apparent change in frequency (or pitch) of waves due to the relative motion between the source and the observer (listener).

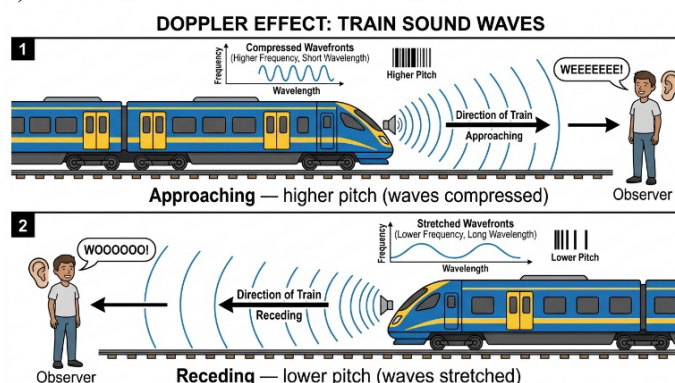
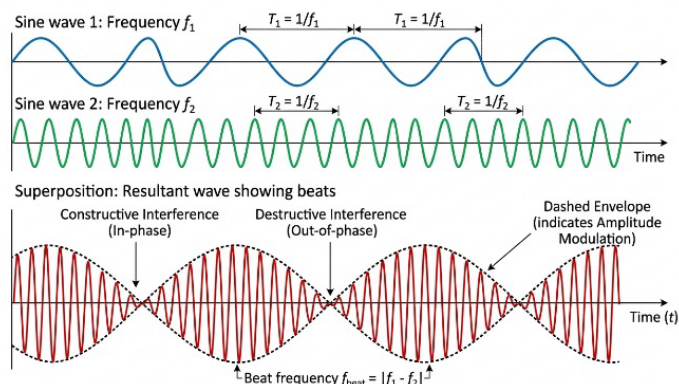
Everyday Examples

- Train whistle:** As a train approaches, the pitch sounds higher; as it moves away, the pitch sounds lower
- Race car pass-by:** Engine noise drops in pitch as the car passes
- Siren on an ambulance:** Pitch changes as the ambulance passes

Explanation

When the source moves toward a stationary observer:

- The waves are "compressed" (wavelength decreases)





where θ is the half-angle of the Mach cone.

Mach number = v_s/v

The sonic boom is produced when the shock wave passes a detector.

Applications of the Doppler Effect

Application	How It Works
Radar (RADIO Detection And Ranging)	Radio waves reflected from moving objects (cars, airplanes) show frequency shifts; the shift reveals speed
Sonar	Sound waves reflected from submarines or fish show Doppler shifts
Astronomy (Redshift)	Light from distant galaxies is redshifted (frequency decreased), indicating they are moving away—evidence for the expanding universe
Doppler Echocardiography	Ultrasound waves reflected from blood cells in the heart show frequency shifts that reveal blood flow velocity
Satellite Navigation	Doppler shifts help determine satellite positions and velocities
Weather Radar	Doppler shifts in reflected microwaves reveal wind velocity and precipitation patterns

Core Principles and Key Equations

Concept	Key Equation	Physical Meaning
SHM restoring force	$F = -kx$	Force proportional to displacement, opposite direction
SHM period (spring)	$T = 2\pi\sqrt{m/k}$	Period independent of amplitude
Simple pendulum	$T = 2\pi\sqrt{L/g}$	Period depends only on length and gravity
Physical pendulum	$T = 2\pi\sqrt{I/mgh}$	Period depends on rotational inertia
Wave speed on string	$v = \sqrt{\tau/\mu}$	Speed determined by medium properties
Sound speed	$v = \sqrt{B/\rho}$	Faster in stiffer, less dense media
Universal wave equation	$v = f\lambda$	Fundamental relationship for all waves
Wave power	$P_{\text{avg}} = \frac{1}{2}\mu v \omega^2 y_m^2$	Proportional to square of amplitude and frequency
Sound intensity (point source)	$I = P/4\pi r^2$	Inverse square law
Sound level	$\beta = 10\log(I/I_0)$ dB	Logarithmic scale for human hearing
Standing wave (string/open pipe)	$f_n = nv/2L$	All harmonics present
Standing wave (closed pipe)	$f_n = nv/4L$ (odd n)	Only odd harmonics
Beat frequency		Difference of frequencies
Doppler effect	$f' = f(v \pm v_D)/(v \pm v_S)$	Relative motion changes frequency
Mach cone angle	$\sin \theta = v/v_S$	For supersonic sources

Important Distinctions

Concept	Key Points
SHM vs. General Oscillation	SHM requires $F \propto -x$; not all oscillations are SHM
Transverse vs. Longitudinal	Vibration direction: perpendicular vs. parallel to wave travel
Mechanical vs. Electromagnetic	Mechanical needs medium; electromagnetic does not
Constructive vs. Destructive Interference	Phase difference: 0° vs. 180° (or $\lambda/2$ path difference)

Oscillations and Waves: ONE-LINERS

Introduction to Oscillations

1. Oscillatory motion is the repeated back-and-forth movement of a body about a fixed central position called the equilibrium position.
2. Oscillatory motion requires periodicity, a restoring force toward equilibrium, and inertial overshoot.
3. Displacement in oscillation is the distance of the oscillating body from the mean position at any instant.
4. Amplitude is the maximum displacement from the mean position.
5. Time period is the time taken to complete one full oscillation.
6. Frequency is the number of complete oscillations per unit time.
7. Angular frequency is related to frequency by $\omega = 2\pi f = 2\pi/T$.
8. The fundamental relationship between period and frequency is $T = 1/f$ and $f = 1/T$.

Simple Harmonic Motion (SHM)

9. Simple Harmonic Motion is defined as oscillatory motion in which acceleration is directly proportional to displacement and always directed toward the mean position.
10. Mathematically, SHM is defined by $a = -\omega^2 x$, where ω is the angular frequency.
11. The restoring force in SHM obeys Hooke's law: $F = -kx$.
12. The force in SHM is proportional to displacement, opposite in direction, and linear.
13. Not every oscillatory motion is SHM; the restoring force must be strictly proportional to displacement.
14. The differential equation of SHM is $d^2x/dt^2 + (k/m)x = 0$.
15. The general solution of the SHM differential equation is a sinusoidal function.
16. The displacement function in SHM is $x(t) = x_m \cos(\omega t + \phi)$.
17. The phase constant ϕ determines the position at $t = 0$.
18. The velocity in SHM is $v(t) = -\omega x_m \sin(\omega t + \phi)$.
19. The velocity amplitude (maximum speed) is $v_m = \omega x_m$.
20. The acceleration in SHM is $a(t) = -\omega^2 x_m \cos(\omega t + \phi) = -\omega^2 x(t)$.
21. The acceleration amplitude is $a_m = \omega^2 x_m$.
22. In SHM, acceleration is proportional to displacement but opposite in sign: $a = -\omega^2 x$.
23. For a mass-spring system: $\omega = \sqrt{k/m}$ and $T = 2\pi\sqrt{m/k}$.
24. A stiffer spring (larger k) gives larger ω and shorter period.
25. A larger mass m gives smaller ω and longer period.
26. The time period of a mass-spring system is independent of amplitude.
27. SHM is the projection of uniform circular motion onto a diameter.
28. In SHM, velocity leads displacement by 90° ($\pi/2$ radians).
29. Acceleration is 180° out of phase with displacement.

Energy in SHM

30. Potential energy in SHM: $U = \frac{1}{2} kx^2 = \frac{1}{2} kx_m^2 \cos^2(\omega t + \phi)$.
31. Kinetic energy in SHM: $K = \frac{1}{2} mv^2 = \frac{1}{2} kx_m^2 \sin^2(\omega t + \phi)$.

135. Mach number is v_s/v .
136. The sonic boom is produced when the shock wave passes a detector.
137. Radar uses the Doppler effect to detect the speed of moving objects.
138. Astronomical redshift indicates galaxies are moving away, providing evidence for the expanding universe.
139. Doppler echocardiography uses ultrasound to measure blood flow velocity.
140. Weather radar uses Doppler shifts to reveal wind velocity and precipitation patterns.

Practice MCQs

- M** 1. Oscillatory motion is characterized by:
- K** A) Motion in a straight line only
- B) Repeated back-and-forth movement about an equilibrium position
- C) Motion with constant velocity
- P** D) Motion with increasing amplitude
- C** Correct Answer: B
- R** 2. The maximum displacement from the mean position in SHM is called:
- E** A) Frequency
- P** B) Wavelength
- A** C) Amplitude
- R** D) Period
- A** Correct Answer: C
- T** 3. The time taken to complete one full oscillation is called:
- I** A) Frequency
- O** B) Amplitude
- N** C) Wavelength
- S** D) Time period
- Correct Answer: D
4. Frequency is defined as:
- A) The time for one oscillation
- B) The number of oscillations per unit time
- C) The maximum displacement
- D) The distance between two wave crests
- Correct Answer: B
5. The SI unit of frequency is:
- A) Second
- B) Hertz
- C) Radian
- D) Meter
- Correct Answer: B

6. Angular frequency is related to frequency by:

- A) $\omega = 2\pi f$
- B) $\omega = f/2\pi$
- C) $\omega = 2\pi/f$
- D) $\omega = f$

Correct Answer: A

7. The relationship between period and frequency is:

- A) $T = f$
- B) $T = 1/f$
- C) $T = 2\pi f$
- D) $T = f^2$

Correct Answer: B

8. In SHM, acceleration is:

- A) Directly proportional to displacement and in the same direction
- B) Directly proportional to displacement and opposite in direction
- C) Inversely proportional to displacement
- D) Constant

Correct Answer: B

9. The mathematical condition for SHM is:

- A) $a = \omega^2 x$
- B) $a = -\omega^2 x$
- C) $a = \omega x$
- D) $a = -\omega x$

Correct Answer: B

10. The restoring force in SHM obeys:

- A) Newton's law of gravitation
- B) Hooke's law
- C) Coulomb's law

Chapter 8: Physical Optics

Introduction

Geometrical optics, which describes light in terms of rays that travel in straight lines and bend only at interfaces, provides a remarkably successful framework for understanding lenses, mirrors, and optical instruments. However, this ray model is fundamentally an approximation. Light, like sound and water waves, is a wave phenomenon, and when we examine situations where the wave nature becomes evident, we observe effects that cannot be explained by geometrical optics alone.

Consider a simple example: when monochromatic light passes through a narrow slit and falls on a screen, geometrical optics predicts a sharp image of the slit—a bright region of the same width as the slit. What we actually observe is quite different: a broad central bright fringe surrounded by alternating dark and less intense bright fringes. Similarly, when light reflects from a thin

film of oil on water, we see vibrant colors that shift as we change our viewing angle, and when sunlight passes through a narrow gap, it spreads into the region of geometric shadow. These phenomena—**interference** (the superposition of waves from two or more sources), **diffraction** (the spreading of waves around obstacles or through apertures), and **polarization** (the restriction of transverse wave vibrations to a specific plane)—are the hallmarks of the wave nature of light.

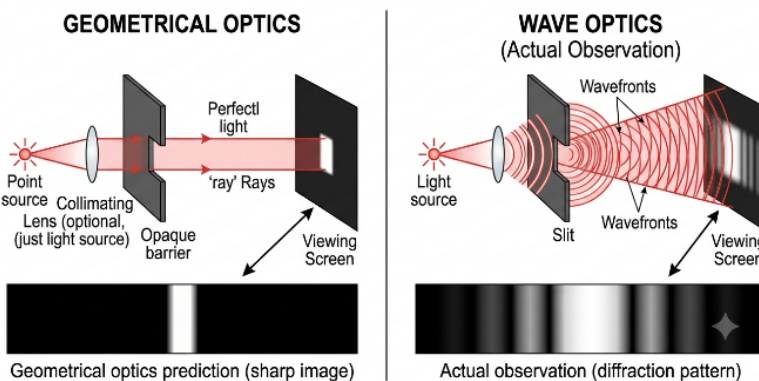
The study of light has fascinated scientists for centuries. Is light a stream of particles, as Isaac Newton proposed, or a wave, as Christiaan Huygens suggested? The answer, as we now understand, is both. This chapter explores the wave nature of light through these fundamental phenomena. Interference occurs when two or more waves overlap and combine, producing regions of enhanced or diminished intensity. Diffraction describes the bending of waves around obstacles or through apertures. Polarization reveals the transverse nature of light waves by restricting their vibrations to a single plane. Together, these phenomena provide compelling evidence that light behaves as a wave.

Beyond light, the wave principles developed here apply to sound waves, water waves, and even matter waves in quantum mechanics. The electron double-slit experiment, discussed in this chapter, bridges classical wave optics with the strange world of quantum physics. Finally, we extend these wave concepts to a modern frontier of physics—**gravitational waves**—ripples in the fabric of spacetime itself, predicted by Einstein and now detected directly by laser interferometers.

Fundamental Concepts: Wave Fronts and Huygens' Principle

Before exploring interference and diffraction, we must establish foundational concepts that describe how waves propagate.

COMPARISON: GEOMETRICAL VS. WAVE OPTICS (Single Slit Diffraction)



For small angles ($y \ll L$):

$$I(y) = 4I_0 \cos^2 \left(\frac{\pi dy}{\lambda L} \right)$$

The intensity varies between:

- Maximum $I_{\max} = 4I_0$ when $\phi = 0, \pm 2\pi, \pm 4\pi, \dots$ (even multiples of π)
- Zero when $\phi = \pm\pi, \pm 3\pi, \pm 5\pi, \dots$ (odd multiples of π)

The average intensity is $2I_0$, which is twice the intensity from a single slit—consistent with energy conservation (interference redistributes energy but does not create or destroy it).

Example: Light of wavelength 633 nm from a He-Ne laser passes through slits separated by 0.100 mm. The screen is 2.00 m away. Find the fringe spacing.

Solution: $\Delta y = \lambda L/d = (633 \times 10^{-9} \text{ m})(2.00 \text{ m})/(1.00 \times 10^{-4} \text{ m}) = 0.0127 \text{ m} = 1.27 \text{ cm}$.

Interference in Thin Films

The vibrant colors seen in soap bubbles, oil slicks on water, and the iridescent wings of butterflies arise from interference of light waves reflected from the two surfaces of a thin transparent film.

Basic Mechanism

Consider a thin film of thickness L and index of refraction n_2 , surrounded by media with indices n_1 (above) and n_3 (below). Incident light ray i is partially reflected at the top surface (ray r_1) and partially transmitted. The transmitted light reflects from the bottom surface and emerges as ray r_2 . The two reflected rays r_1 and r_2 interfere.

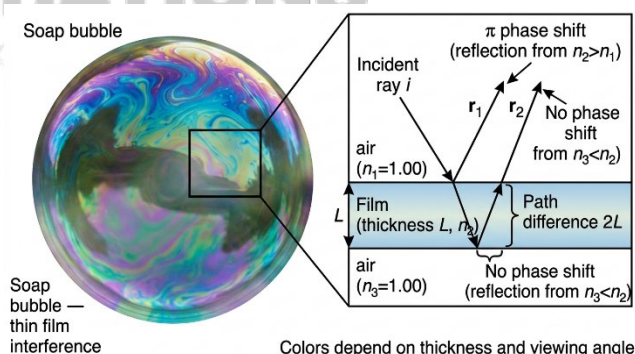
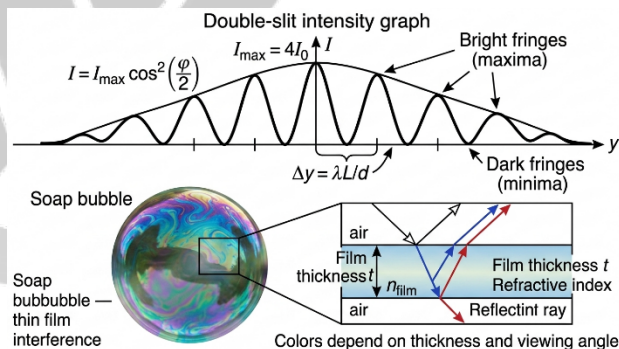
Phase Changes on Reflection

A crucial phenomenon occurs when light reflects from an interface: the reflected wave may undergo a phase shift of π radians (half a wavelength) depending on the indices of refraction.

Rule for reflection phase shift:

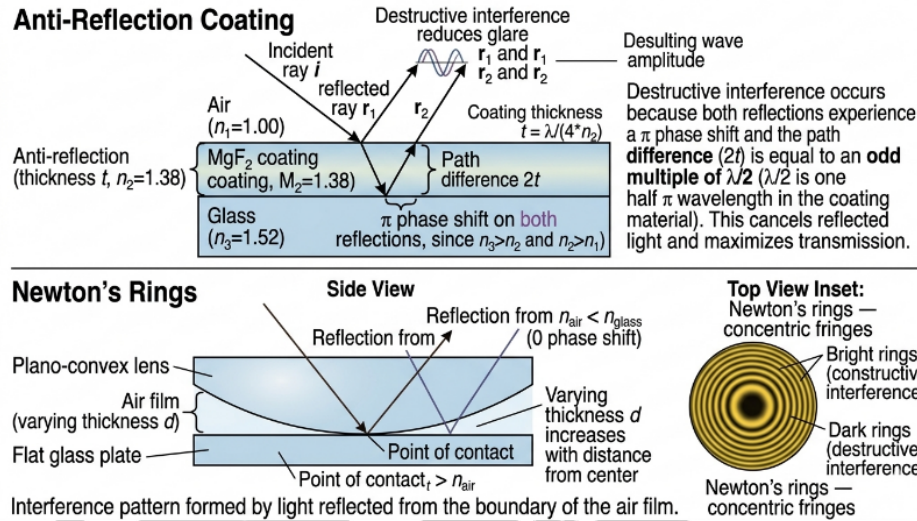
- If the incident wave travels in the medium with the **smaller** index of refraction (faster speed) and reflects from a medium with **larger** index (slower speed), the reflected wave undergoes a **half-cycle phase shift** (π rad, equivalent to $\lambda/2$).
- If the incident wave travels in the medium with the **larger** index and reflects from a medium with **smaller** index, **no phase shift** occurs.
- Refraction does **not** cause any phase shift.

This rule can be remembered as: "**Higher means half**" — reflection from a higher-index medium produces a half-wavelength phase shift.



Newton's Rings

When a plano-convex lens is placed on a flat glass plate, a thin air film of varying thickness is formed between them. Monochromatic light reflected from this arrangement produces concentric circular fringes called **Newton's rings**. The rings provide a sensitive method for testing the flatness of optical surfaces.



The Michelson Interferometer

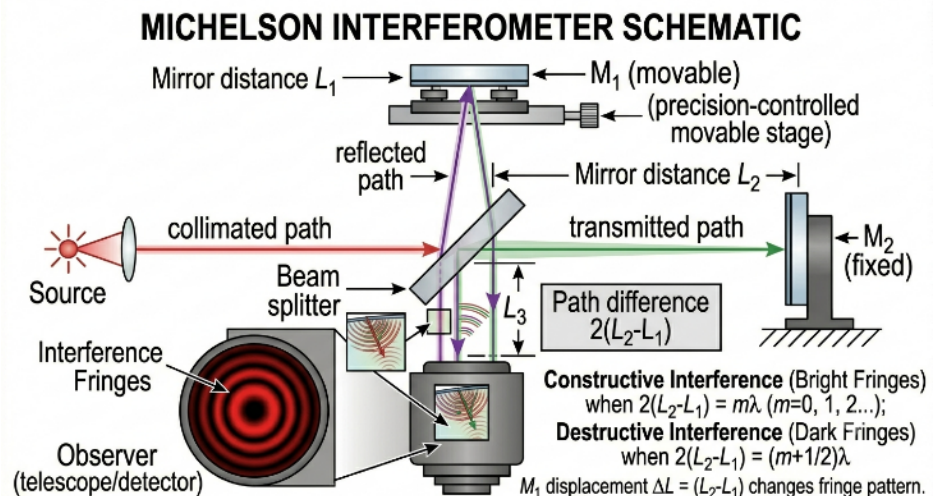
The Michelson interferometer is a precision instrument that uses interference to measure wavelengths and small distances with extraordinary accuracy.

Operating Principle

A beam splitter divides light from a monochromatic source into two perpendicular beams. These beams reflect from movable mirrors M_1 and M_2 and recombine at the beam splitter, where they interfere. The observer sees an interference pattern determined by the path difference $2(L_2 - L_1)$.

Fringe Shift

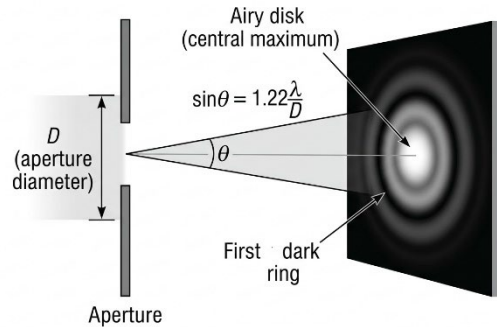
When mirror M_2 is moved a distance Δx , the path difference changes by $2\Delta x$. Each change of path difference by one wavelength causes the fringe pattern to shift by one fringe. Thus:



For a circular aperture of diameter D , the first dark ring occurs at:

$$\sin \theta = 1.22 \frac{\lambda}{D}$$

The central bright spot is called the **Airy disk**, containing approximately 85% of the total light energy. The factor 1.22 arises from the circular geometry (a Bessel function, rather than the sine function for a rectangular slit).



Comparison: Double Slit vs. Single Slit

Feature	Double Slit	Single Slit
Pattern	Equally spaced bright fringes	Central bright fringe wider, decreasing intensity side maxima
Minima condition	$d \sin \theta = (m + \frac{1}{2})\lambda$	$a \sin \theta = m\lambda$ ($m = \pm 1, \pm 2, \dots$)
Intensity	All bright fringes nearly equal	Central maximum much brighter

The Diffraction Grating

A **diffraction grating** is an optical component containing a large number of equally spaced parallel slits (or grooves)—typically thousands per centimeter. Typical gratings have 300–600 lines/mm.

Grating Element

The **grating element d** (slit spacing) is:

$$d = \frac{\text{Unit length of grating}}{\text{Number of lines on that length}}$$

For example, a grating with 5000 lines per cm has:

$$d = \frac{1 \text{ cm}}{5000} = 2 \times 10^{-4} \text{ cm} = 2 \mu\text{m}$$

Grating Equation

When monochromatic light is incident normally on a grating, each slit produces diffraction. Waves from adjacent slits interfere. Constructive interference (principal maxima) occurs when the path difference between adjacent slits is an integer multiple of wavelength:

$$d \sin \theta = m\lambda$$

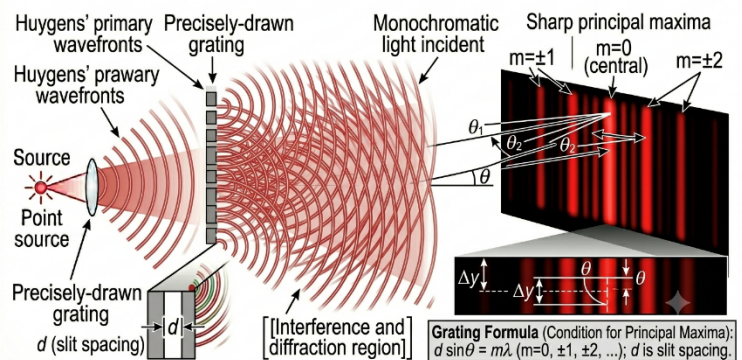
where:

- d = grating element (slit spacing)
- θ = angle of diffraction (measured from the normal)
- m = order number ($m = 0, \pm 1, \pm 2, \dots$)
- λ = wavelength of light

Intensity Pattern and Resolving Power

The intensity pattern from a grating differs significantly from a double slit:

DIFFRACTION GRATING DIAGRAM AND INTERFERENCE PATTERN



Method 3: Polarization by Double Refraction (Birefringence)

Certain crystals (e.g., calcite, quartz) have two different refractive indices for different polarization directions. When unpolarized light enters such a crystal, it splits into two beams—the ordinary ray (obeys Snell's law) and the extraordinary ray—each polarized perpendicularly.

Malus's Law

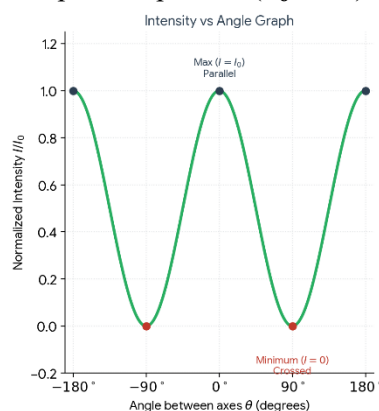
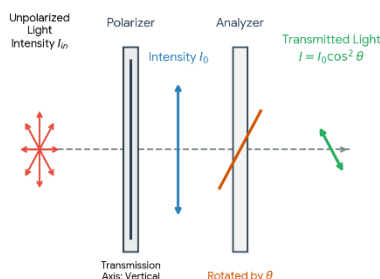
When plane-polarized light of intensity I_0 passes through a second polarizer (analyzer) whose transmission axis makes an angle θ with the polarization direction of the incident light, the transmitted intensity I is:

$$I = I_0 \cos^2 \theta$$

Derivation: The incident electric field amplitude E_0 can be resolved into components parallel ($E_0 \cos \theta$) and perpendicular ($E_0 \sin \theta$) to the analyzer's axis. Only the parallel component is transmitted. Since intensity is proportional to the square of amplitude, $I \propto (E_0 \cos \theta)^2 = I_0 \cos^2 \theta$.

Special cases:

- $\theta = 0^\circ$ (parallel axes): $I = I_0$ (maximum transmission)
- $\theta = 90^\circ$ (crossed axes): $I = 0$ (complete extinction)
- $\theta = 45^\circ$: $I = I_0/2$

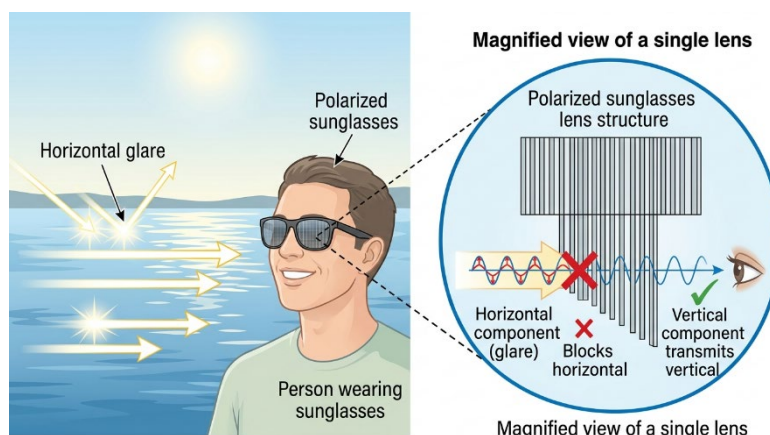


Example 7.3: Plane-polarized light of intensity 100 W/m^2 passes through a polarizer at 60° to the polarization direction. Find the transmitted intensity.

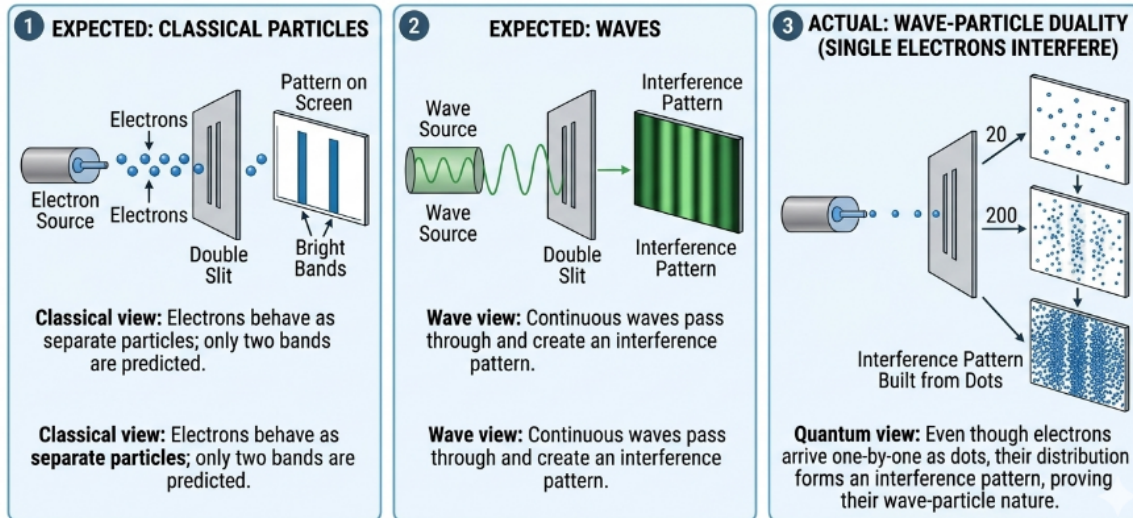
Solution: $I = I_0 \cos^2 \theta = (100 \text{ W/m}^2) \times \cos^2(60^\circ) = 100 \times (0.5)^2 = 100 \times 0.25 = 25 \text{ W/m}^2$.

Applications of Polarization

Application	Principle
Polarized sunglasses	Horizontally polarized glare from roads or water is blocked by vertically oriented polarizers



- The electron is described by a **wave function** ψ , which gives the probability of finding the electron at any location via $|\psi|^2$.
- The act of **measurement** (detection on the screen) causes the wave function to **collapse** to a definite position.



Key features: Indeterminism (only probabilities are predictable), complementarity (wave and particle aspects are mutually exclusive but both necessary), and the central role of measurement.

Criticisms: The "collapse" mechanism is ad hoc; it does not explain what happens to the other possibilities.

The Many-Worlds Interpretation

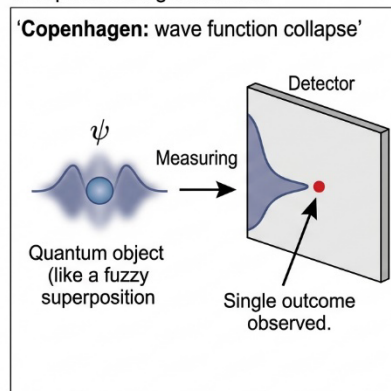
Proposed by Hugh Everett III in 1957, the Many-Worlds Interpretation (MWI) eliminates wave function collapse:

- All possibilities in the superposition are equally real.
- When a measurement occurs, the universe **splits** into parallel branches—one for each possible outcome.
- In each branch, the observer sees one definite outcome (e.g., the electron at a particular point).
- The interference pattern arises from contributions across all branches.

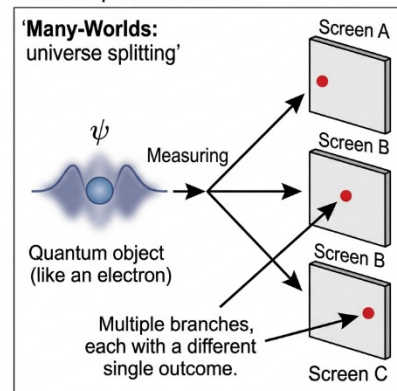
Key features: Deterministic evolution of the wave function (no collapse), no special role for measurement, all possibilities realized in some branch.

Criticisms: The existence of countless parallel universes is experimentally untestable; the interpretation is ontologically extravagant.

Panel 1: Copenhagen — wave function collapses to single outcome



Panel 2: Many-Worlds — universe splits into multiple branches



Single-slit intensity	$I = I_0 \left(\frac{\sin \beta/2}{\beta/2} \right)^2$	$\beta = \frac{2\pi a \sin \theta}{\lambda}$
Circular aperture minimum	$\sin \theta = 1.22 \frac{\lambda}{D}$	$D = \text{aperture diameter}$
Rayleigh criterion	$\theta_R = 1.22 \frac{\lambda}{D}$	
Grating equation	$d \sin \theta = m\lambda$	$d = 1/(\text{lines per unit length})$
Grating resolving power	$R = \frac{\lambda}{\Delta\lambda} = Nm$	$N = \text{number of slits}$
Thin film constructive (air, $n_2 > 1$)	$2L = (m + \frac{1}{2}) \frac{\lambda}{n_2}$	$L = \text{thickness}$
Thin film destructive (air, $n_2 > 1$)	$2L = m \frac{\lambda}{n_2}$	
Malus's law	$I = I_0 \cos^2 \theta$	$\theta = \text{angle between polarizers}$
Brewster's angle	$\tan i_p = \frac{n_2}{n_1}$	$i_p = \text{polarizing angle}$
Bragg's law	$2d \sin \theta = m\lambda$	$d = \text{crystal plane spacing}$

Conceptual Summary

Concept	Core Idea	Key Equation
Interference	Superposition of waves from coherent sources	$\Delta r = m\lambda$ (constructive)
Diffraction	Bending of waves around obstacles	$a \sin \theta = m\lambda$ (single-slit min)
Diffraction grating	Many slits produce sharp maxima	$d \sin \theta = m\lambda$
Polarization	Restriction of transverse wave vibrations to one plane	$I = I_0 \cos^2 \theta$ (Malus)
Brewster's angle	Angle for complete polarization by reflection	$\tan i_p = n_2/n_1$
Thin-film interference	Colors from path differences and reflection phase shifts	$2L = (m + \frac{1}{2})\lambda/n_2$ (bright)
Quantum interference	Electrons (and all matter) exhibit wave-particle duality	Copenhagen vs. Many-Worlds
Gravitational waves	Ripples in spacetime from accelerating masses	Detected by LIGO interferometer

LET'S MAKE IT HAPPEN

Physical Optics: One-Liners

Introduction to Physical Optics

1. Geometrical optics describes light in terms of rays that travel in straight lines and bend only at interfaces.
2. Physical optics examines phenomena that cannot be explained by geometrical optics alone, including interference, diffraction, and polarization.
3. When monochromatic light passes through a narrow slit, geometrical optics predicts a sharp image, but actual observation shows a broad central bright fringe with alternating dark and bright fringes.
4. Interference is the superposition of waves from two or more sources producing regions of enhanced or diminished intensity.
5. Diffraction is the spreading of waves around obstacles or through apertures.
6. Polarization is the restriction of transverse wave vibrations to a specific plane.
7. The wave principles developed for light also apply to sound waves, water waves, and even matter waves in quantum mechanics.

Wave Fronts and Huygens' Principle

8. A wave front is the locus of all points in a medium that are in the same phase of vibration.
9. For a point source of light in a homogeneous medium, wave fronts are concentric spheres centered on the source.
10. At a large distance from the source, a small portion of a spherical wave front approximates a plane wave front.
11. Rays are lines drawn perpendicular to wave fronts, indicating the direction of energy propagation.
12. Huygens' Principle: Every point on a wave front acts as a source of secondary spherical wavelets that spread outward in the forward direction.
13. The new wave front at a later time is the envelope (tangent surface) of all secondary wavelets.
14. Huygens' Principle can derive Snell's law: $n_1 \sin \theta_1 = n_2 \sin \theta_2$.
15. The index of refraction is defined as $n = c/v$, where c is the speed of light in vacuum.
16. When light enters a medium with a higher index of refraction, its wavelength decreases: $\lambda_n = \lambda/n$.
17. Frequency of light does not change upon entering a different medium.

Interference of Waves

18. Interference occurs when two or more waves traveling through the same medium superimpose.
19. The principle of superposition states that the resulting wave displacement is the vector sum of individual displacements.
20. Constructive interference occurs when waves arrive in phase; amplitudes add, producing greater intensity.
21. Destructive interference occurs when waves arrive 180° out of phase; amplitudes subtract, producing smaller intensity.
22. For a stable interference pattern, sources must be coherent (constant phase relationship).
23. Ordinary light sources are not coherent because atoms emit light in random, independent bursts of approximately 10^{-8} s duration.
24. Coherent light can be obtained by splitting a single wavefront or using a laser.

134. Gravitational waves are ripples in the fabric of spacetime, propagating at the speed of light from accelerating massive objects.
135. They were predicted by Albert Einstein in 1916 as a consequence of his General Theory of Relativity.
136. When massive objects accelerate, the curvature of spacetime changes and propagates outward as waves.
137. Just as a moving charged particle emits electromagnetic waves, a moving mass emits gravitational waves.
138. Sources of gravitational waves include rotating neutron stars, binary black hole inspirals, supernova explosions, and stochastic backgrounds.
139. The strongest gravitational waves come from merging black holes and neutron stars.
140. The LIGO interferometer consists of two 4 km long arms arranged perpendicularly with mirrors at each end.
141. A gravitational wave stretches one arm while compressing the other, producing a measurable interference signal.
142. The spacetime distortion from gravitational waves is minuscule — about 10^{-18} m over a 4 km baseline.
143. On September 14, 2015, LIGO made the first direct detection of gravitational waves from merging black holes.
144. Rainer Weiss, Barry Barish, and Kip Thorne received the 2017 Nobel Prize in Physics for this discovery.
145. Pakistani-American physicist Dr. Nergis Mavalvala played a key role in the LIGO collaboration.
146. The 2017 detection of a neutron star merger was observed across the electromagnetic spectrum, ushering in multi-messenger astronomy.
147. Gravitational waves allow us to "see" objects that emit no light (black holes) and probe the strong-field regime of gravity.

Practice MCQs

1. Physical optics studies phenomena that:

- A) Can be explained by ray optics
- B) Cannot be explained by geometrical optics alone
- C) Only occur in vacuum
- D) Only occur with mechanical waves

Correct Answer: B

2. A wave front is defined as:

- A) A line perpendicular to wave propagation
- B) The locus of all points in the same phase of vibration
- C) The direction of energy flow
- D) The maximum displacement of a wave

Correct Answer: B

3. Huygens' Principle states that:

- A) Light travels in straight lines
- B) Every point on a wave front acts as a source of secondary wavelets
- C) Waves cannot interfere
- D) Light is a particle

Correct Answer: B

4. The new wave front according to Huygens' Principle is:

- A) The line connecting all sources
- B) The envelope of all secondary wavelets
- C) A straight line
- D) A circle

Correct Answer: B



Part III: Thermal Physics



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Chapter 9: Heat, Thermodynamics and Statistical Mechanics

Introduction

Thermodynamics is the branch of physics that studies the relationships between heat, work, temperature, and energy—the fundamental principles that govern energy transformations in physical systems. The word itself derives from the Greek *therme* (heat) and *dynamis* (power), reflecting its historical origins in the study of heat engines during the Industrial Revolution.

At its core, thermodynamics addresses profound questions: How does heat flow? How much work can be extracted from a given amount of heat? What limits the efficiency of engines? Why do some processes occur spontaneously while others do not? The laws of thermodynamics provide universal answers—answers that apply not only to steam engines and refrigerators but also to biological systems, chemical reactions, chemical reactions, and even the evolution of the universe itself.

Complementing the macroscopic perspective of thermodynamics is the **kinetic theory of gases**, which explains macroscopic behaviour in terms of the microscopic motion of atoms and molecules. This dual approach—

phenomenological thermodynamics and statistical mechanics—provides a complete understanding of thermal physics. The present chapter develops both viewpoints systematically, beginning with fundamental concepts of temperature and heat, progressing through the kinetic-molecular model, and culminating in the powerful laws that govern all thermal processes.

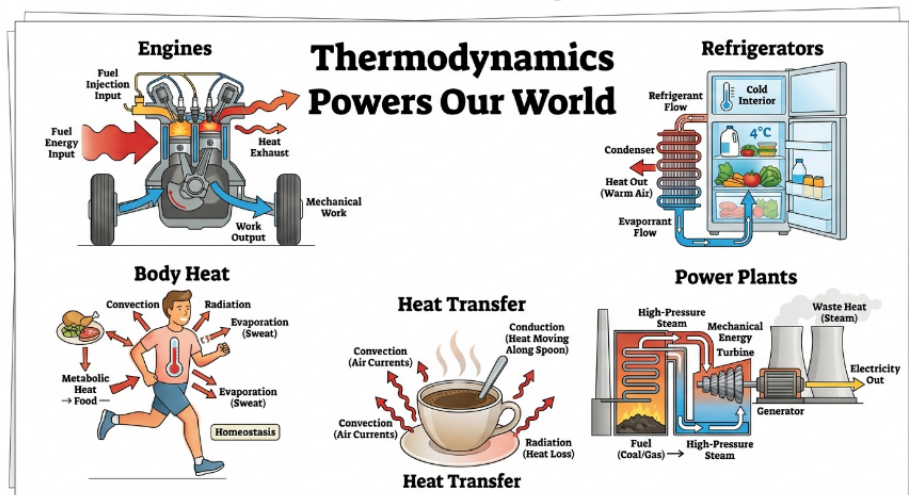
The practical importance of thermodynamics cannot be overstated. Automobile engines convert the chemical energy of fuel into mechanical work through controlled combustion. Refrigerators and air conditioners use work to transfer heat from cold regions to hot regions. The human body maintains a near-constant temperature through complex heat-exchange mechanisms. Power plants generate electricity by boiling water into steam that turns turbines. All these systems obey the same fundamental principles, which we shall develop in this chapter.

Fundamental Concepts: System, Surroundings, and Thermal Equilibrium

System, Surroundings, and Boundary

Before developing the mathematical machinery of thermodynamics, we must establish precise definitions of the fundamental quantities that appear throughout this chapter.

In any thermodynamic analysis, we first identify the **system**—the portion of the universe under investigation. The **surroundings** constitute everything outside the system, and the **boundary** separates the two. Boundaries may be classified according to their properties:



Thermal Expansion of Matter

Linear Expansion

Most materials expand when heated. For a solid rod of initial length L_0 at temperature T_0 , the change in length ΔL when the temperature changes by ΔT is approximately proportional to both L_0 and ΔT :

$$\Delta L = \alpha L_0 \Delta T$$

where α is the **coefficient of linear expansion** (units: K^{-1} or $(^\circ\text{C})^{-1}$). The final length is:

$$L = L_0(1 + \alpha \Delta T)$$

Conceptual Insight: The proportionality to L_0 means that all linear dimensions of an object—including the diameters of holes—expand by the same fractional amount. A common misconception is that a hole in a material shrinks when heated; in fact, the hole expands just as if it were filled with the same material.

Volume Expansion

For solids and liquids, the volume change follows a similar relationship:

$$\Delta V = \beta V_0 \Delta T$$

where β is the **coefficient of volume expansion**. For isotropic solids, β is related to the linear expansion coefficient by:

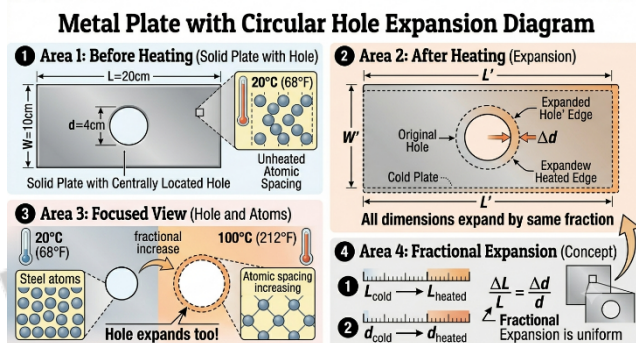
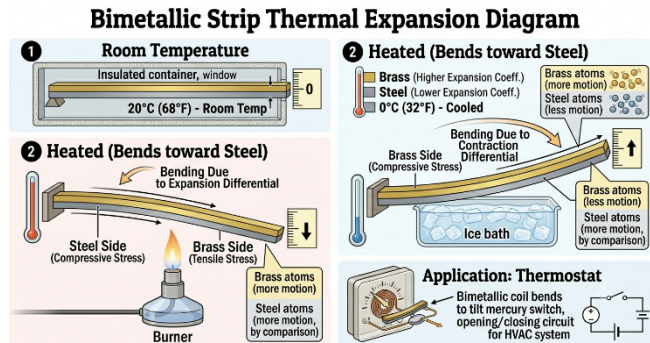
$$\beta = 3\alpha$$

Special Case —

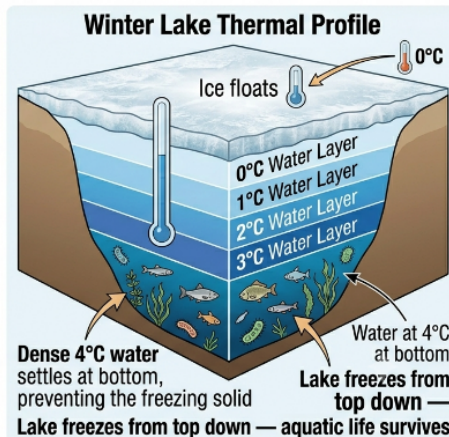
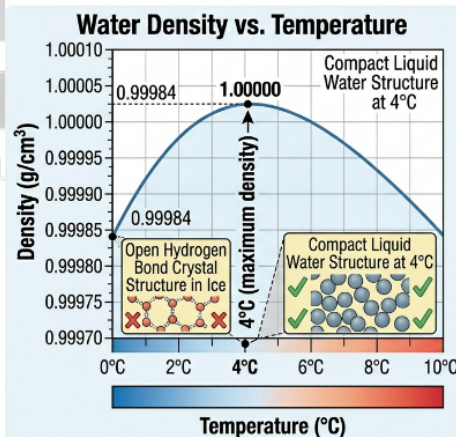
Water: Water exhibits anomalous behaviour. Between 0°C and 4°C , water **contracts** as temperature increases (negative β). At 4°C , water has its maximum density. This property is crucial for aquatic life: lakes freeze from the top down, allowing water beneath the ice to remain liquid at 4°C .

Thermal Stress

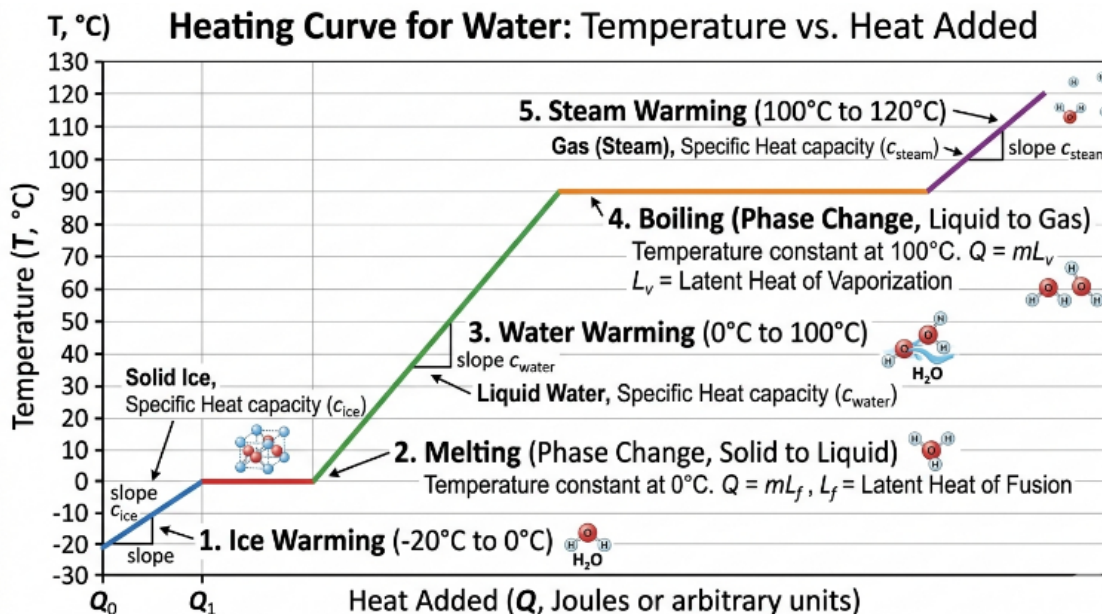
If a rod is clamped rigidly at both ends and its temperature changes, thermal stresses develop. The rod would like to expand or contract, but the clamps prevent this. The resulting stress is:



Water Density Anomaly and Lake Thermal Stratification



- Heat of vaporization (boiling): $L_v = 2.256 \times 10^6 \text{ J/kg}$



Physical Interpretation: The heat of fusion represents the energy required to overcome the attractive forces holding molecules in the rigid structure of a solid. The heat of vaporization is even larger because it must completely separate molecules from the liquid phase into the gas phase.

Calorimetry Problems

In calorimetry, we apply conservation of energy: when two or more objects exchange heat in an isolated system, the sum of all heat transfers is zero:

$$\sum Q = 0$$

Problem-Solving Strategy:

- Identify all objects that exchange heat
- Determine whether each object undergoes temperature change, phase change, or both
- Write expressions for each Q using $Q = mc\Delta T$ for temperature changes and $Q = \pm mL$ for phase changes
- Apply $\sum Q = 0$ and solve for the unknown

Caution: When a phase change may occur, it is often necessary to first determine whether the phase change is partial or complete. This requires checking whether the calculated final temperature would fall within the phase transition region.

Phase Transitions And The Clausius-Clapeyron Equation

Phase transitions (solid $\leftrightarrow$ liquid $\leftrightarrow$ gas) are ubiquitous in nature. The **Clausius-Clapeyron equation** describes how the phase transition temperature changes with pressure.

The Clausius-Clapeyron Equation

For a phase transition from phase 1 to phase 2 at temperature T and pressure P , the chemical potentials are equal:

$$\mu_1(T, P) = \mu_2(T, P)$$

This gives:

$$\frac{dP}{dT} = \frac{L}{T(V_2 - V_1)}$$

where:

- L = latent heat of the transition (J/kg or J/mol)
- V_1, V_2 = molar volumes of the two phases

Special Cases

Liquid-Vapor Transition

Since $V_{\text{gas}} \gg V_{\text{liquid}}$, and using the ideal gas law $V_{\text{gas}} = RT/P$:

$$\frac{dP}{dT} = \frac{L}{T(RT/P)} = \frac{LP}{RT^2}$$

Rearranging:

$$\frac{1}{P} \frac{dP}{dT} = \frac{L}{RT^2}$$

Integrating

(assuming L constant):

$$\ln\left(\frac{P_2}{P_1}\right) = -\frac{L}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

or:

$$P = P_0 e^{-L/RT}$$

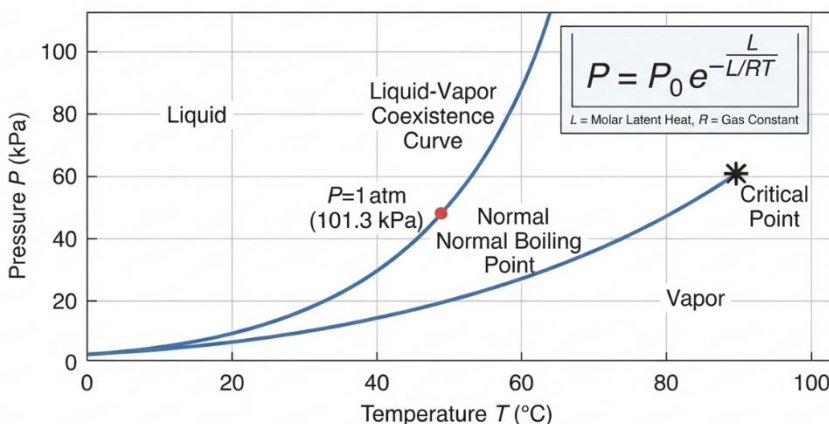
This shows that vapor pressure increases exponentially with temperature.

Solid-Liquid Transition

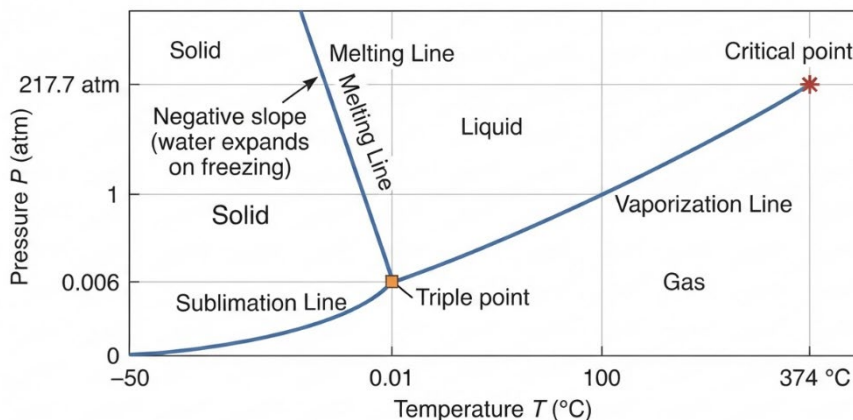
Since $V_{\text{liquid}} \approx V_{\text{solid}}$ for most materials, $V_2 - V_1$ is small, so a large change in pressure produces only a small change in melting temperature.

Key Insight: For water, $V_{\text{liquid}} < V_{\text{solid}}$ (water expands on freezing!), so dP/dT is negative—increasing pressure lowers the melting point. This is why ice skates work: pressure melts the ice beneath the blade.

Vapor Pressure Curve (Clausius-Clapeyron)



Phase Diagram of Water



Molecular Speeds and the Maxwell-Boltzmann Distribution

Root-Mean-Square Speed

The **root-mean-square (rms) speed** v_{rms} is defined as $\sqrt{\langle v^2 \rangle}$. From $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT$:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$$

where $M = mN_A$ is the molar mass.

In terms of pressure and density:

$$v_{\text{rms}} = \sqrt{\frac{3p}{\rho}}$$

Example: At room temperature (300 K), oxygen molecules ($M = 0.032 \text{ kg/mol}$) have $v_{\text{rms}} \approx 483 \text{ m/s}$ —faster than a speeding bullet!

The Maxwell-Boltzmann Speed Distribution

Not all molecules in a gas have the same speed. The **Maxwell-Boltzmann speed distribution** gives the fraction of molecules with speeds between v and $v + dv$:

$$P(v) dv = 4\pi \left(\frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-Mv^2/2RT} dv$$

Key characteristics of the distribution:

- $P(v)$ is zero at $v = 0$ and as $v \rightarrow \infty$
- It rises to a maximum at the **most probable speed** v_p
- The area under the entire curve equals 1 (all molecules have some speed)

Characteristic Speeds

Three speeds characterize the distribution:

Most probable speed (where $dP/dv = 0$):

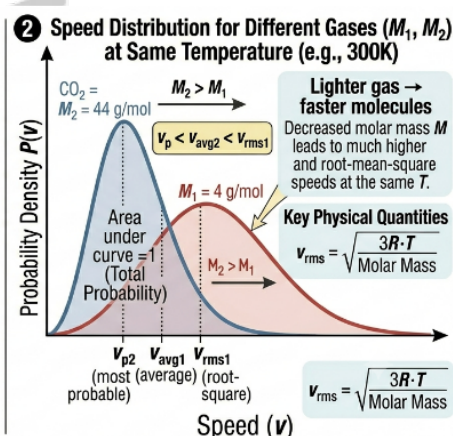
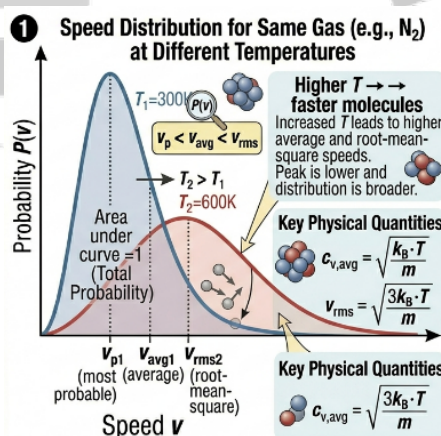
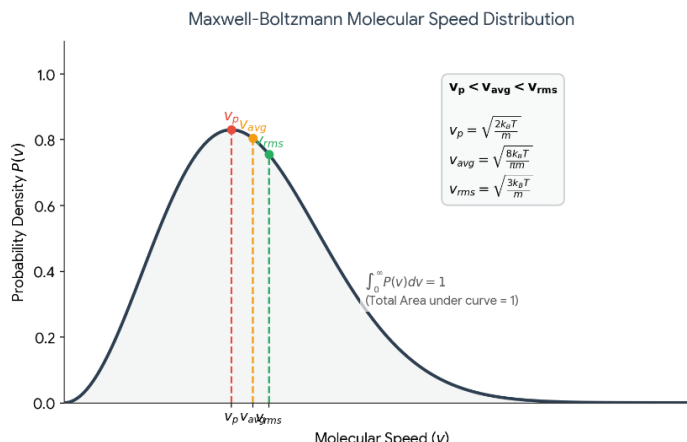
$$v_p = \sqrt{\frac{2RT}{M}}$$

Average speed:

$$v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}}$$

Root-mean-square speed (already derived):

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$



Polyatomic	6 (3 trans + 3 rot)	3R	4R	$\frac{4}{3} \approx 1.33$
-------------------	---------------------	----	----	----------------------------

For **any** process involving an ideal gas, the change in internal energy is:

$$\Delta U = nC_V\Delta T$$

This holds because internal energy depends only on temperature. Thus, even for constant-pressure or adiabatic processes, ΔU can be calculated using C_V .

Thermodynamic Potentials

Thermodynamic potentials are state functions that allow us to determine the equilibrium conditions of a system under different constraints. Each potential is naturally suited to a particular set of independent variables.

1. Internal Energy $U(S, V)$

Natural variables: Entropy S , Volume V



$$U = TS - PV + \mu N$$

$$\text{Also } dU = TdS - PdV + \mu dN$$

2. Enthalpy $H(S, P)$

Natural variables: Entropy S , Pressure P



$$H = U + PV$$

$$dH = TdS + VdP + \mu dN$$

3. Helmholtz Free Energy $F(T, V)$

Natural variables: Temperature T , Volume V



$$F = U - TS$$

$$dF = -SdT - PdV + \mu dN$$

4. Gibbs Free Energy $G(T, P)$

Natural variables: Temperature T , Pressure P



$$G = H - TS$$

$$dG = -SdT + VdP + \mu dN$$

The Four Thermodynamic Potentials

Potential	Symbol	Natural Variables	Differential Form
Internal Energy	U	S, V, N	$dU = TdS - PdV + \mu dN$
Enthalpy	H	S, P, N	$dH = TdS + VdP + \mu dN$
Helmholtz Free Energy	F or A	T, V, N	$dF = -SdT - PdV + \mu dN$
Gibbs Free Energy	G	T, P, N	$dG = -SdT + VdP + \mu dN$

1. Internal Energy $U(S, V, N)$

Definition: The sum of all forms of energy in the system (kinetic + potential + chemical).

Natural variables: Entropy S , Volume V , Number of particles N

Differential form:

$$dU = TdS - PdV + \mu dN$$

Physical Interpretation:

Term	Meaning
TdS	Heat added to the system
$-PdV$	Work done by the system
μdN	Chemical energy change

Useful relations:

$$T = \left(\frac{\partial U}{\partial S}\right)_{V, N}, -P = \left(\frac{\partial U}{\partial V}\right)_{S, N}, \mu = \left(\frac{\partial U}{\partial N}\right)_{S, V}$$

For an isolated system in equilibrium, all accessible microstates are equally probable.

Definitions

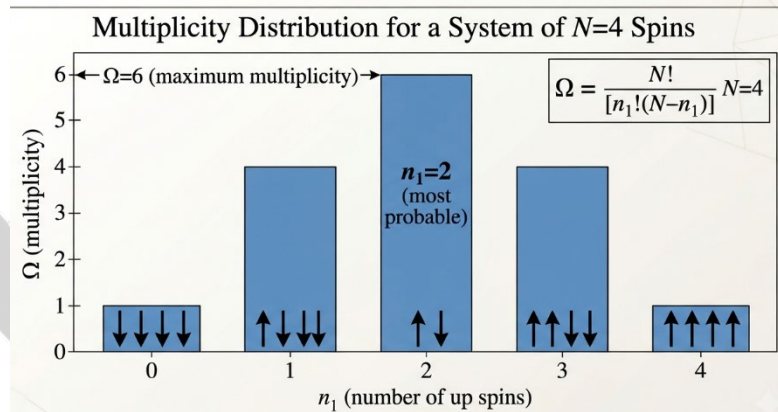
Term	Definition	Example
Microstate	A complete specification of the positions and momenta of all particles in the system	The exact position and velocity of every gas molecule
Macrostate	A specification of macroscopic properties (T, P, V, etc.)	A gas at 300 K and 1 atm
Accessible states	All microstates consistent with the macroscopic constraints (fixed E, V, N)	All possible arrangements of molecules with the same total energy

Microstates and Multiplicity

The Multiplicity Function (Ω)

The **multiplicity** Ω (also called the **statistical weight**) is the number of microstates corresponding to a given macrostate.

For a system of N particles with total energy E and volume V , the multiplicity is:



$$\Omega(E, V, N) = \text{Number of microstates with energy } E, \text{ volume } V, \text{ and } N \text{ particles}$$

Physical Interpretation

Property	Meaning
Large Ω	Many ways to arrange the system $\rightarrow$ High entropy
Small Ω	Few ways to arrange the system $\rightarrow$ Low entropy
Equilibrium	The macrostate with the maximum Ω is the most probable

Example: Two-State System

Consider N independent spins, each of which can be either "up" ($+1$) or "down" (-1). If n_1 spins are up and n_2 are down, then $n_1 + n_2 = N$.

The multiplicity is:

$$\Omega(n_1) = \frac{N!}{n_1! n_2!} = \frac{N!}{n_1! (N - n_1)!}$$

For $N = 4$:

n_1	n_2	Ω	Probability
0	4	1	1/16
1	3	4	4/16
2	2	6	6/16 (most probable)
3	1	4	4/16
4	0	1	1/16

Key Insight: The most probable macrostate (equal numbers of up and down spins) has the largest multiplicity and thus the highest entropy.

Heat Engines and the Second Law of Thermodynamics

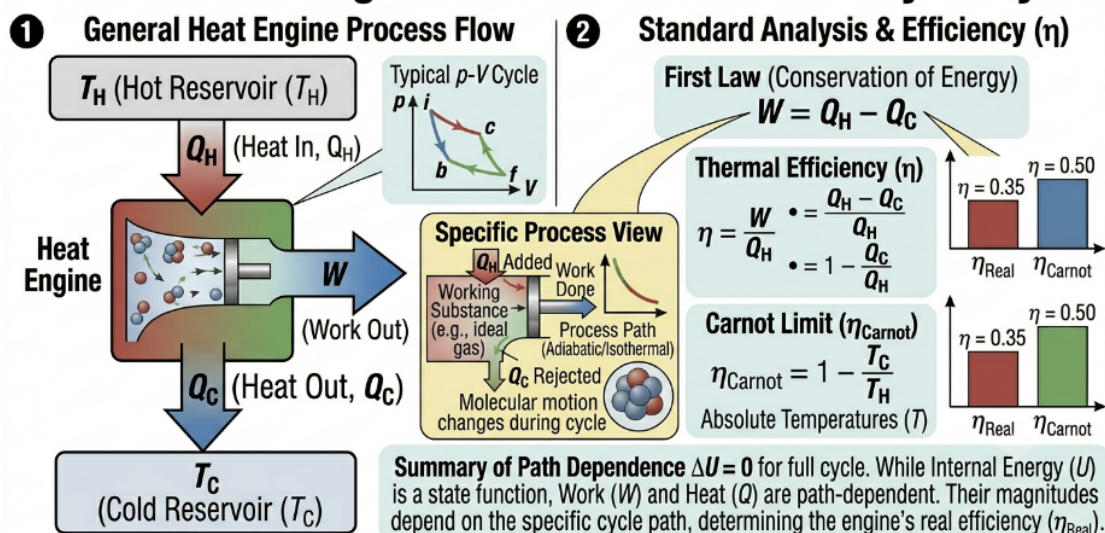
The Heat Engine Concept

A **heat engine** is a device that converts thermal energy into mechanical work. All heat engines operate on a cycle—the working substance (gas, steam, etc.) returns to its initial state after each cycle, allowing continuous operation.

Every heat engine requires three components:

1. **Hot reservoir** (heat source) at temperature T_H : Supplies heat Q_H to the engine
2. **Cold reservoir** (heat sink) at temperature T_C : Receives waste heat Q_C from the engine
3. **Working substance**: The material that undergoes the cyclic process

Standard Heat Engine Schematic and Efficiency Analysis



During one complete cycle, the engine:

- Absorbs heat Q_H from the hot reservoir
- Rejects heat Q_C to the cold reservoir
- Performs net work $W = Q_H - Q_C$

Since the working substance returns to its initial state, $\Delta U = 0$ over the cycle, and the First Law gives $W = Q_H - Q_C$.

Efficiency of a Heat Engine

The thermal efficiency η is the fraction of input heat that is converted to useful work:

$$\eta = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

Efficiency is often expressed as a percentage: $\eta\% = \eta \times 100\%$.

Key observation: For any real engine, $Q_C > 0$, so $\eta < 1$ (less than 100%). Some heat must always be rejected to the cold reservoir.

The Second Law of Thermodynamics

The First Law tells us that energy is conserved, but it does not forbid processes that never occur in nature. For example, the First Law would allow heat to flow spontaneously from a cold body to a hot body, or a

The Entropy Statement of the Second Law

The Second Law can be restated in terms of entropy:

The total entropy of an isolated system can never decrease. For reversible processes, $\Delta S_{\text{total}} = 0$; for irreversible processes, $\Delta S_{\text{total}} > 0$.

Mathematically:

$$\Delta S_{\text{universe}} = \Delta S_{\text{system}} + \Delta S_{\text{surroundings}} \geq 0$$

The equality holds for reversible processes; the inequality holds for irreversible (natural) processes.

Entropy and Disorder

Entropy is a measure of the **disorder** or **randomness** of a system. A highly ordered state (e.g., ice at 0°C) has low entropy; a disordered state (e.g., water at 0°C) has higher entropy.

Examples of entropy increase:

- Melting of ice: Molecules change from ordered lattice to disordered liquid
- Mixing of gases: Molecules become more randomly distributed
- Heat flow from hot to cold: Energy becomes more evenly distributed

Entropy and the Degradation of Energy

Every time entropy increases, the opportunity to convert heat into work is lost. When hot and cold water are mixed, the resulting warm water cannot be spontaneously separated back into hot and cold layers—some energy has become **unavailable** for doing work. This phenomenon is called the **degradation of energy**.

Physical interpretation: Energy is conserved (First Law), but its **quality** degrades over time. Organized energy (mechanical work, electrical energy, concentrated heat) can be converted into disorganized energy (low-temperature heat), but the reverse is not possible without external work.

Microscopic Interpretation of Entropy: Boltzmann's Formula

Ludwig Boltzmann provided a microscopic interpretation of entropy that connects thermodynamics to statistical mechanics:

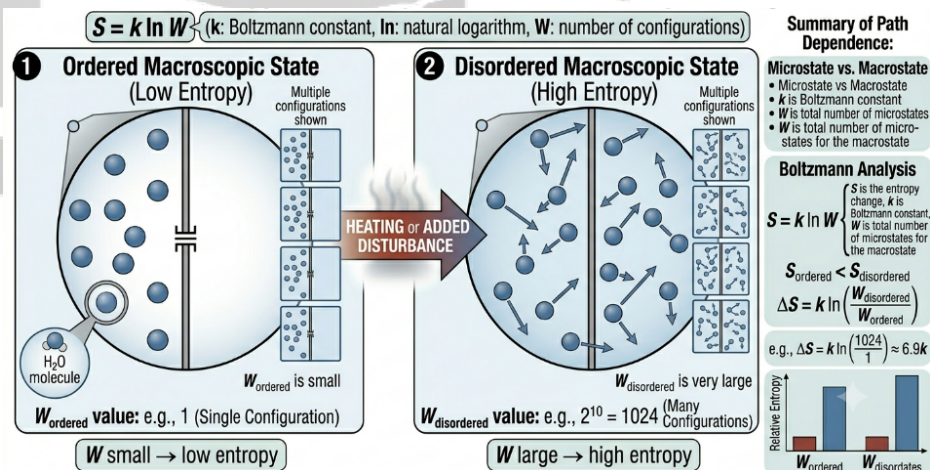
$$S = k \ln W$$

where:

- S = entropy (J/K)
- k = Boltzmann constant
 $1.38 \times 10^{-23} \text{ J/K}$
- W = the number of configurations (microstates) corresponding to a given macroscopic state

For N molecules distributed between two halves of a box, the multiplicity (number of microstates) is:

$$W = \frac{N!}{n_1! n_2!}$$



Multiplicity	$\Omega = \frac{N!}{n_1! n_2!}$	Number of microstates for a given macrostate
Boltzmann entropy	$S = k_B \ln \Omega$	Entropy is the logarithm of the number of microstates
Maxwell-Boltzmann	$f(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_B T}$	Speed distribution for classical particles
Bose-Einstein	$\langle n \rangle = \frac{1}{e^{(\epsilon - \mu)/k_B T} - 1}$	Distribution for bosons
Fermi-Dirac	$\langle n \rangle = \frac{1}{e^{(\epsilon - \mu)/k_B T} + 1}$	Distribution for fermions (Pauli exclusion)
Planck's law	$u(\nu) = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/k_B T} - 1}$	Blackbody radiation (photons)
Helmholtz free energy	$F = U - TS$	Useful work at constant T, V
Gibbs free energy	$G = H - TS = U + PV - TS$	Useful work at constant T, P
Maxwell relation	$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$	Relates thermodynamic properties
Clausius-Clapeyron	$\frac{dP}{dT} = \frac{L}{T(V_2 - V_1)}$	Phase equilibrium

Key Equations Reference Sheet

$$\begin{aligned} \langle E_k \rangle &= \frac{3}{2} kT & p &= \frac{1}{3} \rho v_{\text{rms}}^2 & v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} \\ pV &= nRT & \Delta U &= Q - W & W &= \int p dV \\ \Delta U &= nC_V \Delta T & W_{\text{isothermal}} &= nRT \ln \frac{V_f}{V_i} & pV^\gamma &= \text{constant} \\ \eta &= 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H} & \Delta S &= \int \frac{dQ_{\text{rev}}}{T} & S &= k \ln W \\ \left(p + \frac{an^2}{V^2}\right) &(V - nb) & &= nRT \end{aligned}$$

Heat And Thermodynamics: One-Liners

Fundamental Concepts and Temperature

1. Thermodynamics studies the relationships between heat, work, temperature, and energy in physical systems.
2. The system is the portion of the universe under investigation; the surroundings constitute everything outside the system.
3. An isolated system exchanges neither matter nor energy with its surroundings.
4. A closed system exchanges energy but not matter with its surroundings.
5. An open system exchanges both matter and energy with its surroundings.
6. State variables are measurable quantities such as pressure, volume, temperature, and number of moles.
7. A state function depends only on the current state of the system, not on how that state was achieved.
8. Internal energy and entropy are state functions; heat and work are path-dependent quantities.
9. The Zeroth Law of Thermodynamics: If two systems are each in thermal equilibrium with a third system, then they are in thermal equilibrium with each other.
10. The Zeroth Law establishes that temperature is a meaningful and measurable property.
11. Temperature is related to the average kinetic energy of the molecules of a substance.
12. The Kelvin scale is based on the triple point of water, defined as $T_3 = 273.16 \text{ K}$.
13. Absolute zero is $0 \text{ K} = -273.15^\circ\text{C}$, the temperature of minimum possible total energy.
14. The relationship between Kelvin and Celsius is $T_K = T_C + 273.15$.
15. Celsius to Fahrenheit conversion: $T_F = \frac{9}{5}T_C + 32^\circ$.
16. Temperature differences are the same on Kelvin and Celsius scales; $1 \text{ K} = 1^\circ\text{C}$.

Thermal Expansion

17. Linear expansion: $\Delta L = \alpha L_0 \Delta T$, where α is the coefficient of linear expansion.
18. The final length is $L = L_0(1 + \alpha \Delta T)$.
19. All linear dimensions of an object—including holes—expand by the same fractional amount when heated.
20. Volume expansion: $\Delta V = \beta V_0 \Delta T$, where β is the coefficient of volume expansion.
21. For isotropic solids, $\beta = 3\alpha$.
22. Water exhibits anomalous expansion; between 0°C and 4°C , it contracts as temperature increases.
23. Water has maximum density at 4°C .
24. Thermal stress in a clamped rod: $F/A = -Y\alpha\Delta T$, where Y is Young's modulus.

Heat and Calorimetry

25. Heat is energy transferred between a system and its environment solely because of a temperature difference.
26. The SI unit of heat is the joule (J); $1 \text{ cal} = 4.1868 \text{ J}$.
27. Specific heat: $Q = mc\Delta T$, where c is the specific heat capacity.
28. Molar heat capacity: $Q = nC\Delta T$, where C is molar heat capacity.
29. For gases, heat capacities at constant volume (C_V) and constant pressure (C_p) differ.
30. Phase change heat: $Q = \pm mL$, where L is the latent heat of transformation.

- C) Isolated system
D) Adiabatic system

Correct Answer: C

3. A closed system exchanges:

- A) Matter but not energy
B) Energy but not matter
C) Both matter and energy
D) Neither matter nor energy

Correct Answer: B

4. A state function depends on:

- A) The path taken to reach the state
B) The current state only
C) The history of the system
D) The surroundings only

Correct Answer: B

5. Which of the following is a state function?

- A) Heat
B) Work
C) Internal energy
D) Both A and B

Correct Answer: C

6. The Zeroth Law of Thermodynamics establishes:

- A) Conservation of energy
B) The direction of heat flow
C) Temperature as a meaningful property
D) The impossibility of perpetual motion

Correct Answer: C

7. The triple point of water is defined as:

- A) 273.15 K
B) 273.16 K
C) 373.15 K
D) 0 K

Correct Answer: B

8. Absolute zero is:

- A) 0°C
B) -273.15°C
C) 273.15 K
D) -273.15 K

Correct Answer: B

9. The relationship between Kelvin and Celsius is:

$$A) T_K = T_C - 273.15$$

$$B) T_K = T_C + 273.15$$

$$C) T_K = T_C$$

$$D) T_K = T_C + 273$$

Correct Answer: B

10. Celsius to Fahrenheit conversion is:

$$A) T_F = \frac{9}{5}T_C + 32$$

$$B) T_F = \frac{5}{9}(T_C - 32)$$

$$C) T_F = T_C + 32$$

$$D) T_F = \frac{9}{5}T_C - 32$$

Correct Answer: A

11. Linear expansion is given by:

$$A) \Delta L = \alpha L_0 \Delta T$$

$$B) \Delta L = \beta L_0 \Delta T$$

$$C) \Delta L = \alpha L_0 T$$

$$D) \Delta L = \alpha \Delta T / L_0$$

Correct Answer: A

12. For isotropic solids, the volume expansion coefficient is:

$$A) \beta = \alpha$$

$$B) \beta = 2\alpha$$

$$C) \beta = 3\alpha$$

$$D) \beta = \alpha/3$$

Correct Answer: C

13. Water has maximum density at:

$$A) 0^\circ\text{C}$$

$$B) 4^\circ\text{C}$$

$$C) 100^\circ\text{C}$$

$$D) -4^\circ\text{C}$$

Correct Answer: B

14. Heat is defined as:

- A) The total energy of a system
B) Energy transferred due to temperature difference
C) The temperature of a system
D) The work done by a system

Correct Answer: B

15. The SI unit of heat is the:

- A) Calorie
B) Joule
C) Watt



Part IV: Electromagnetism



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Chapter 10: Electrostatics

Introduction

Electrostatics — the study of electric charges at rest — forms the foundation of electromagnetism and much of modern physics. From the simple observation that amber rubbed with wool attracts lightweight particles (a phenomenon known to the ancient Greeks over 2,500 years ago) to the sophisticated operation of photocopiers, laser printers, touch screens, and defibrillators, electrostatic principles govern a vast range of physical phenomena and technological applications.

The significance of electrostatics extends far beyond classroom exercises. The chemical bonds that hold atoms together to form molecules are fundamentally electric in nature. The normal force that prevents your feet from sinking through the floor arises from electric repulsion between the electrons in your shoes and those in the floor. The

shock you feel after walking across a carpet on a dry day, the transmission of nerve signals in your body, and even the behavior of lightning — all are manifestations of electrostatic principles.

One of the primary goals of physics is to find simple ways of solving what appear

to be labor-intensive problems. In electrostatics, we often need to determine the electric field or potential due to extended charged objects. Directly summing the contributions from every charged particle using Coulomb's law can be impractical. Two powerful tools simplify such calculations:

1. **Gauss's law** — relates the electric field on a closed surface to the net charge enclosed.
2. **Electric potential** — a scalar quantity that simplifies energy calculations.

This chapter develops these tools step by step, from the fundamental properties of electric charge to the storage of energy in capacitors. By the end of this chapter, you will understand not only *what* happens when charges interact, but *why* — from the atomic level to large-scale natural phenomena like lightning.

The Nature of Electric Charge

Electric charge is a fundamental property of matter, existing in two types: **positive** and **negative**.

Following Benjamin Franklin's convention, the charge on a glass rod rubbed with silk is called positive; the charge on a plastic rod rubbed with fur is called negative.

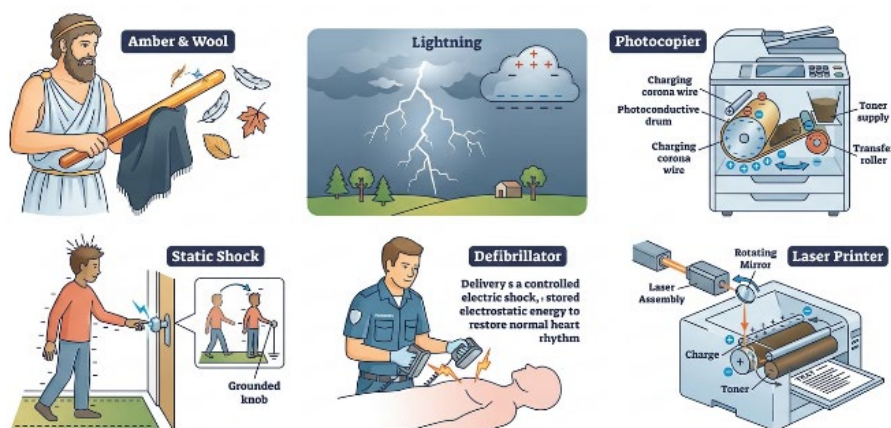
Like charges repel; unlike charges attract. This simple statement is the cornerstone of all electrostatic phenomena.

The Structure of Matter and the Origin of Charge

To understand what electric charge is and why objects can become charged, we must look inside the atom.

Ordinary matter consists of three fundamental particles:

Particle	Charge	Mass (kg)	Location
Electron	$-e = -1.602 \times 10^{-19} \text{ C}$	9.109×10^{-31}	Outside nucleus



from ground. Removing the ground, then the charged object, leaves the conductor permanently charged — opposite in sign to the inducing charge.

Polarization in Insulators

When a charged object is brought near an insulator, the charges within the insulator cannot flow, but they can **redistribute slightly** — a phenomenon called **polarization**.

The positive charges shift slightly toward the negative object, and negative charges shift away. This creates an induced dipole, resulting in a net attractive force even though the insulator is neutral overall.

This explains why a charged balloon sticks to a neutral wall, and why a charged comb attracts small pieces of paper.

COULOMB'S LAW — THE FORCE BETWEEN CHARGES

Statement of the Law

After extensive experiments using a torsion balance, Charles Augustin de Coulomb established in 1784 the fundamental law governing the electrostatic force between charged particles.

Coulomb's Law: The magnitude of the electrostatic force between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.

Mathematical Formulation

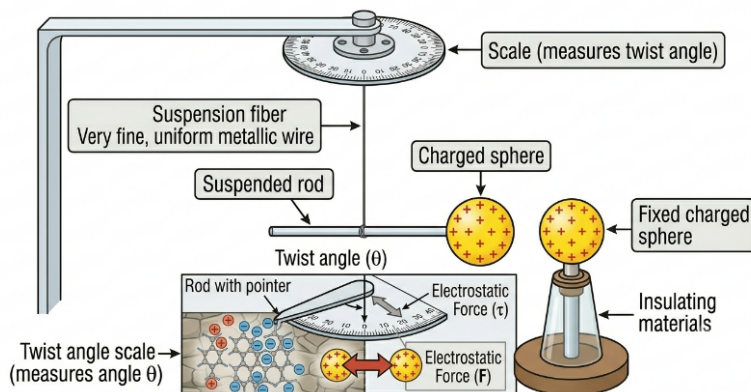
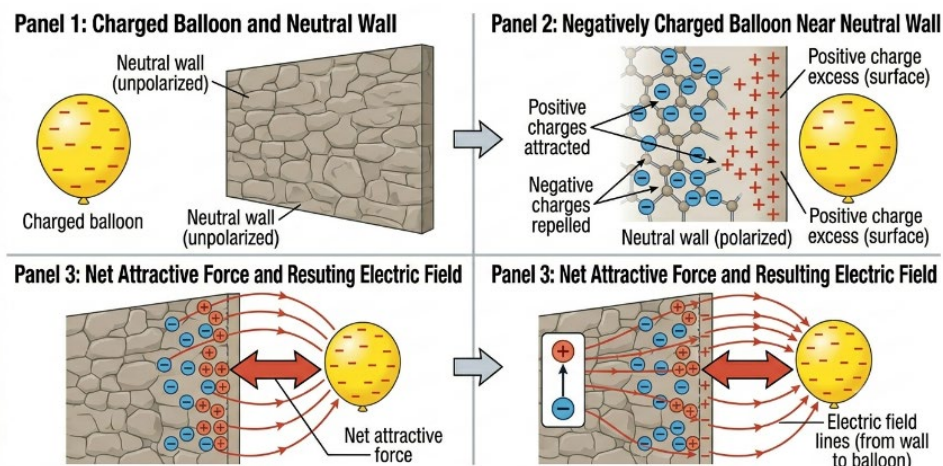
$$F = k \frac{|q_1 q_2|}{r^2}$$

In vector form, the force on charge q_2 due to q_1 is:

$$\vec{F}_{21} = k \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

where:

- F = magnitude of electrostatic force (N)



THE ELECTRIC FIELD

How does one charged object know that another charged object is nearby? The concept of **action-at-a-distance** — the idea that forces act instantaneously across empty space — was philosophically troubling to many physicists. Michael Faraday introduced a more satisfying picture: a charged object modifies the space around it, creating

an **electric field**. A second charged object then experiences a force because it is in this field.

Definition: An electric field is a region of space around a charged object in which another charged object experiences an electrostatic force.

Electric Field Intensity

Electric field intensity $\vec{E}$ at a point is the force per unit positive test charge placed at that point:

$$\vec{E} = \frac{\vec{F}}{q_0}$$

where q_0 is a small positive test charge (small enough not to disturb the original field).

Key points:

- The test charge q_0 must be small enough that it does not disturb the charge distribution that creates the field.
- The electric field exists at a point **whether or not** a test charge is present there.
- The SI unit of electric field is the newton per coulomb (N/C) or, equivalently, the volt per meter (V/m).

If the electric field $\vec{E}$ at a point is known, the force on any charge q placed at that point is:

$$\vec{F} = q\vec{E}$$

Electric Field Due to a Point Charge

From Coulomb's law, the force on a test charge q_0 at distance r from source charge Q is:

$$F = \frac{1}{4\pi\epsilon_0} \frac{|Qq_0|}{r^2}$$

Dividing by q_0 gives the electric field magnitude:

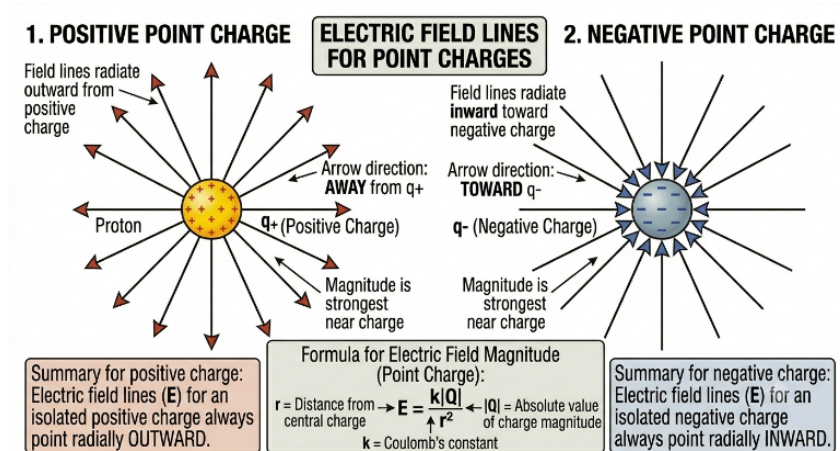
$$E = \frac{1}{4\pi\epsilon_0} \frac{|Q|}{r^2}$$

In vector form (for a positive source charge):

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

Key points:

- E decreases as $1/r^2$ (inverse square law)



ELECTRIC FLUX AND GAUSS'S LAW

Electric Flux

Before stating Gauss's law, we must understand **electric flux** — a measure of the number of electric field lines passing through a surface. Despite the analogy to fluid flow, nothing actually flows; flux is a mathematical construct.

For a uniform field $\vec{E}$ and a flat surface of area A with unit normal $\hat{n}$:

$$\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \theta$$

where θ is the angle between $\vec{E}$ and the normal to the surface.

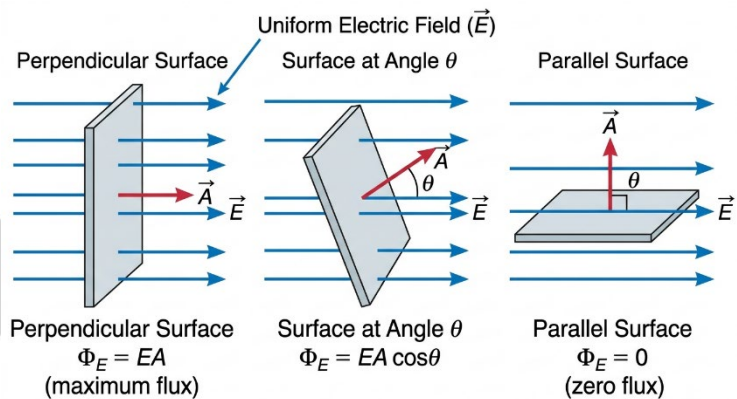
For a general (non-uniform) field and a curved surface, we divide the surface into infinitesimal elements $d\vec{A}$ and integrate:

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

For a **closed surface**, we adopt the convention that $d\vec{A}$ points **outward**.

Then:

- $\Phi_E > 0$ means net outward flux (field lines leaving the enclosed volume)
- $\Phi_E < 0$ means net inward flux (field lines entering the enclosed volume)
- $\Phi_E = 0$ means field lines are tangent to the surface or equal numbers enter and leave



Special cases for a flat surface in uniform field:

- $\theta = 0^\circ$ (surface perpendicular to field): $\Phi_E = EA$ (maximum)
- $\theta = 90^\circ$ (surface parallel to field): $\Phi_E = 0$ (minimum)

Gauss's Law:

Gauss's law relates the net electric flux through a closed surface to the total charge enclosed:

The net electric flux through any closed surface is equal to the total charge enclosed divided by ϵ_0 .

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

Equivalently:

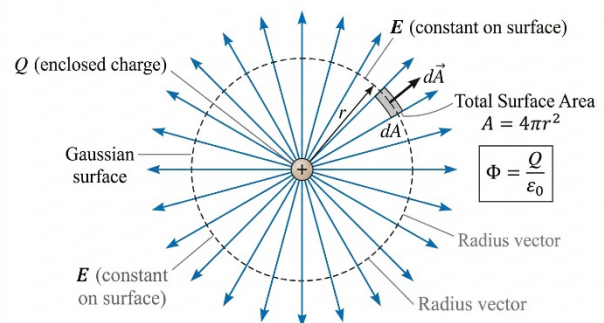
$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}$$

The circle on the integral sign indicates integration

over a **closed surface** (called a Gaussian surface). The Gaussian surface is imaginary; we are free to choose any closed surface that simplifies our calculation.

Key points:

- Q_{enc} is the algebraic sum of all charges **inside** the surface.
- Charges **outside** the surface contribute zero net flux.
- The field $\vec{E}$ in the integral is due to **all** charges (inside and outside).



$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^3} r$$

The field increases linearly with r inside the sphere.

Cylindrical Symmetry

For an infinite line of charge or infinite cylindrical shell:

- Gaussian surface: coaxial cylinder of radius r and length L
- Flux through curved surface: $E \cdot 2\pi r L$
- Flux through ends: zero (field is parallel to the ends)

For infinite line of charge (linear density λ):

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Inside a uniformly charged solid cylinder (volume density ρ , radius R , $r < R$):

$$E = \frac{\rho r}{2\epsilon_0} = \frac{\lambda_{\text{enc}}}{2\pi\epsilon_0 r}$$

where $\lambda_{\text{enc}} = \rho\pi r^2$ is the charge per unit length enclosed.

Planar Symmetry

For an infinite sheet of charge:

- Gaussian surface: cylinder (pillbox) with axis perpendicular to sheet
- Flux through two ends: $EA + EA = 2EA$
- Flux through curved side: zero (field is parallel to the curved surface)

For a single infinite nonconducting sheet (surface density σ):

$$E = \frac{\sigma}{2\epsilon_0}$$

Direction: away from sheet if $\sigma > 0$, toward sheet if $\sigma < 0$.

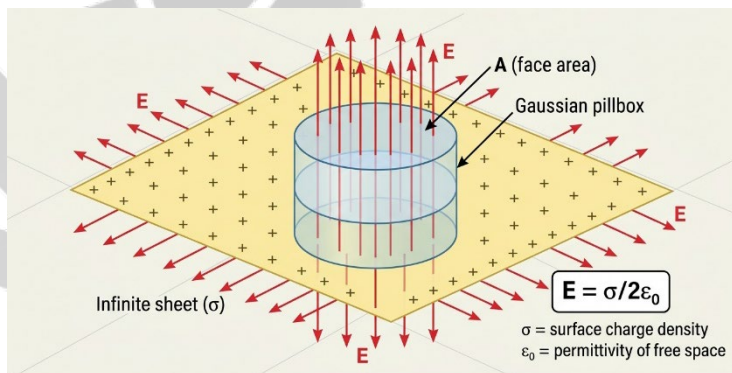
For two oppositely charged parallel plates (surface charge densities $+\sigma$ and $-\sigma$):

- Between plates: fields add, $E = \sigma/\epsilon_0$ (directed from positive to negative)
- Outside plates: fields cancel, $E = 0$

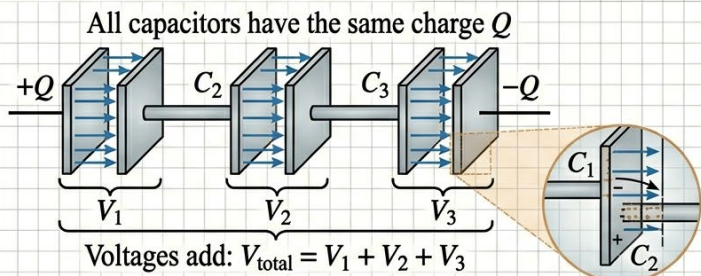
This is the **uniform field** we discussed earlier.

Results from Gauss's Law

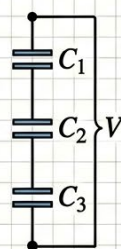
Charge Distribution	Gaussian Surface	Result
Point charge q	Sphere of radius r	$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$
Infinite line charge λ	Cylinder of radius r , length L	$E = \frac{\lambda}{2\pi\epsilon_0 r}$
Infinite sheet σ	Pillbox through sheet	$E = \frac{\sigma}{2\epsilon_0}$
Solid conducting sphere (charge Q , radius R) — outside ($r > R$)	Sphere	$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$
Solid conducting sphere — inside ($r < R$)	Sphere	$E = 0$



PANEL 1: SERIES COMBINATION



Series

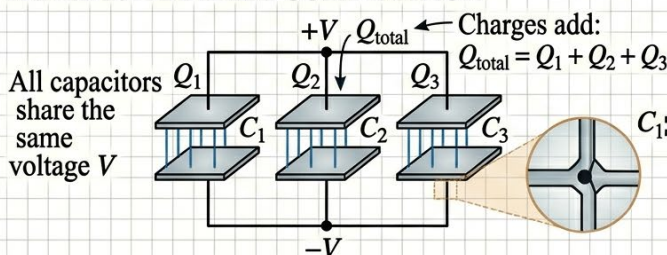


Key Relationships:

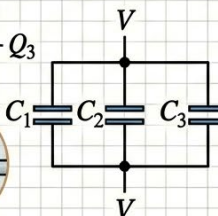
- All series capacitors have identical charge ($Q_{\text{total}} = Q_1 = Q_2 = Q_3$).
- Total voltage equals the sum of individual voltages ($V_{\text{total}} = V_1 + V_2 + V_3$).

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

PANEL 2: PARALLEL COMBINATION



Parallel



Key Relationships:

- All parallel capacitors share the same voltage ($V_{\text{total}} = V_1 = V_2 = V_3$).
- Total charge equals the sum of individual charges ($Q_{\text{total}} = Q_1 + Q_2 + Q_3$).

$$C_{\text{eq}} = C_1 + C_2 + C_3$$

Energy Stored in a Capacitor

When charging a capacitor, work is done to move charge from one plate to the other against the growing electric field. This work is stored as **electric potential energy**.

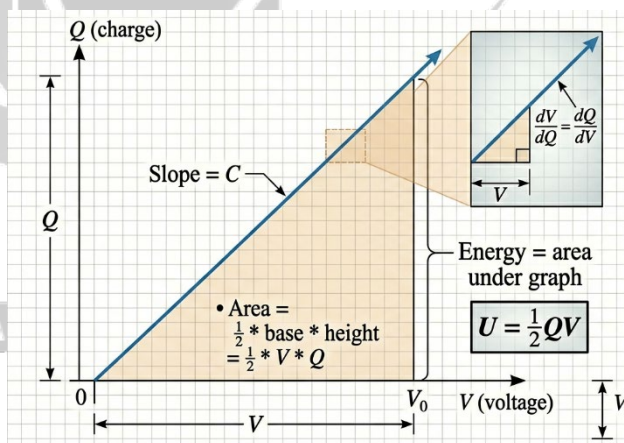
Consider charging from 0 to final charge Q and final voltage V . The average voltage during charging is $V/2$. Therefore:

$$U = \frac{1}{2} QV$$

Using $Q = CV$ and $V = Q/C$, we obtain three equivalent forms:

$$U = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$$

Graphical interpretation: On a Q vs. V graph, the energy is the area under the straight line (a triangle):



$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times Q \times V$$

Energy density (energy per unit volume) in the electric field between plates:

$$u = \frac{U}{Ad} = \frac{1}{2} \epsilon_0 \epsilon_r E^2$$

This is a fundamental result: **energy is stored in the electric field itself**, not merely on the plates. This expression is universal — it applies to any electric field configuration, not just parallel-plate capacitors.

DIELECTRICS AND POLARIZATION

What is a Dielectric?

- **Torque** tends to align the dipole with the field:

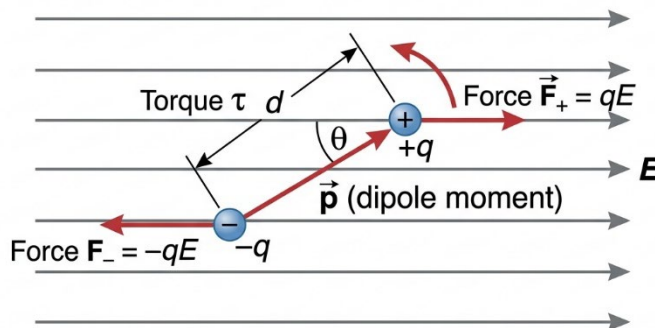
$$\vec{\tau} = \vec{p} \times \vec{E}$$

Magnitude: $\tau = pE \sin \theta$, where θ is the angle between $\vec{p}$ and $\vec{E}$.

Potential energy of the dipole in the field:

$$U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$$

- Minimum energy ($U = -pE$) occurs when $\vec{p}$ is parallel to $\vec{E}$ (stable equilibrium).
- Maximum energy ($U = +pE$) occurs when $\vec{p}$ is antiparallel to $\vec{E}$ (unstable equilibrium).



Formulas: $\tau = pE \sin \theta$
 $U = -pE \cos \theta$

$F_{\text{net}} = 0$, but $\tau_{\text{net}} \neq 0$

Field of a Dipole

For points far from the dipole ($r \gg d$):

- On the axis: $E \approx \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$
- On the perpendicular bisector: $E \approx \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$

The dipole field falls off as $1/r^3$, faster than the $1/r^2$ field of a point charge. This is because the net charge is zero, and the fields of the two nearly cancel at large distances.

CHARGING AND DISCHARGING OF CAPACITORS (RC CIRCUITS)

The RC Circuit

When a capacitor charges or discharges through a resistor R , the process is **exponential**, not linear. The time behavior is governed by the **time constant** $\tau = RC$.

Charging a Capacitor

Consider a battery of emf $\mathcal{E}$, resistor R , and capacitor C in series. At $t = 0$, the capacitor is uncharged. The charge as a function of time is:

$$q(t) = Q_0(1 - e^{-t/RC})$$

where $Q_0 = C\mathcal{E}$ is the final charge.

Current during charging:

$$i(t) = I_0 e^{-t/RC}, I_0 = \frac{\mathcal{E}}{R}$$

Discharging a Capacitor

If a charged capacitor (initial charge Q_0) discharges through a resistor R :

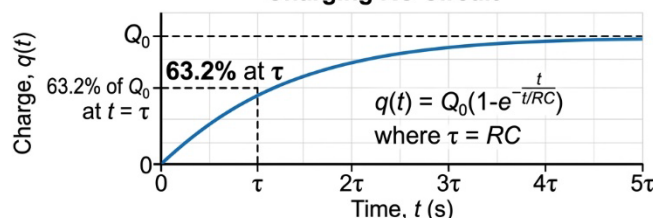
$$q(t) = Q_0 e^{-t/RC}$$

Time Constant $\tau = RC$

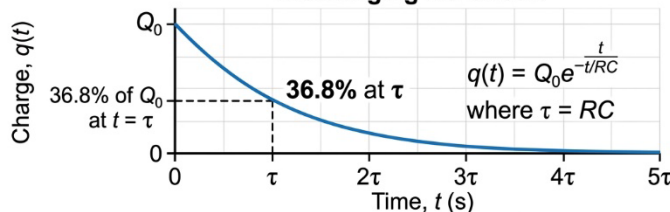
The **time constant** $\tau = RC$ has the following significance:

- **During charging:** After time τ , the capacitor reaches 63.2% of its final charge

Charging RC Circuit



Discharging RC Circuit





Electric flux	$\Phi_E = EA \cos \theta$	$\text{N} \cdot \text{m}^2/\text{C}$
Gauss's law	$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$	—
Electric potential (point charge)	$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$	V
Potential difference	$\Delta V = \frac{W}{q_0}$	V
Field-potential relation	$E = -\frac{dV}{dr}$	V/m
Capacitance	$C = \frac{Q}{V}$	F
Parallel-plate capacitor	$C = \frac{\epsilon_0 \epsilon_r A}{d}$	F
Series capacitors	$\frac{1}{C_{\text{eq}}} = \sum \frac{1}{C_i}$	F
Parallel capacitors	$C_{\text{eq}} = \sum C_i$	F
Energy stored	$U = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$	J
Energy density	$u = \frac{1}{2} \epsilon_0 \epsilon_r E^2$	J/m^3
RC time constant	$\tau = RC$	s
Charging (q)	$q = Q_0(1 - e^{-t/RC})$	C
Discharging (q)	$q = Q_0 e^{-t/RC}$	C
Dipole moment	$\vec{p} = q\vec{d}$	$\text{C} \cdot \text{m}$
Dipole torque	$\vec{\tau} = \vec{p} \times \vec{E}$	$\text{N} \cdot \text{m}$
Dipole potential energy	$U = -\vec{p} \cdot \vec{E}$	J

Electrostatics: One-Liners

The Nature of Electric Charge

1. Electric charge is a fundamental property of matter, existing in two types: positive and negative.
2. Like charges repel; unlike charges attract.
3. The electron has charge $-e = -1.602 \times 10^{-19}$ C and mass 9.109×10^{-31} kg.
4. The proton has charge $+e = +1.602 \times 10^{-19}$ C and mass 1.673×10^{-27} kg.
5. The neutron has zero charge and mass 1.675×10^{-27} kg.
6. In a neutral atom, the number of electrons equals the number of protons.
7. A positive ion is formed when an atom loses one or more electrons.
8. A negative ion is formed when an atom gains one or more extra electrons.
9. Ionization is the process of gaining or losing electrons.
10. Charging by rubbing involves transferring electrons from one material to another.
11. A negatively charged object has an excess of electrons; a positively charged object has a deficit of electrons.
12. The algebraic sum of all electric charges in any closed system is constant—conservation of charge.
13. Every observable amount of electric charge is an integer multiple of the elementary charge e : $q = ne$.
14. The elementary charge is $e = 1.602 \times 10^{-19}$ C.
15. Charge is quantized; quarks have charges of $\pm e/3$ and $\pm 2e/3$ but are never observed in isolation.
16. Conductors contain free electrons that can move freely throughout the material.
17. Insulators have no free electrons; all electrons are tightly bound to atoms.
18. Semiconductors have properties intermediate between conductors and insulators.
19. Charging by friction (triboelectric effect) transfers electrons between two dissimilar materials.
20. Charging by contact (conduction) transfers charge when a charged object touches a neutral conductor.
21. Charging by induction charges an object without direct contact using charge separation and grounding.
22. Polarization in insulators occurs when charges within the insulator redistribute slightly in response to an external field.
23. A charged balloon sticks to a neutral wall due to polarization and electrostatic attraction.

Coulomb's Law

24. Coulomb's Law: The magnitude of the electrostatic force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.
25. Mathematically: $F = k |q_1 q_2| / r^2 = (1/4\pi\epsilon_0) |q_1 q_2| / r^2$.
26. Coulomb constant: $k = 8.99 \times 10^9$ N · m²/C².
27. Permittivity of free space: $\epsilon_0 = 8.85 \times 10^{-12}$ C²/N · m².
28. The force between two point charges acts along the line joining them.
29. Same sign charges repel; opposite sign charges attract.



The Electric Dipole

135. An electric dipole consists of two charges $+q$ and $-q$ separated by distance d .
136. Dipole moment: $\vec{p} = q\vec{d}$, pointing from negative to positive.
137. In a uniform field: net force is zero; torque $\vec{\tau} = \vec{p} \times \vec{E}$.
138. Torque magnitude: $\tau = pE \sin \theta$.
139. Potential energy: $U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$.
140. Minimum energy at $\theta = 0$ (parallel, stable equilibrium); maximum at $\theta = 180^\circ$ (antiparallel, unstable equilibrium).
141. Dipole field falls off as $1/r^3$, faster than point charge field.

RC Circuits

142. Time constant: $\tau = RC$, with units of seconds.
143. Charging: $q(t) = Q_0(1 - e^{-t/RC})$, $i(t) = I_0 e^{-t/RC}$.
144. Discharging: $q(t) = Q_0 e^{-t/RC}$.
145. After one time constant, capacitor reaches 63.2% of final charge (charging) or falls to 36.8% (discharging).
146. After approximately 5τ , capacitor is considered fully charged or discharged.
147. Applications: camera flashes, timing circuits, filters, defibrillators.

Applications of Electrostatics

148. Electrostatic precipitators remove dust and smoke particles from industrial exhaust gases.
149. Photocopiers and laser printers use xerography: charging, exposure, developing, transfer, and fusing.
150. Electropainting uses electrostatic attraction for even coverage and reduced overspray.
151. Lightning rods protect buildings by providing a path for lightning to ground.
152. The resting potential across a cell membrane is approximately -70 mV.
153. Action potentials are brief reversals of membrane potential that travel along nerve cells.
154. The electric eel can generate up to 860 V to stun prey.
155. Lightning is a massive electrostatic discharge caused by charge separation in thunderclouds.
156. Dielectric breakdown of air occurs at approximately 3×10^6 V/m.

Practice MCQs

1. Electric charge is a fundamental property of matter that exists in:

- A) One type only
- B) Two types: positive and negative
- C) Three types
- D) Four types

Correct Answer: B

2. Like charges _____ and unlike charges _____.

- A) Attract, repel
- B) Repel, attract
- C) Attract, attract

D) Repel, repel

Correct Answer: B

3. The charge on an electron is:

- A) $+1.602 \times 10^{-19}$ C
- B) -1.602×10^{-19} C
- C) 1.602×10^{-19} C
- D) Zero

Correct Answer: B

4. The charge on a proton is:

- A) -1.602×10^{-19} C
- B) $+1.602 \times 10^{-19}$ C
- C) 1.602×10^{-19} C

Chapter 11: Current and Electricity

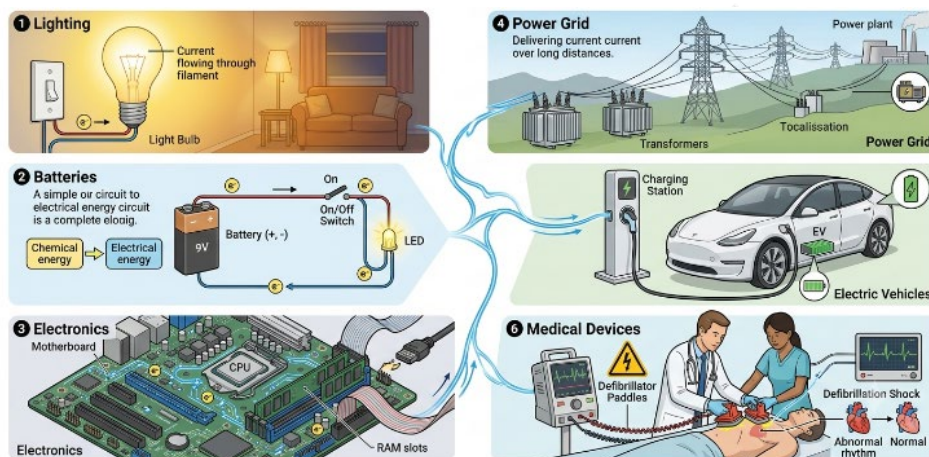
Introduction

Current electricity is the study of electric charges in motion—a phenomenon that powers virtually every aspect of modern life. From the moment you switch on a light bulb to the charging of an electric vehicle, the principles governing the flow of electric charge are at work. In the preceding chapters, we studied electrostatics—the physics of stationary charges. We now turn our attention to charges in motion.

An **electric current** consists of charges moving from one region to another. When these charges follow a conducting path that forms a closed loop, the path is called an **electric circuit**.

Electric circuits are fundamentally a means for conveying energy from one source to another location. As charged particles move within a circuit, electric potential energy is transferred from a source (such as a battery or generator) to a device where that energy is either stored or converted into another form: into sound in a stereo system, into light in a lamp, or into heat in a toaster.

Examples of electric currents abound and involve many professions:



- **Meteorologists** study lightning and the slow flow of charge through the atmosphere.
- **Biologists and physiologists** investigate nerve currents that control muscles.
- **Electrical engineers** design power systems, information storage systems, and communication networks.
- **Space engineers** monitor charged particles from the Sun that can disrupt telecommunications.

This chapter develops a comprehensive understanding of how electric currents are established, maintained, and utilized in electrical circuits. We begin by examining the fundamental concept of electric current and the mechanisms of electrical conduction in materials, particularly metals. We then explore the relationship between voltage, current, and resistance through Ohm's law, and investigate how factors such as material properties, dimensions, and temperature affect resistance. The chapter progresses to analyze how resistors and sources of electromotive force (emf) combine in series and parallel configurations, and introduces essential measurement techniques using ammeters, voltmeters, potentiometers, and Wheatstone bridges. Finally, we examine the practical applications of these principles in household wiring, electrical safety, and power distribution.

Throughout this chapter, we emphasize both conceptual understanding and mathematical rigor, ensuring that you develop the ability to analyze and solve real-world circuit problems. We restrict our discussion

to **direct-current (DC) circuits**, in which the direction of the current does not change with time. (Alternating current, or AC, where the current oscillates back and forth, will be examined in a later chapter.)

Electric Current: The Flow of Charge

Definition of Electric Current

Consider a conductor—a copper wire, for example—that contains a vast number of free electrons capable of moving throughout the material. Under normal conditions, these electrons move randomly in all directions, and their net displacement over any time interval is zero. However, when an external influence (such as a battery) is applied, these charges acquire a directed motion.

Electric current is defined as the time rate of flow of electric charge through a cross-sectional area of a conductor.

Mathematically, if a net charge dQ passes through a given cross-section in a time interval dt , the current I is given by:

$$I = \frac{dQ}{dt} \quad (7.1)$$

For situations where the current is constant (steady current), we write:

$$I = \frac{Q}{t} \quad (7.2)$$

Important distinction: If charge dQ is positive, the current direction is the same as the motion of positive charge. Current is a **scalar** quantity. Although we often draw arrows to indicate current direction, these arrows are not vectors and do not require vector addition; rather, currents in branches of a circuit add algebraically (with proper signs) according to Kirchhoff's current law.

The Ampere

The SI unit of electric current is the **ampere** (abbreviated A). One ampere is defined as the current that exists when one coulomb of charge flows through a cross-sectional area in one second:

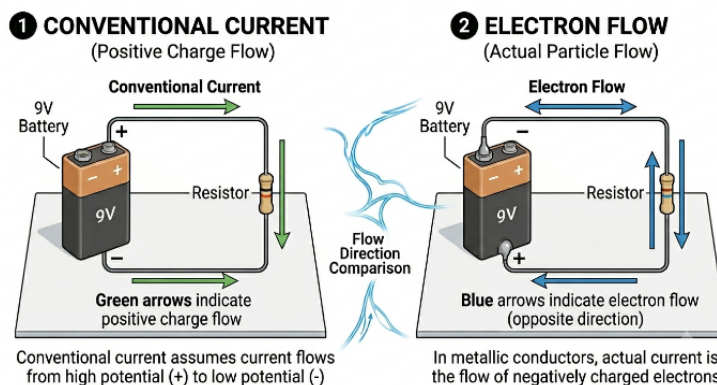
$$1 \text{ A} = 1 \frac{\text{C}}{\text{s}}$$

The ampere is a fundamental SI unit, meaning that the coulomb is actually derived from it ($1 \text{ C} = 1 \text{ A} \cdot \text{s}$). In practice, the ampere is a relatively large unit, so smaller submultiples are commonly used:

- **Milliampere (mA):** $1 \text{ mA} = 10^{-3} \text{ A}$
- **Microampere (μA):** $1 \mu\text{A} = 10^{-6} \text{ A}$

Conventional Current vs. Electron Flow

An important historical distinction must be understood. Before the discovery of the electron, scientists assumed that electric current consisted of positive charges moving from the positive terminal to the negative terminal of a battery. This assumption is now known as **conventional current**.

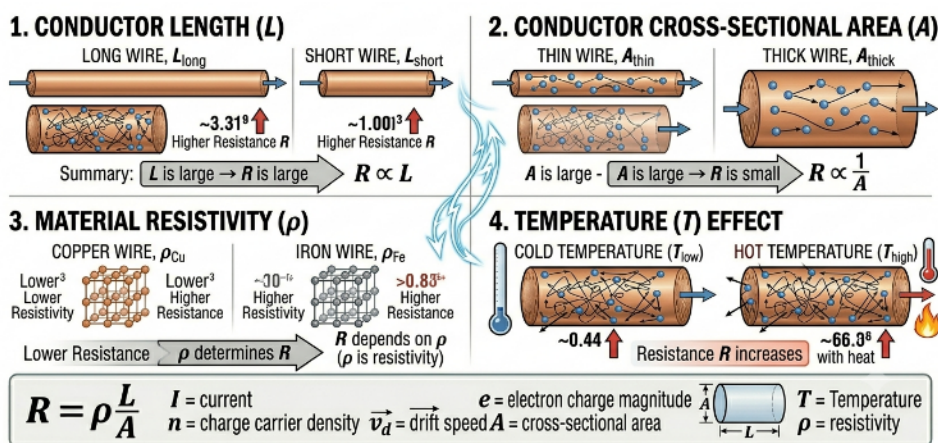


Rubber	$10^{13}-10^{16}$	Insulator
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Factors Affecting Resistance

For a given conductor at constant temperature, the resistance depends on four factors:

- Length (L):** Resistance is directly proportional to length. Longer conductors offer more opposition because charge carriers must travel farther and encounter more collisions. $R \propto L$
- Cross-sectional Area (A):** Resistance is inversely proportional to cross-sectional area. Thicker wires provide more pathways for current, reducing resistance. $R \propto 1/A$
- Material (ρ):** Different materials have different inherent abilities to conduct current, quantified by their **resistivity**.
- Temperature (T):** Resistance changes with temperature, as discussed in Section 7.6.



Conductance

Conductance (G) is the reciprocal of resistance:

$$G = \frac{1}{R} \quad (7.18)$$

The SI unit of conductance is the **siemens** (S), also called the mho (ohm spelled backward). $1 \text{ S} = 1 \Omega^{-1}$.

Temperature Dependence of Resistance

The resistance of most materials changes with temperature, but the direction of change depends on the type of material.

For Metals (Conductors)

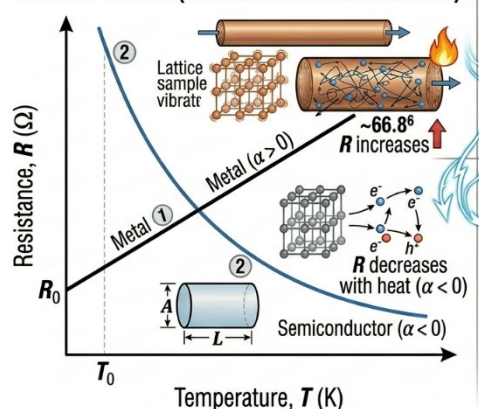
Resistance **increases** with increasing temperature. As temperature rises, lattice ions vibrate more vigorously, increasing the frequency of collisions with moving electrons. The temperature dependence is approximately linear over normal temperature ranges:

$$R_T = R_0(1 + \alpha T) \quad (7.19)$$

where:

- R_0 = resistance at 0°C
- R_T = resistance at temperature T (in $^\circ\text{C}$)
- α = temperature coefficient of resistance (units: $^\circ\text{C}^{-1}$ or K^{-1})

1. R vs T GRAPH (METAL vs SEMICONDUCTOR)

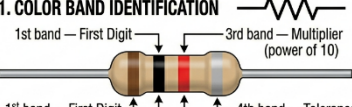


Color Code Table:

Color	Digit	Multiplier	Tolerance
Black	0	$\times 10^0$	—
Brown	1	$\times 10^1$	$\pm 1\%$
Red	2	$\times 10^2$	$\pm 2\%$
Orange	3	$\times 10^3$	—
Yellow	4	$\times 10^4$	—
Green	5	$\times 10^5$	$\pm 0.5\%$
Blue	6	$\times 10^6$	$\pm 0.25\%$
Violet	7	$\times 10^7$	$\pm 0.1\%$
Gray	8	$\times 10^8$	—
White	9	$\times 10^9$	—
Gold	—	$\times 10^{-1}$	$\pm 5\%$
Silver	—	$\times 10^{-2}$	$\pm 10\%$
No band	—	—	$\pm 20\%$

RESISTOR COLOR CODE DIAGRAM

1. COLOR BAND IDENTIFICATION



Resistance, $R = (\text{Digit 1 Digit 2}) \times 10^{\text{Multiplier}} \pm \text{Tolerance\%}$

3. CALCULATED EXAMPLE

1st: Brown (1) 2nd: Black (0) 3rd: Red ($\times 100$) 4th: Silver ($\pm 10\%$)
 Calculation: $10 \times 100 \Omega = 1000 \Omega$
 Final Value: **1 k Ω $\pm 10\%$**
 Tolerance: $\pm 100 \Omega$

2. COLOR CODE LOOKUP TABLE

Color	Digit	Multiplier (10^n)	Tolerance (%)
Black	0	10^0	—
Brown	1	10^1	$\pm 1\%$
Red	2	10^2	$\pm 2\%$
Orange	3	10^3	—
Yellow	4	10^4	—
Green	5	10^5	$\pm 0.5\%$
Blue	6	10^6	$\pm 0.25\%$
Violet	7	10^7	$\pm 0.1\%$
Gray	8	10^8	—
White	9	10^9	—
Gold	—	10^{-1}	$\pm 5\%$
Silver	—	10^{-2}	$\pm 10\%$

4. MNEMONIC DEVICE

BB ROY of Great Britain had a Very Good Wife

B=Black=0 6=Brown=6
 R=Red=2 7=Green=8
 G=Grey=4 4=Silver=9

Reading Procedure:

1. Hold the resistor with color bands to the left (metallic bands, if present, are always on the right).
2. First band = first digit
3. Second band = second digit
4. Third band = multiplier (power of 10)
5. Fourth band = tolerance (if present)

Mnemonic for color code order: "BB ROY of Great Britain had Very Good Wife" (Black, Brown, Red, Orange, Yellow, Green, Blue, Violet, Gray, White).

Example: Brown-Black-Red-Silver = $10 \times 10^2 \Omega = 1000 \Omega = 1 \text{ k}\Omega$ with $\pm 10\%$ tolerance.

The Potential Divider (Voltage Divider)

A potential divider is a fundamental circuit that produces an output voltage that is a fraction of its input voltage. It consists of two resistors (or a single variable resistor with a sliding contact) connected in series across a voltage source.

Consider a voltage V_{in} applied across two resistors R_1 and R_2 connected in series. The same current I flows through both resistors:

$$I = \frac{V_{in}}{R_1 + R_2}$$

The output voltage V_{out} (taken across R_2) is:

$$V_{out} = IR_2 = \frac{R_2}{R_1 + R_2} V_{in} \quad (7.24)$$

When a variable resistor (potentiometer) is used, the sliding contact allows R_2 to vary from 0 to the total resistance, giving V_{out} from 0 to V_{in} .

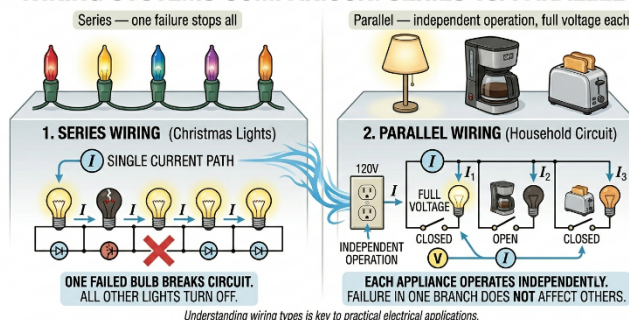
Applications of Potential Dividers:

- Volume control knobs in audio equipment
- Sensor circuits (using LDRs or thermistors as one resistor)

Why household circuits are wired in parallel:

1. Each appliance receives the full supply voltage (220 V in Pakistan).
2. Appliances can be switched on/off independently.
3. If one appliance fails, others continue to operate.
4. Adding more appliances does not affect the voltage across existing ones (though it increases total current).

WIRING SYSTEMS COMPARISON: SERIES vs. PARALLEL



Combinations of emf Sources

Just as resistors can be combined, sources of electromotive force (batteries, cells) can be connected in series or parallel to achieve desired voltage and current ratings.

Series Combination of emf Sources

When cells are connected with the positive terminal of one connected to the negative terminal of the next, they are in **series**.

For n cells in series:

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \dots + \varepsilon_n \quad (7.31)$$

If the cells are identical (each with emf ε and internal resistance r), then:

$$\varepsilon_{eq} = n\varepsilon, r_{eq} = nr \quad (7.32)$$

Advantages: Higher voltage output.

Disadvantages: If one cell fails (goes open), the entire circuit stops. If cells are mismatched, weaker cells may be forced into reverse charging.

Application: Series connections are used when a higher voltage is required than any single cell can provide (e.g., 12 V battery from six 2 V cells).

Parallel Combination of emf Sources

When the positive terminals of all cells are connected together and the negative terminals are connected together, the cells are in **parallel**.

For identical cells in parallel:

$$\varepsilon_{eq} = \varepsilon (\text{same as one cell}) \quad (7.33)$$

$$r_{eq} = \frac{r}{n} (\text{internal resistance decreases}) \quad (7.34)$$

Advantages: Higher current capacity (ampere-hour rating increases). Lower internal resistance allows greater current delivery without excessive voltage drop.

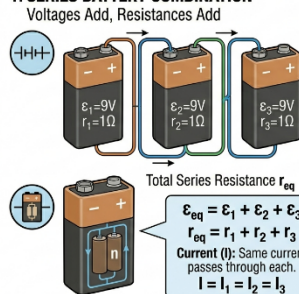
Disadvantages: No increase in voltage. Cells must be well-matched; mismatched cells can circulate current between themselves.

Application: Parallel connections are used when higher current is required at the same voltage (e.g., backup battery banks, electric vehicle battery packs).

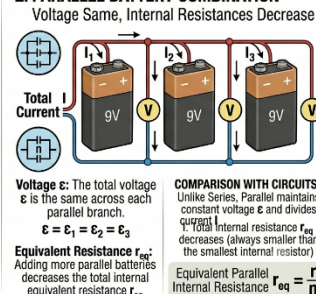
Practical Considerations for Battery Combinations

BATTERY COMBINATIONS DIAGRAM

1. SERIES BATTERY COMBINATION



2. PARALLEL BATTERY COMBINATION



Typical specifications: full-scale current $I_{fs} \sim 1 \text{ mA}$, coil resistance $R_c \sim 20 \Omega$.

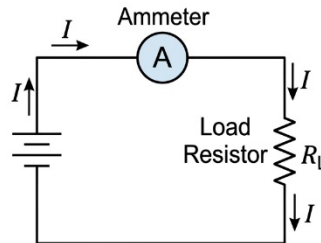
The Ammeter

An **ammeter** is a device used to measure electric current.

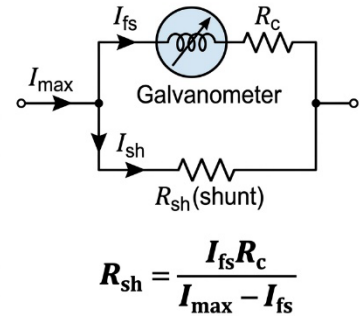
Connection: Ammeters must be connected **in series** with the component whose current is to be measured. This ensures that all the current flowing through the component also flows through the ammeter.

Ideal Ammeter: An ideal ammeter has **zero resistance** so that it does not affect the circuit current. Real ammeters have very low resistance (typically 1Ω or less).

① Ammeter — series connection (low resistance)



② Range extension using shunt



Range Extension using Shunt Resistor: To extend the range of a galvanometer to measure larger currents, a **shunt resistor** R_{sh} is connected in parallel with the meter:

$$R_{sh} = \frac{I_{fs} R_c}{I_{max} - I_{fs}} \quad (7.38)$$

where I_{fs} is the full-scale current of the galvanometer, R_c is its coil resistance, and I_{max} is the desired full-scale current for the ammeter.

Range Selection: Ammeters are available with different current ranges:

- Microammeter (μA) — for very small currents
- Milliammeter (mA) — for small currents
- Ammeter (A) — for larger currents

The Voltmeter

A **voltmeter** is a device used to measure potential difference (voltage).

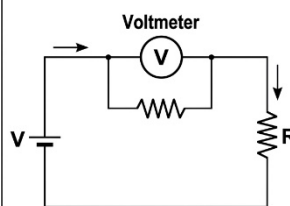
Connection: Voltmeters must be connected **in parallel** with the component across which the voltage is to be measured. This means the voltmeter is connected to the two points between which the potential difference is desired.

Ideal Voltmeter: An ideal voltmeter has **infinite resistance** so that it draws no current from the circuit. Real voltmeters have very high resistance (typically $10 \text{ M}\Omega$ or more).

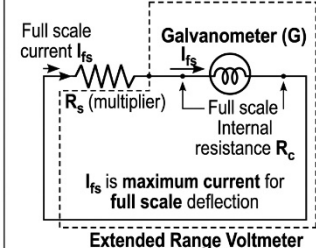
Range Extension using Multiplier Resistor: To extend the range of a galvanometer to measure larger voltages,

a **multiplier resistor** R_s is connected in series:

① Voltmeter — parallel connection (high resistance)



② Range extension using multiplier



$$R_s = \frac{V_{max}}{I_{fs}} - R_c \quad (7.39)$$

For a battery with emf ε and internal resistance r :

- Rate of chemical energy conversion: $P_{\text{chem}} = \varepsilon I$
- Rate of internal dissipation: $P_{\text{internal}} = I^2 r$
- Net electrical power output: $P_{\text{output}} = \varepsilon I - I^2 r = V_{ab} I$

When a battery is being **charged** (current forced from + to -), electrical energy is converted to chemical energy: $P_{\text{input}} = \varepsilon I + I^2 r$.

Maximum Power Transfer Theorem

In many applications (e.g., connecting an amplifier to speakers, or a transmitter to an antenna), we want to deliver the **maximum possible power** to the load. The maximum power transfer theorem states:

Maximum power is delivered to a load when the load resistance equals the internal resistance of the source.

Consider a source with emf ε and internal resistance r connected to a load resistance R .

The current is:

$$I = \frac{\varepsilon}{R + r}$$

Power delivered to the load is:

$$P = I^2 R = \frac{\varepsilon^2 R}{(R + r)^2}$$

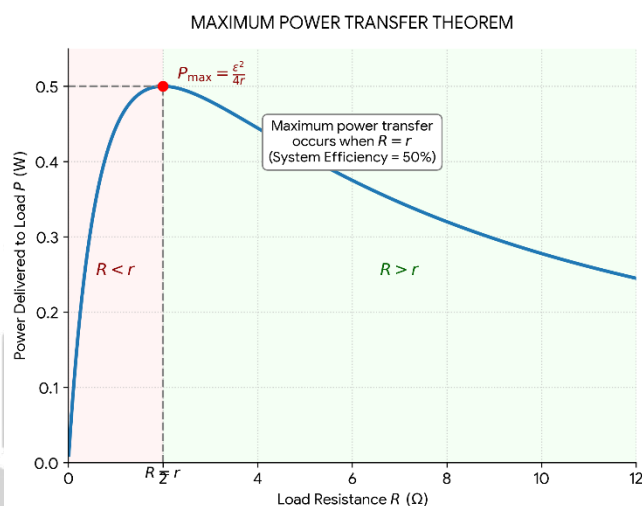
To find the value of R that maximizes P , differentiate with respect to R and set to zero.

The result is:

$$\boxed{R = r} \quad (7.47)$$

The maximum power delivered is:

$$\boxed{P_{\text{max}} = \frac{\varepsilon^2}{4r} = \frac{\varepsilon^2}{4R}} \quad (7.48)$$



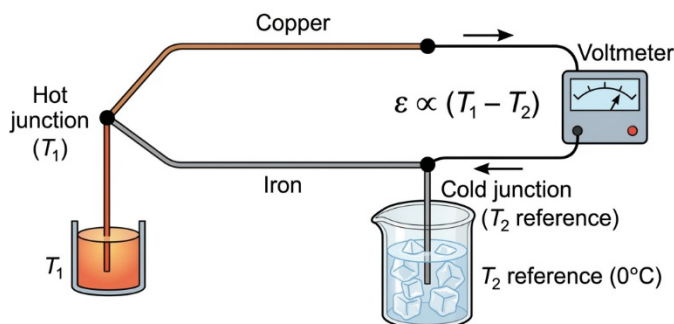
Important: Maximum power transfer does NOT mean maximum efficiency. At $R = r$, the efficiency is only 50% (half the power is dissipated in the internal resistance). For applications requiring efficiency (like power transmission), we want $R \gg r$.

Thermoelectric Effects

The Thermocouple

A **thermocouple** is a temperature-measuring device that converts thermal energy directly into electrical energy through the Seebeck effect (discovered by Thomas Seebeck in 1821).

Construction: A thermocouple consists of two dissimilar metal wires (e.g., copper and iron) joined at two ends to form two junctions.



Applying Kirchhoff's loop rule:

$$\varepsilon - iR - \frac{q}{C} = 0 \text{ with } i = \frac{dq}{dt}$$

This gives the differential equation:

$$R \frac{dq}{dt} + \frac{q}{C} = \varepsilon$$

Solving with initial condition $q(0) = 0$:

$$q(t) = C\varepsilon(1 - e^{-t/RC}) \quad (7.50)$$

$$i(t) = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/RC} \quad (7.51)$$

Key observations:

- At $t = 0$: $q = 0$, $i = \varepsilon/R$ (initial current)
- As $t \rightarrow \infty$: $q \rightarrow C\varepsilon$ (final charge), $i \rightarrow 0$

Time Constant

The product RC has dimensions of time and is called the **capacitive time constant**:

$$\tau = RC \quad (7.52)$$

At $t = \tau$: $q = C\varepsilon(1 - e^{-1}) \approx 0.632 C\varepsilon$ (63% of final)

- At $t = \tau$: $i = (\varepsilon/R)e^{-1} \approx 0.368 (\varepsilon/R)$ (36.8% of initial)

After 5τ , the capacitor is essentially fully charged ($> 99\%$).

Discharging a Capacitor

Now suppose the capacitor has initial charge $q_0 = C\varepsilon$ and is discharged through resistor R (switch moved to position b). The loop rule with $\varepsilon = 0$ gives:

$$R \frac{dq}{dt} + \frac{q}{C} = 0$$

With initial condition $q(0) = q_0$:

$$q(t) = q_0 e^{-t/RC} \quad (7.53)$$

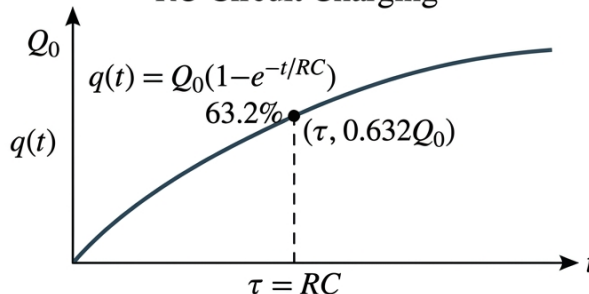
$$i(t) = \frac{dq}{dt} = -\frac{q_0}{RC} e^{-t/RC} \quad (7.54)$$

The negative sign indicates that current flows opposite to the charging direction.

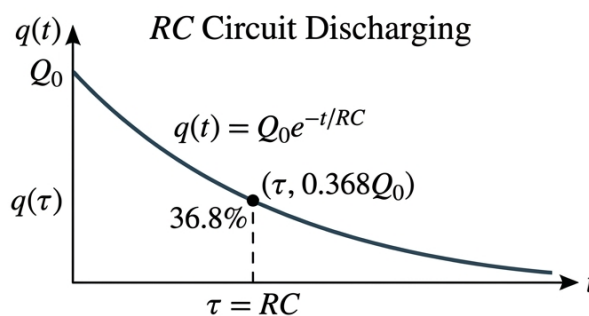
Applications of RC Circuits

- Pacemakers and defibrillators:** RC circuits are used to generate regular electrical pulses. A capacitor charges through a resistor and discharges through the heart tissue at programmed intervals.
- Timing circuits:** The time constant determines delay periods in electronic timers.
- Filters:** RC circuits act as low-pass or high-pass filters in audio and communication systems.

RC Circuit Charging

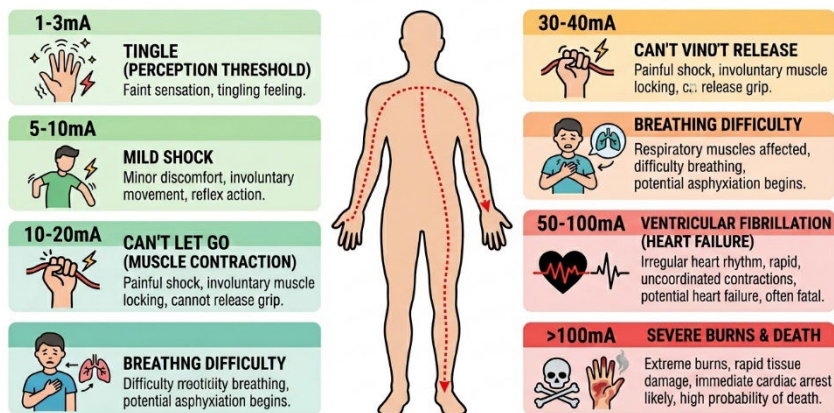


RC Circuit Discharging



Factors affecting shock severity:

ELECTRICAL SAFETY DIAGRAM: EFFECTS OF CURRENT ON HUMAN BODY



- **Current magnitude** — The most critical factor (determined by voltage and body resistance)
- **Path through body** — Hand-to-hand or hand-to-foot paths are most dangerous (pass through heart)
- **Duration of contact** — Longer exposure increases damage
- **Frequency** — AC is generally more dangerous than DC at the same voltage
- **Skin condition** — Wet skin dramatically reduces resistance (from ~100 kΩ dry to ~1 kΩ wet)

Safety Practices:

1. **Never touch electrical switches with wet hands** — Water reduces skin resistance, increasing current flow.
2. **Use proper insulation** — Ensure wires and appliances have intact insulation.
3. **Proper grounding** — All metal-cased appliances should be earthed.
4. **Avoid overloading outlets** — Multiple high-power devices on one outlet can cause overheating.
5. **Use correct fuse ratings** — Never replace a fuse with a higher-rated one.
6. **Install GFCIs** — Especially in bathrooms, kitchens, and outdoor areas.
7. **Regular inspection** — Check for damaged cords, loose connections, or signs of overheating.

Key Formulas

Concept	Definition	Key Equation
Electric Current	Rate of flow of charge	$I = dQ/dt$
Current Density	Current per unit area	$J = I/A = nqv_d$
Drift Speed	Net speed of charge carriers	$v_d = J/(nq)$
Potential Difference	Work per unit charge between two points	$V = W/Q$
Electromotive Force (emf)	Energy per unit charge supplied by a source	$\mathcal{E} = dW/dq$
Resistance	Opposition to current flow	$R = V/I$
Resistivity	Intrinsic resistance of a material	$\rho = E/J = RA/L$
Ohm's Law	$V \propto I$ at constant temperature	$V = IR$ (with R constant)
Series Resistance	Resistors in same current path	$R_{eq} = \sum R_i$
Parallel Resistance	Resistors across same voltage	$1/R_{eq} = \sum 1/R_i$
Power	Rate of energy transfer	$P = IV = I^2R = V^2/R$
Energy	Power $\times$ time	$E = IVt = I^2Rt = V^2t/R$

KCL	Conservation of charge at a junction	$\sum I_{in} = \sum I_{out}$
KVL	Conservation of energy around a loop	$\sum \mathcal{E} = \sum IR$
Internal Resistance	Resistance within an emf source	$V = \mathcal{E} - Ir$
Maximum Power	Occurs when $R_L = r$	$P_{max} = \mathcal{E}^2/4r$
RC Time Constant	Characteristic time for charge/discharge	$\tau = RC$
RC Charging	Exponential approach to final charge	$q(t) = C\mathcal{E}(1 - e^{-t/RC})$
RC Discharging	Exponential decay of charge	$q(t) = q_0 e^{-t/RC}$

Current And Electricity: One-Liners

Electric Current

1. Electric current is the time rate of flow of electric charge through a cross-sectional area of a conductor: $I = dQ/dt$.
2. The SI unit of current is the ampere (A): $1 \text{ A} = 1 \text{ C/s}$.
3. Conventional current is defined as the direction of flow of positive charges, from positive to negative terminal.
4. In metallic conductors, current is actually due to the flow of negatively charged electrons from negative to positive terminal.
5. Direct current (DC) flows steadily in a single direction; alternating current (AC) periodically reverses direction.
6. AC can be easily transformed to higher voltages for efficient long-distance transmission.
7. Drift velocity is the average velocity with which free electrons drift in a conductor under an applied electric field.
8. Typical drift velocities are on the order of 10^{-4} m/s —slower than a snail.
9. The electric field propagates through a conductor at nearly the speed of light, causing almost instantaneous current flow.
10. Current relates to drift velocity: $I = neAv_d$, where n is the free electron density.
11. Current density: $J = I/A = nev_d = \sigma E$.
12. The current density vector points in the direction of the velocity of positive charge carriers.

Potential Difference and Electromotive Force

13. Potential difference is the work done per unit charge in moving a test charge between two points: $V = W/Q$.
14. The SI unit of potential difference is the volt (V): $1 \text{ V} = 1 \text{ J/C}$.
15. Electromotive force (emf) is the energy supplied by a source per unit charge: $\mathcal{E} = dW/dq$.
16. Despite its name, emf is not a force—it is a potential difference measured in volts.
17. Sources of emf convert non-electrical energy into electrical energy (chemical, mechanical, radiant, thermal).
18. Every real source possesses internal resistance.
19. Terminal voltage: $V = \mathcal{E} - Ir$, where r is internal resistance.
20. In an open circuit ($I = 0$), terminal voltage equals emf.
21. In a short circuit ($R = 0$), current is limited only by internal resistance: $I = \mathcal{E}/r$.

Resistance and Resistivity

22. Resistance is defined as $R = V/I$, with SI unit ohm (Ω).
23. Resistivity is a material property: $\rho = E/J = RA/L$, with SI unit $\Omega \cdot \text{m}$.

Practice MCQs

1. Electric current is defined as:

- A) The rate of flow of charge
- B) The force on a charge
- C) The energy per unit charge
- D) The resistance to flow

Correct Answer: A

2. The SI unit of electric current is the:

- A) Coulomb
- B) Volt
- C) Ampere
- D) Ohm

Correct Answer: C

3. One ampere is equal to:

- A) 1 J/C
- B) 1 C/s
- C) 1 V/Ω
- D) Both B and C

Correct Answer: D

4. Conventional current flows from:

- A) Negative to positive terminal
- B) Positive to negative terminal
- C) Both directions simultaneously
- D) Depends on the material

Correct Answer: B

5. In metallic conductors, current is due to the flow of:

- A) Protons
- B) Electrons
- C) Positive ions
- D) Neutrons

Correct Answer: B

6. Direct current (DC) is characterized by:

- A) Periodic reversal of direction
- B) Flow in a single direction
- C) Changing magnitude only
- D) Zero voltage

Correct Answer: B

7. Alternating current (AC) is characterized by:

- A) Flow in a single direction
- B) Periodic reversal of direction

C) Constant magnitude

D) No current flow

Correct Answer: B

8. Drift velocity is the:

- A) Speed of light in a conductor
- B) Average velocity of electrons under an applied field
- C) Thermal speed of electrons
- D) Speed of sound in a conductor

Correct Answer: B

9. Typical drift velocities are on the order of:

- A) 10^8 m/s
- B) 10^5 m/s
- C) 10^{-4} m/s
- D) 10^{-10} m/s

Correct Answer: C

10. The electric field in a conductor propagates at approximately:

- A) The drift velocity
- B) The speed of sound
- C) The speed of light
- D) The thermal speed

Correct Answer: C

11. The relationship between current and drift velocity is:

- A) $I = neAv_d$
- B) $I = neA/v_d$
- C) $I = nev_d/A$
- D) $I = eA/nv_d$

Correct Answer: A

12. Current density is defined as:

- A) $J = I/A$
- B) $J = A/I$
- C) $J = IA$
- D) $J = I^2/A$

Correct Answer: A

13. The SI unit of current density is:

- A) A/m
- B) A/m²
- C) A·m²

Chapter 12: Electromagnetism

Introduction

Electromagnetism stands as one of the most profound intellectual achievements in the history of physics. It unifies what were once thought to be two distinct phenomena—electricity and magnetism—into a single, coherent framework. From the lodestones of ancient Magnesia to the powerful electromagnets in modern MRI machines, from electric generators that power our cities to the tiny speakers in our headphones, the principles of electromagnetism underlie countless technological innovations that shape our modern world.

This chapter presents a comprehensive, unified treatment of electromagnetism, beginning with the fundamental nature of magnetic fields and progressing systematically through their sources, their effects on moving charges and currents, the phenomenon of electromagnetic induction, and culminating in Maxwell's synthesis—the complete set of equations that

describe all classical electromagnetic phenomena.

The structure follows a logical progression. We begin by examining how moving charges create magnetic fields, developing the Biot–Savart law and Ampere's law. We then analyze the magnetic fields produced by specific current configurations: long straight wires, circular loops, solenoids, and toroids. Having established how magnetic fields are created, we

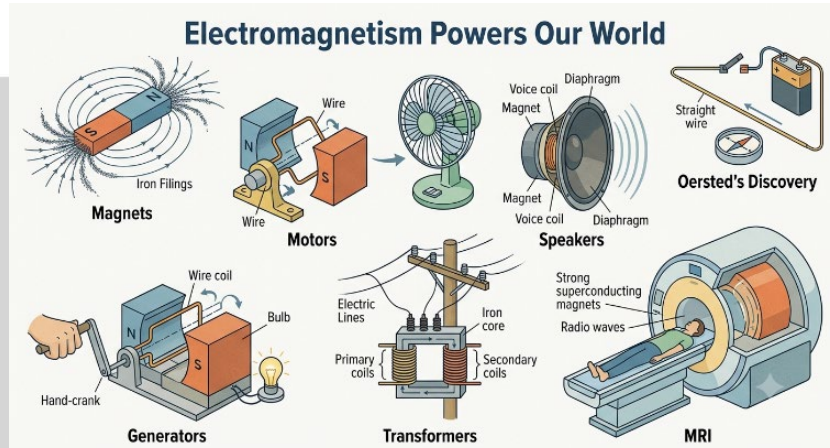
study how these fields affect moving charges and current-carrying conductors, including the Hall effect, torque on current loops, and the operation of DC motors.

The second half of the chapter focuses on electromagnetic induction—the production of electric fields and currents by changing magnetic fields. We describe Faraday's experiments, formulate Faraday's law, and introduce Lenz's law for determining the direction of induced currents. We analyze motional emf, induced electric fields, eddy currents, and the principles of inductors and RL circuits. Finally, we present Maxwell's great synthesis: the displacement current and the complete set of Maxwell's equations, which unify electricity and magnetism and predict the existence of electromagnetic waves traveling at the speed of light.

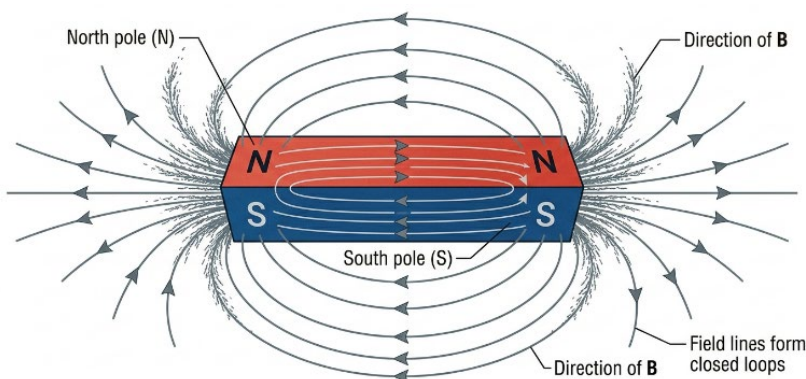
By the end of this chapter, the reader will possess a unified conceptual framework for understanding the interplay between electricity and magnetism—a framework essential for any serious student of physics and for preparation for examinations such as the FPSC Lecturer Physics.

What is a Magnetic Field?

A **magnetic field** is a vector field that permeates space, created by moving electric charges (electric currents) and by intrinsic magnetic moments of elementary particles (such as electron spin). The magnetic field is denoted by the symbol $\vec{B}$. Its SI unit is the **tesla** (T), where



- Where field lines are close together, the magnetic field is strong; where they are far apart, the field is weak.
- For a bar magnet, field lines emerge from the north pole and enter the south pole (inside the magnet, they continue from south to north, forming closed loops).



Magnetic flux Φ_B through a surface is defined analogously to electric flux:

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \cos \phi \, dA$$

where ϕ is the angle between $\vec{B}$ and the area vector $d\vec{A}$ (perpendicular to the surface). For a uniform field $\vec{B}$ over a flat area A ,

$$\Phi_B = BA \cos \phi$$

The SI unit of magnetic flux is the **weber** (Wb): $1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$.

Oersted's Fundamental Discovery

In 1820, Hans Christian Oersted made a discovery that revolutionized physics. During a classroom demonstration, he noticed that a compass needle deflected when brought near a wire carrying an electric current. This simple observation revealed for the first time that electricity and magnetism are not separate phenomena but are intimately linked.

Experimental Observation: When a steady current passes through a straight conducting wire, a magnetic field is established in the region surrounding the wire. The magnetic field exists only while current flows; when the current ceases, the magnetic field disappears.

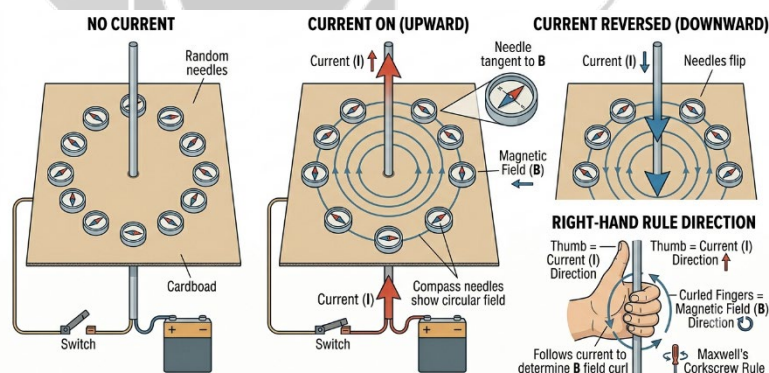
Key Conclusions:

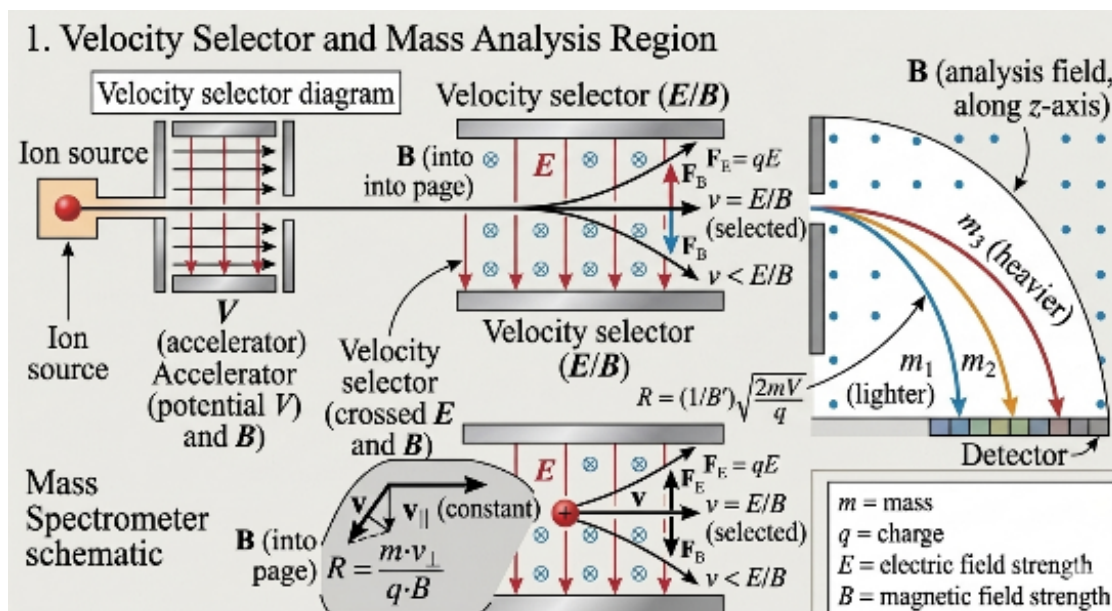
1. A current-carrying conductor produces a magnetic field in its surrounding space.
2. The magnetic field lines form concentric circles centered on the wire.
3. The direction of the magnetic field depends on the direction of the current.
4. The field is stronger near the conductor and weaker farther away.

Magnetic Field of a Moving Point Charge

Consider a point charge q moving with constant velocity $\vec{v}$ (small compared to the speed of light).

Experiments show that the magnetic field $\vec{B}$ at a point a distance r from the charge, along a unit vector $\hat{r}$ from the charge to the field point, is given by





Determination of e/m for the electron (Thomson's experiment). An electron beam is accelerated through a known potential difference V , giving kinetic energy $\frac{1}{2}mv^2 = eV$, so $v = \sqrt{2eV/m}$. The beam then enters a region of uniform magnetic field B perpendicular to the velocity, following a circular path of radius r :

$$evB = \frac{mv^2}{r} \Rightarrow \frac{e}{m} = \frac{v}{Br} = \frac{2V}{B^2 r^2}$$

The accepted value is $e/m = 1.7588 \times 10^{11} \text{ C/kg}$.

Magnetic Force on a Current-Carrying Conductor

For a straight wire segment of length L carrying current I in a uniform magnetic field $\vec{B}$,

$$\vec{F} = I\vec{L} \times \vec{B}$$

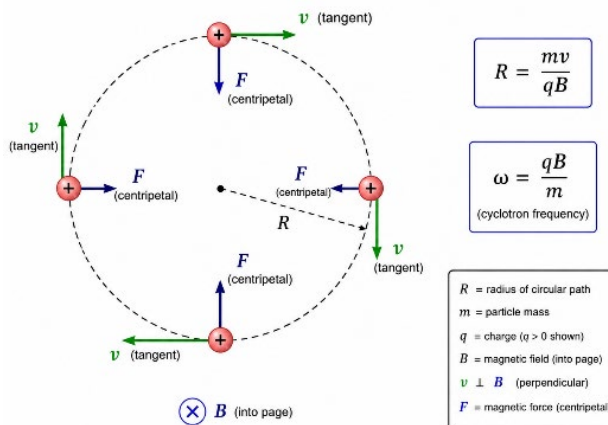
For an infinitesimal segment $d\vec{l}$,

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

The magnitude is $F = ILB \sin \phi$, where ϕ is the angle between the wire and $\vec{B}$.

Fleming's Left-Hand Rule (for conventional current):

Stretch the thumb, forefinger, and middle finger of your left hand so that they are mutually perpendicular. If the **forefinger** points in the direction of the **magnetic field** (B) and the **middle finger** points in the direction of the **current** (I), then the **thumb** indicates the direction of the **force** (F) on the conductor.



Torque on a Current Loop

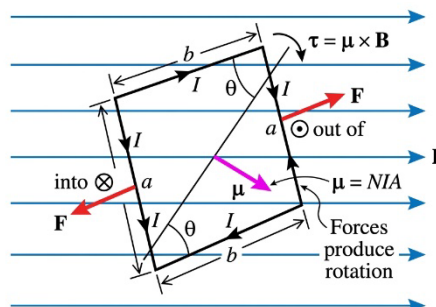
A planar current loop of area A and N turns carrying current I in a uniform magnetic field $\vec{B}$ experiences a torque. The magnetic dipole moment is $\vec{\mu} = NI\vec{A}$. The torque is

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

with magnitude $\tau = \mu B \sin \phi$, where ϕ is the angle between $\vec{\mu}$ and $\vec{B}$. The potential energy of the dipole is

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \phi$$

- Minimum energy: $\phi = 0^\circ$ ($\vec{\mu}$ aligned with $\vec{B}$), $U = -\mu B$ (stable equilibrium)
- Maximum energy: $\phi = 180^\circ$ ($\vec{\mu}$ opposite to $\vec{B}$), $U = +\mu B$ (unstable equilibrium)



The Direct-Current Motor

A DC motor uses the torque on a current-carrying coil in a magnetic field to produce rotational motion. The commutator reverses the current every half-cycle so that the torque always acts in the same direction. The back emf $\mathcal{E}$ induced in the rotating coil opposes the applied voltage. For a series-wound motor,

$$V_{ab} = \mathcal{E} + Ir$$

where r is the internal resistance. Mechanical power output is $P_{\text{mech}} = \mathcal{E}I$.

The Hall Effect

When a current-carrying conductor is placed in a magnetic field perpendicular to the current, a transverse potential difference (Hall voltage) develops. This occurs because the magnetic force deflects charge carriers to one side, creating an electric field that opposes further deflection. The Hall coefficient allows determination of the sign and density of charge carriers.

ELECTROMAGNETIC INDUCTION

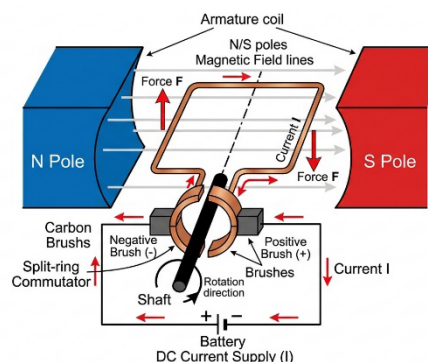
Faraday's Law of Induction

The fundamental experimental observation: A changing magnetic flux through a circuit induces an electromotive force (emf) and, if the circuit is closed, a current. **Faraday's law** states:

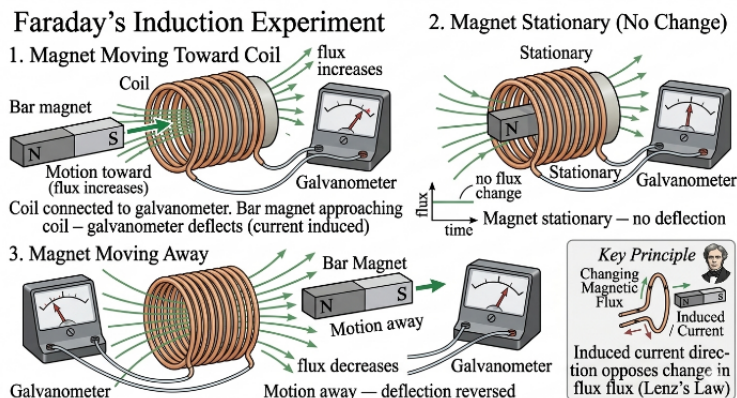
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

For a coil with N identical turns,

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$



Faraday's Induction Experiment



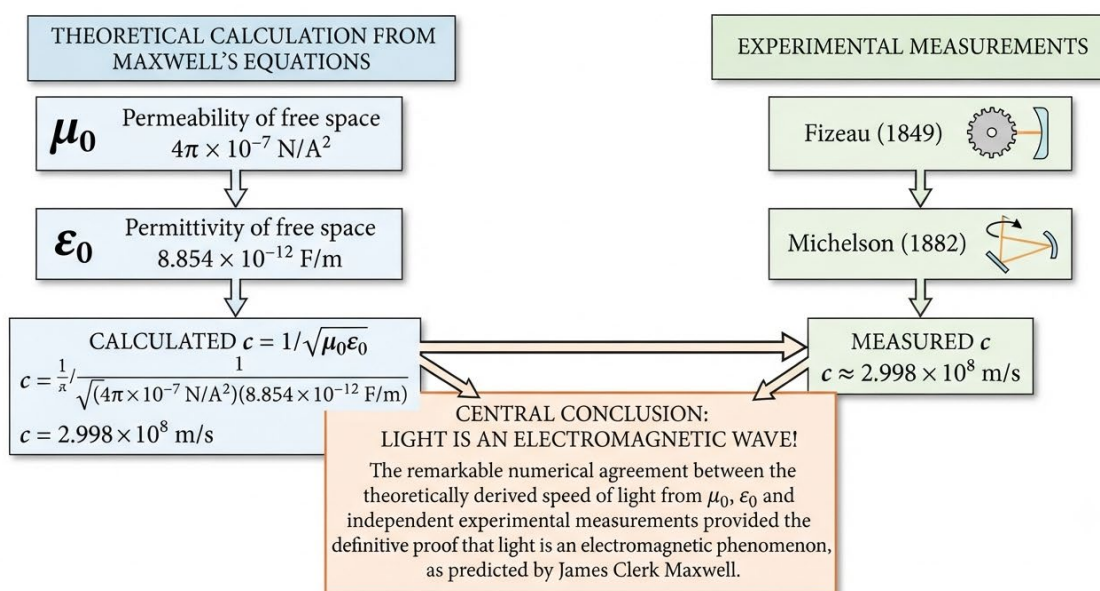
$$v = \frac{1}{\sqrt{1.112 \times 10^{-17}}} = 2.998 \times 10^8 \text{ m/s}$$

Profound Conclusion

$$v = c = 2.998 \times 10^8 \text{ m/s}$$

Light is an electromagnetic wave!

Historical Significance: Maxwell wrote in 1865: "The velocity of transverse undulations in our hypothetical medium, calculated from the electromagnetic experiments of MM. Kohlrausch and Weber, agrees so exactly with the velocity of light calculated from the optical experiments of M. Fizeau, that we can scarcely avoid the inference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena."



Planes Wave Solutions

The General Plane Wave

A plane electromagnetic wave propagating in the x -direction can be written as:

$$\mathbf{E}(x, t) = \mathbf{E}_0 \cos(kx - \omega t)$$

$$\mathbf{B}(x, t) = \mathbf{B}_0 \cos(kx - \omega t)$$

where:

Symbol	Meaning	Relation
$\mathbf{E}_0$	Electric field amplitude	—
$\mathbf{B}_0$	Magnetic field amplitude	—
$k = 2\pi/\lambda$	Wave number	$k = \omega/c$
$\omega = 2\pi f$	Angular frequency	$\omega = ck$

Relationship Between E and B Amplitudes

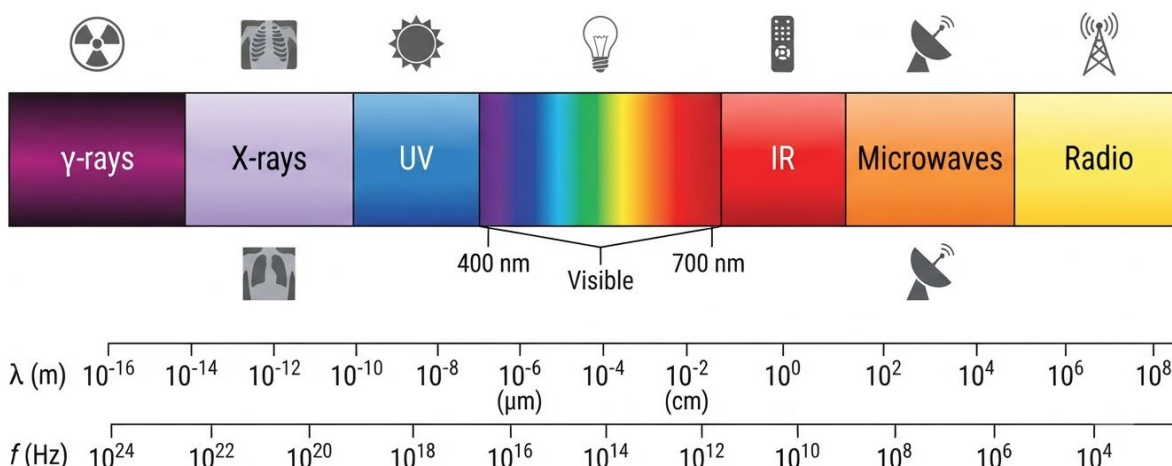
From Faraday's law:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Key Relationships

$$f = \frac{c}{\lambda}, E = hf = \frac{hc}{\lambda}$$

where $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$ is Planck's constant.



12. Electromagnetism

BOUNDARY CONDITIONS FOR EM FIELDS

When electromagnetic waves encounter boundaries between different media, boundary conditions determine how the fields behave. These conditions are essential for understanding reflection, refraction, and transmission.

General Boundary Conditions

At the interface between two media (1 and 2), the fields must satisfy:

Field Component	Boundary Condition	Meaning
Normal E	$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$	Normal component of D is continuous
Tangential E	$E_{1\parallel} = E_{2\parallel}$	Tangential component of E is continuous
Normal B	$B_{1\perp} = B_{2\perp}$	Normal component of B is continuous
Tangential B	$\frac{B_{1\parallel}}{\mu_1} = \frac{B_{2\parallel}}{\mu_2}$	Tangential component of H is continuous

Physical Interpretation

Condition	Physical Reason
Tangential E continuous	From Faraday's law (no voltage buildup along the interface)
Normal B continuous	From Gauss's law for magnetism (no magnetic monopoles)
Normal D discontinuous	Surface charge at the interface
Tangential H discontinuous	Surface current at the interface

REFLECTION AND TRANSMISSION AT NORMAL INCIDENCE

When a wave strikes a boundary at normal incidence, part is reflected and part is transmitted.

Application to EM Waves

Concept	Time Domain	Frequency Domain
Short pulse	$\delta(t)$	Constant spectrum
Continuous wave	$\cos(\omega_0 t)$	$\delta(\omega - \omega_0)$
Wave packet	$A(t)e^{i\omega_0 t}$	$F(\omega - \omega_0)$

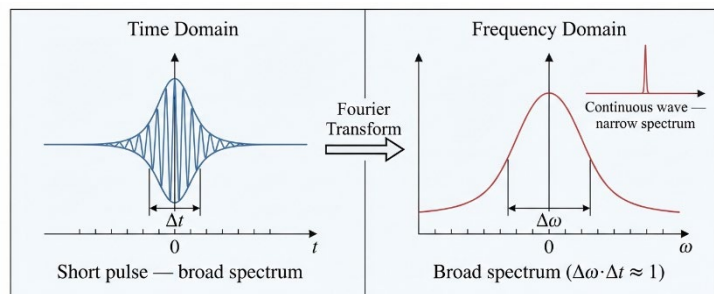
The Uncertainty Principle in Waves

A wave packet of finite duration Δt has a frequency spread $\Delta\omega$:

$$\Delta\omega\Delta t \approx 1$$

In terms of frequency: $\Delta f\Delta t \approx 1$

Physical Interpretation: A short pulse contains a wide range of frequencies; a long continuous wave has a narrow frequency spectrum. This is the Fourier analog of the Heisenberg uncertainty principle.



MAGNETIC MATERIALS

Atomic Origins of Magnetism

The magnetic properties of materials originate from electrons:

1. **Spin magnetic dipole moment:** Electrons have an intrinsic spin angular momentum $\vec{S}$ and an associated spin magnetic dipole moment:

$$\vec{\mu}_s = -\frac{e}{m} \vec{S}$$

The measured component along any axis is quantized: $\mu_{s,z} = \pm\mu_B$, where $\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24} \text{ J/T}$ is the **Bohr magneton**.

2. **Orbital magnetic dipole moment:** Electrons orbiting the nucleus also have orbital angular momentum $\vec{L}_{\text{orb}}$ and an associated orbital magnetic moment:

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m} \vec{L}_{\text{orb}}$$

Classification of Magnetic Materials

Magnetic materials respond to applied magnetic fields in three distinct ways:

Diamagnetism:

- Exhibited by all materials, but very weak
- Atoms have no permanent magnetic moment
- An external field induces a magnetic moment **opposite** to the field
- Relative permeability $K_m < 1$ (typically 0.99990–0.99999)
- Diamagnetic materials are **repelled** from regions of stronger magnetic field
- Examples: bismuth, copper, water

Paramagnetism:

- Atoms have permanent magnetic moments, but they are randomly oriented due to thermal motion
- An external field partially aligns the moments **with** the field
- Relative permeability $K_m > 1$ but close to 1 (typically 1.00001–1.003)
- Paramagnetic materials are **attracted** to regions of stronger field

Problem: A metal rod of length 0.250 m slides on frictionless rails at 3.00 m/s perpendicular to a uniform magnetic field of 0.400 T. The resistance of the circuit is 2.00 Ω . Calculate the induced emf, the induced current, and the magnetic force on the rod.

Solution:

- Induced emf: $\mathcal{E} = vBL = (3.00)(0.400)(0.250) = 0.300 \text{ V}$
- Induced current: $I = \mathcal{E}/R = 0.300/2.00 = 0.150 \text{ A}$
- Magnetic force: $F = ILB = (0.150)(0.250)(0.400) = 0.0150 \text{ N}$ opposing the motion

Example 6: RL Circuit Time Constant

Problem: An RL circuit has $R = 50.0 \Omega$ and $L = 0.200 \text{ H}$. The battery emf is 12.0 V. (a) Calculate the time constant. (b) Find the current after one time constant. (c) How long does it take for the current to reach 99% of its final value?

Solution:

(a) $\tau_L = L/R = 0.200/50.0 = 0.00400 \text{ s} = 4.00 \text{ ms}$

(b) $I(\tau_L) = (\mathcal{E}/R)(1 - e^{-1}) = (12.0/50.0)(0.632) = (0.240)(0.632) = 0.152 \text{ A}$

(c) For $I = 0.99(\mathcal{E}/R)$: $0.99 = 1 - e^{-t/\tau_L} \Rightarrow e^{-t/\tau_L} = 0.01 \Rightarrow t/\tau_L = -\ln(0.01) = 4.605 \Rightarrow t = 4.605 \times 4.00 \text{ ms} = 18.4 \text{ ms}$

Key Formulas and Equations

Quantity	Symbol	SI Unit	Key Equation(s)
Magnetic field	$\vec{B}$	tesla (T)	$\vec{F} = q\vec{v} \times \vec{B}$
Magnetic flux	Φ_B	weber (Wb)	$\Phi_B = \int \vec{B} \cdot d\vec{A}$
Magnetic constant	μ_0	T·m/A	$\mu_0 = 4\pi \times 10^{-7}$
Magnetic dipole moment	$\vec{\mu}$	A·m ²	$\vec{\mu} = NI\vec{A}$
Magnetization	$\vec{M}$	A/m	$\vec{M} = \vec{\mu}_{\text{total}}/V$
Induced emf	$\mathcal{E}$	volt (V)	$\mathcal{E} = -d\Phi_B/dt$
Inductance	L	henry (H)	$\mathcal{E}_L = -L dI/dt, U_B = \frac{1}{2}LI^2$
Displacement current	I_d	ampere (A)	$I_d = \epsilon_0 d\Phi_E/dt$
Wave equation (E)	$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$		EM waves propagate at speed c
Wave equation (B)	$\nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$		B-field also propagates as a wave
Speed of light	$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$		Maxwell's great synthesis
E-B relationship	$E_0 = cB_0$		Ratio of amplitudes
Plane wave	$\mathbf{E} = \mathbf{E}_0 \cos(kx - \omega t)$		Transverse wave solution
Poynting vector	$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$		Energy flow per unit area
Average intensity	$S_{\text{avg}} = \frac{1}{2} \epsilon_0 c E_0^2$		Time-averaged power per area
Energy density	$u = \epsilon_0 E^2$		Energy per unit volume
Radiation pressure (absorb)	$P = S_{\text{avg}}/c$		Pressure from absorbed light
Radiation pressure (reflect)	$P = 2S_{\text{avg}}/c$		Pressure from reflected light

Fundamental Constants

$$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2, \epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2, c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 2.998 \times 10^8 \text{ m/s}$$

Electromagnetism: One-Liners

Magnetic Fields and Their Sources

1. A magnetic field is a vector field created by moving electric charges and intrinsic magnetic moments.
2. The SI unit of magnetic field is the tesla (T): $1 \text{ T} = 1 \text{ N}/(\text{A} \cdot \text{m})$.
3. One tesla equals 10^4 gauss (G); Earth's magnetic field is approximately 0.5 G.
4. Magnetic fields are produced only by moving charges or intrinsic magnetic moments; stationary charges produce only electric fields.
5. Magnetic field lines form continuous closed loops; they never have endpoints, reflecting the absence of magnetic monopoles.
6. Magnetic force on a moving charge is perpendicular to both velocity and magnetic field.
7. Magnetic forces do no work on charges; they change direction but not speed.
8. Magnetic flux: $\Phi_B = \int \vec{B} \cdot d\vec{A} = BA \cos \phi$ for uniform field.
9. The SI unit of magnetic flux is the weber (Wb): $1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$.
10. Oersted discovered in 1820 that a current-carrying wire deflects a compass needle, linking electricity and magnetism.
11. The magnetic field of a current-carrying wire forms concentric circles centered on the wire.
12. Magnetic field of a moving point charge: $\vec{B} = (\mu_0/4\pi)(q\vec{v} \times \hat{r})/r^2$.
13. The magnetic constant (permeability of free space) is $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A}$.
14. The speed of light relates to μ_0 and ϵ_0 : $c = 1/\sqrt{\mu_0 \epsilon_0}$.
15. The Biot-Savart law: $d\vec{B} = (\mu_0/4\pi)I d\vec{l} \times \hat{r}/r^2$.
16. For an infinitely long straight wire: $B = \mu_0 I/(2\pi r)$, with field lines as concentric circles.
17. The magnetic field of a long straight wire decreases as $1/r$.
18. Force between parallel wires: $F/L = \mu_0 I I'/(2\pi r)$; same direction $\rightarrow$ attractive, opposite $\rightarrow$ repulsive.
19. This force defines the ampere: the current producing $2 \times 10^{-7} \text{ N/m}$ between wires 1 m apart.
20. Magnetic field on the axis of a circular loop: $B = \mu_0 I a^2/[2(x^2 + a^2)^{3/2}]$.
21. At the center of a circular loop: $B = \mu_0 I/(2a)$.
22. Magnetic dipole moment: $\vec{\mu} = NI\vec{A}$, with direction given by the right-hand rule.
23. For a solenoid: $B = \mu_0 nI$, where $n = N/L$ is turns per unit length.
24. The field inside a long solenoid is nearly uniform and parallel to the axis; outside it is approximately zero.
25. For a toroidal solenoid: $B = \mu_0 NI/(2\pi r)$, confined to the interior and varying as $1/r$.

Ampere's Law

26. Ampere's law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$, valid for steady currents.
27. The circulation of the magnetic field around a closed loop equals μ_0 times the enclosed current.
28. For a long straight wire, Ampere's law gives $B = \mu_0 I/(2\pi r)$.

101. A cathode ray oscilloscope displays voltage waveforms using an electron beam deflected by electric fields.

Practice MCQs

1. A magnetic field is produced by:

- A) Stationary electric charges
- B) Moving electric charges
- C) Electric fields only
- D) Gravitational fields

Correct Answer: B

2. The SI unit of magnetic field is the:

- A) Gauss
- B) Tesla
- C) Weber
- D) Henry

Correct Answer: B

3. One tesla is equal to:

- A) 10^2 gauss
- B) 10^4 gauss
- C) 10^6 gauss
- D) 10^8 gauss

Correct Answer: B

4. Magnetic field lines:

- A) Begin on positive charges and end on negative charges
- B) Form continuous closed loops
- C) Are straight lines
- D) Never form loops

Correct Answer: B

5. The absence of magnetic monopoles implies:

- A) $\oint \vec{E} \cdot d\vec{A} = Q/\epsilon_0$
- B) $\oint \vec{B} \cdot d\vec{A} = 0$
- C) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$
- D) $\oint \vec{E} \cdot d\vec{l} = -d\Phi_B/dt$

Correct Answer: B

6. The magnetic force on a moving charge:

- A) Does work on the charge
- B) Does no work on the charge
- C) Changes the speed of the charge
- D) Is parallel to the velocity

Correct Answer: B

7. Oersted's experiment showed that:

- A) Electric charges produce electric fields
- B) A current-carrying wire produces a magnetic field
- C) Magnetic fields produce electric currents
- D) Light is an electromagnetic wave

Correct Answer: B

8. The magnetic field of a long straight wire:

- A) Decreases as $1/r^2$
- B) Decreases as $1/r$
- C) Is constant
- D) Increases as r

Correct Answer: B

9. The Biot-Savart law gives the magnetic field due to:

- A) A point charge
- B) A current element
- C) A changing electric field
- D) A magnetic dipole

Correct Answer: B

10. The permeability of free space μ_0 is:

- A) $4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$
- B) $8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$
- C) $6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$
- D) $1.38 \times 10^{-23} \text{ J/K}$

Correct Answer: A

11. The speed of light is related to μ_0 and ϵ_0 by:

- A) $c = \mu_0 \epsilon_0$
- B) $c = 1/\sqrt{\mu_0 \epsilon_0}$
- C) $c = \sqrt{\mu_0/\epsilon_0}$
- D) $c = \sqrt{\epsilon_0/\mu_0}$

Correct Answer: B

12. For an infinitely long straight wire carrying current I , the magnetic field at distance r is:

- A) $B = \mu_0 I/(2\pi r)$
- B) $B = \mu_0 I/(4\pi r^2)$

Chapter 13: Oscillation and Alternating Current

Introduction

The Significance of Oscillatory Phenomena in Electromagnetism

Throughout our study of physics, we have encountered oscillatory systems in various contexts—from the simple harmonic motion of a mass on a spring to the vibrations of a musical string. In each case, oscillations arise when a system possesses two distinct forms of energy storage, and energy continually transfers between these forms. The mechanical oscillator stores energy as kinetic energy of motion and potential energy of deformation; the restoring force continually converts one form into the other.

In electromagnetism, an analogous oscillatory system exists: the **LC circuit**, consisting of a capacitor (which stores energy in an electric field) and an inductor (which stores energy in a magnetic field). When these two elements are connected, energy oscillates between the electric field of the capacitor and the magnetic field of the inductor, giving rise to **electromagnetic oscillations**. These oscillations form the foundation for alternating current (AC) technology, radio transmission, and countless electronic devices that define modern civilization.

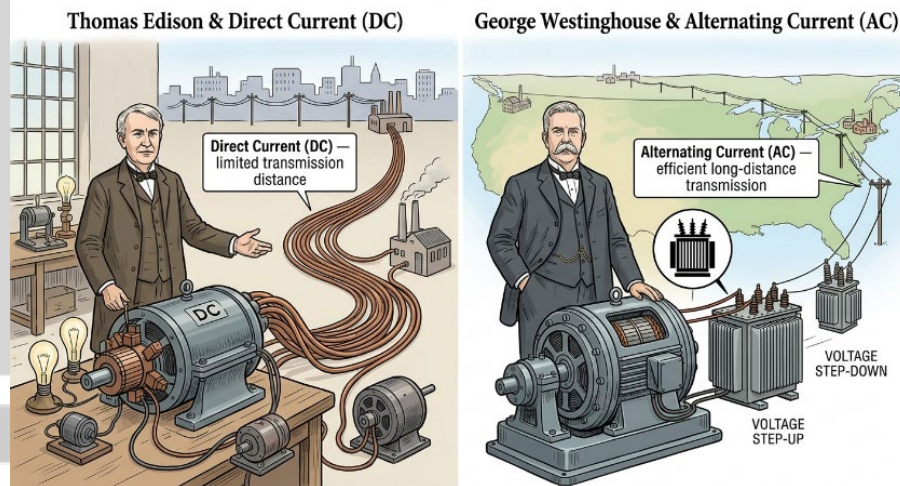
Historical Context: The War of the Currents

In the late 19th century, a fierce debate raged between two inventors over the best method of electric power distribution.

Thomas Edison championed direct current (DC)—steady current that does not vary with time. **George Westinghouse** favored alternating current (AC), with sinusoidally varying voltages and currents.

Westinghouse argued that transformers (which we shall study in this chapter) can be used to step voltage up and down with AC but not with DC. Low voltages are safer for consumer use, but high voltages and correspondingly low currents are best for long-distance power transmission to minimize I^2R losses in transmission cables.

Westinghouse prevailed, and most present-day household and industrial power distribution systems operate with alternating current. Any appliance plugged into a wall outlet uses AC. Circuits in modern communication equipment also make extensive use of AC. Understanding AC circuits is therefore essential not only for physicists but for anyone who uses or designs electrical and electronic systems.

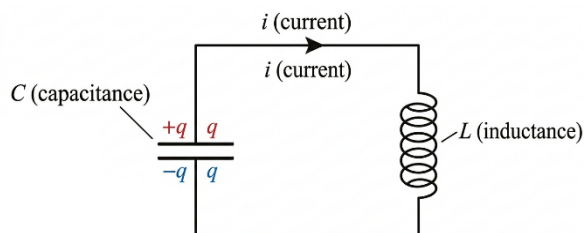


Fundamental Concepts of Oscillatory

Circuits

The LC Circuit: Energy Storage and Transfer

Consider the circuit shown in Figure, consisting of a capacitor of capacitance C and an inductor of inductance L connected in series. This is called an **LC circuit**. The capacitor stores energy in its electric field, given by:



$$U_E = \frac{q^2}{2C} \quad (7.1)$$

where q is the instantaneous charge on the capacitor. The inductor stores energy in its magnetic field, given by:

$$U_B = \frac{1}{2} Li^2 \quad (7.2)$$

where i is the instantaneous current through the inductor.

Drawing Instructions: Draw a simple series loop. On the left side, draw two parallel plates representing a capacitor with charges $+q$ and $-q$ indicated. On the right side, draw a coil representing an inductor of inductance L . Connect them with wires to form a closed loop. Indicate the direction of current with an arrow.

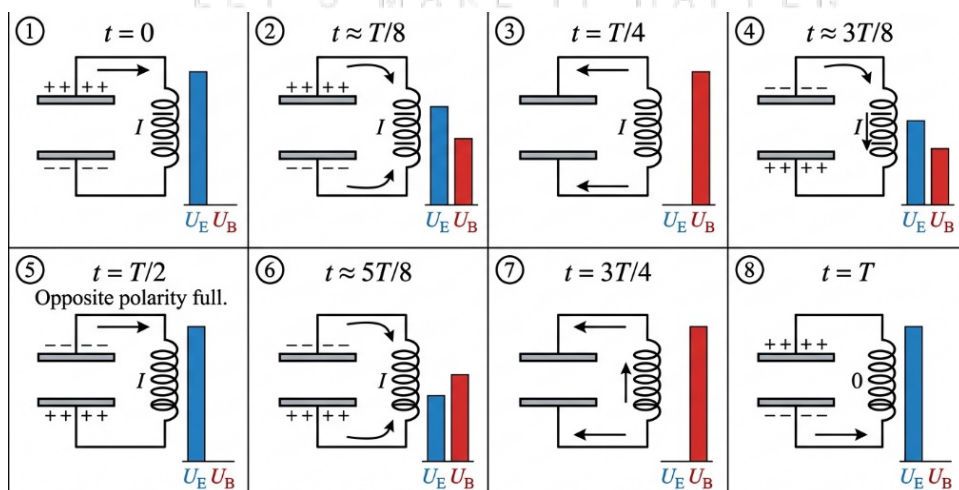
Qualitative Description of Oscillations

Let us examine the sequence of events when a charged capacitor is connected to an inductor. We assume the capacitor initially has maximum charge Q and the current is zero.

Stage 1: The capacitor is fully charged with charge $+Q$ on its upper plate and $-Q$ on its lower plate. All energy is stored in the electric field: $U_E = Q^2/2C$, $U_B = 0$.

Stage 2: The capacitor begins to discharge through the inductor. As charge flows, the current increases from zero. The decreasing electric field energy converts into increasing magnetic field energy. The capacitor charge decreases while the inductor current grows.

Stage 3: When the capacitor has completely discharged ($q = 0$), the current reaches its maximum value I . All energy is now stored in the magnetic field: $U_B = \frac{1}{2} LI^2$, $U_E = 0$.



$$\frac{q}{C} \frac{dq}{dt} + L \frac{dq}{dt} \frac{d^2q}{dt^2} = 0 \quad (7.5)$$

Factoring dq/dt (which is generally not zero except at turning points):

$$\frac{dq}{dt} \left(\frac{q}{C} + L \frac{d^2q}{dt^2} \right) = 0 \quad (7.6)$$

Since dq/dt is not identically zero, the quantity in parentheses must be zero:

$$L \frac{d^2q}{dt^2} + \frac{1}{C} q = 0 \quad (7.7)$$

This is the differential equation for LC oscillations. It has exactly the same mathematical form as the equation for simple harmonic motion:

$$m \frac{d^2x}{dt^2} + kx = 0 \quad (7.8)$$

with the correspondences: $x \leftrightarrow q$, $m \leftrightarrow L$, and $k \leftrightarrow 1/C$.

Solution for Charge and Current

Equation (7.7) is a second-order linear differential equation with constant coefficients. Its general solution is:

$$q(t) = Q \cos(\omega t + \phi) \quad (7.9)$$

where:

- Q is the **charge amplitude** (maximum charge on the capacitor)
- ω is the **angular frequency** of oscillation
- ϕ is the **phase constant** determined by initial conditions

Substituting this solution into equation (7.7) gives:

$$L[-\omega^2 Q \cos(\omega t + \phi)] + \frac{1}{C} [Q \cos(\omega t + \phi)] = 0 \quad (7.10)$$

Factoring $Q \cos(\omega t + \phi)$:

$$\left(-\omega^2 L + \frac{1}{C} \right) Q \cos(\omega t + \phi) = 0 \quad (7.11)$$

For this to hold for all t , the coefficient must be zero:

$$\omega^2 L = \frac{1}{C} \Rightarrow \omega = \frac{1}{\sqrt{LC}} \quad (7.12)$$

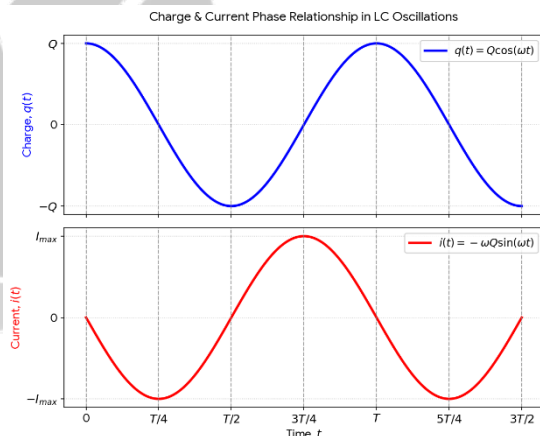
This is the **natural angular frequency** of the LC circuit. The ordinary frequency f and period T are:

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}, T = \frac{1}{f} = 2\pi\sqrt{LC} \quad (7.13)$$

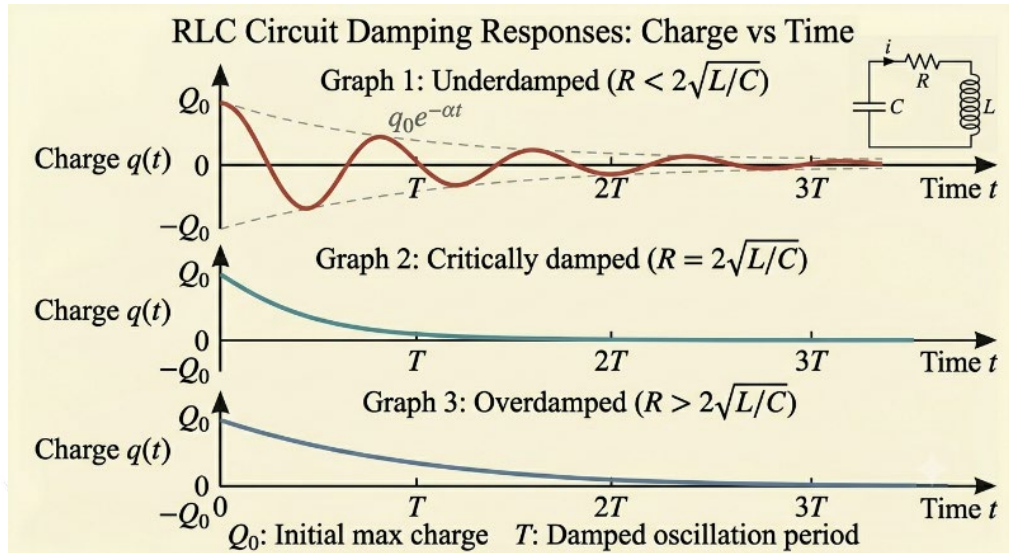
The current is obtained by differentiating the charge:

$$i(t) = \frac{dq}{dt} = -\omega Q \sin(\omega t + \phi) \quad (7.14)$$

The **current amplitude** I is:



3. **Overdamped** ($R > 2\sqrt{L/C}$): The circuit returns to equilibrium slowly without oscillating. This is generally avoided in oscillator design.



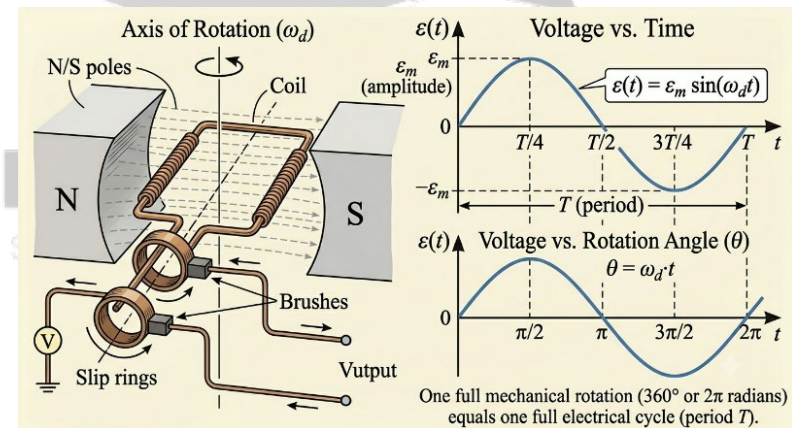
Alternating Current and Forced Oscillations

Introduction to Alternating Current

The oscillations we have studied so far are **free oscillations**—they occur naturally when an LC or RLC circuit is given an initial energy and then left alone. However, these oscillations eventually decay due to resistance. To maintain continuous oscillations, we must supply energy to compensate for the losses. This is accomplished by connecting the circuit to an external source that produces an alternating voltage—an **AC generator** or **alternator**.

Alternating current (AC) is current that varies sinusoidally with time, reversing direction periodically. In most countries, the frequency of AC power is either 50 Hz or 60 Hz. The voltage supplied to household outlets is specified by its **root-mean-square (rms)** value—for example, 120 V in North America or 220-240 V in many other countries.

The AC generator produces a sinusoidal emf:



$$\varepsilon(t) = \varepsilon_m \sin(\omega_d t) \quad (7.28)$$

or, equivalently:

$$v(t) = V_m \cos(\omega_d t) \quad (7.29)$$

where:

- ε_m or V_m is the **amplitude** (maximum value)
- ω_d is the **driving angular frequency** (equal to the angular speed of the generator's coil)
- $f = \omega_d / 2\pi$ is the driving frequency

A **smoothing capacitor** (also called a filter capacitor) can be added across the output to reduce the pulsations (ripple), producing a nearly constant DC voltage. The capacitor charges during the peaks and discharges through the load between peaks.

Response of Individual Circuit Elements to AC

The Resistor in an AC Circuit

Consider a resistor R connected to an AC source. The instantaneous voltage across the resistor is given by:

$$v_R(t) = V_R \cos(\omega_d t) \quad (7.34)$$

From Ohm's law, the instantaneous current is:

$$i_R(t) = \frac{v_R(t)}{R} = \frac{V_R}{R} \cos(\omega_d t) = I_R \cos(\omega_d t) \quad (7.35)$$

where $I_R = V_R/R$ is the current amplitude.

Key observations:

- The current is **in phase** with the voltage (they reach their maxima and zeros at the same times)
- The relationship between amplitudes is the same as in DC circuits: $V_R = I_R R$

The Inductor in an AC Circuit

Now consider a pure inductor L (with negligible resistance) connected to an AC source. The voltage across an inductor is related to the current by:

$$v_L(t) = L \frac{di_L}{dt} \quad (7.36)$$

If the current is $i_L(t) = I_L \cos(\omega_d t)$, then:

$$v_L(t) = L \frac{d}{dt} [I_L \cos(\omega_d t)] = -I_L \omega_d L \sin(\omega_d t) \quad (7.37)$$

Using the identity $-\sin \theta = \cos(\theta + 90^\circ)$:

$$v_L(t) = I_L \omega_d L \cos(\omega_d t + 90^\circ) = V_L \cos(\omega_d t + 90^\circ) \quad (7.38)$$

where $V_L = I_L \omega_d L$.

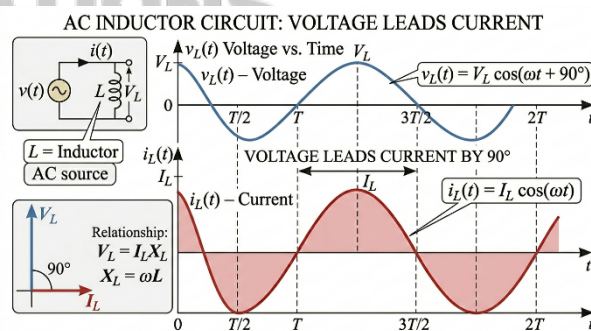
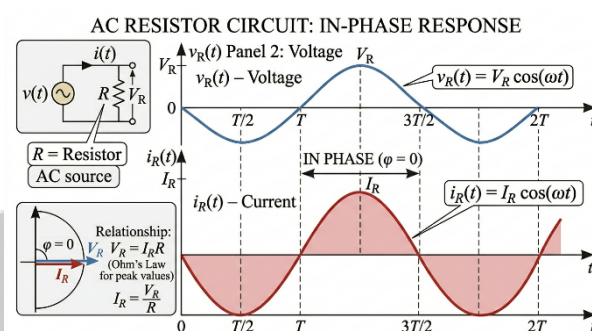
Key observations:

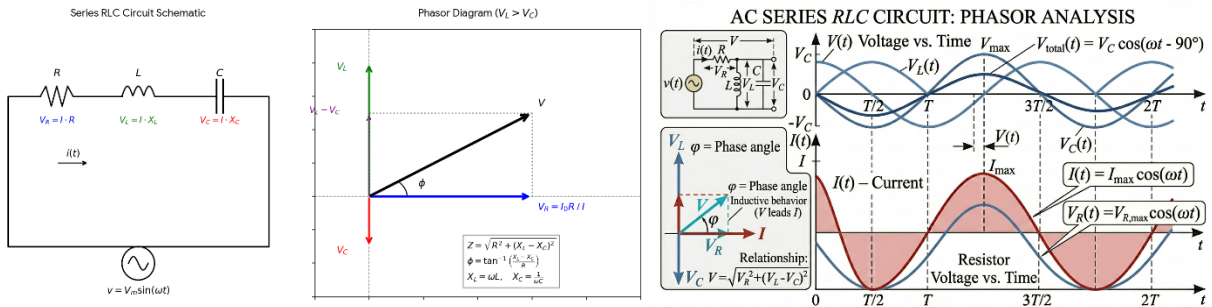
- The voltage **leads** the current by 90° (or one-quarter cycle)
- The amplitude relationship is $V_L = I_L X_L$, where $X_L = \omega_d L$ is the **inductive reactance**
- Inductive reactance increases with frequency; an inductor acts like a short circuit at DC ($X_L = 0$ at $f = 0$) and like an open circuit at very high frequencies

The Capacitor in an AC Circuit

Consider a pure capacitor C connected to an AC source. The charge on the capacitor is related to the voltage by $q = C v_C$. The current is:

$$i_C(t) = \frac{dq}{dt} = C \frac{dv_C}{dt} \quad (7.39)$$





Let the current be:

$$i(t) = I \cos(\omega_d t) \quad (7.43)$$

Then:

$$v_R(t) = IR \cos(\omega_d t) \quad (7.44)$$

$$v_L(t) = IX_L \cos(\omega_d t + 90^\circ) = -IX_L \sin(\omega_d t) \quad (7.45)$$

$$v_C(t) = IX_C \cos(\omega_d t - 90^\circ) = IX_C \sin(\omega_d t) \quad (7.46)$$

By Kirchhoff's loop rule, the source voltage equals the sum of the individual voltages:

$$v_s(t) = v_R(t) + v_L(t) + v_C(t) \quad (7.47)$$

The voltage phasors are:

- V_R in phase with current (horizontal, to the right)
- V_L 90° ahead of current (vertical, up)
- V_C 90° behind current (vertical, down)

Since V_L and V_C are opposite in direction, they subtract. The resultant voltage amplitude is:

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} \quad (7.48)$$

Impedance and Phase Angle

Factor out I from equation (7.48):

$$V = I \sqrt{R^2 + (X_L - X_C)^2} \quad (7.49)$$

We define the **impedance** Z of the circuit as:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2} \quad (7.50)$$

Then:

$$V = IZ \text{ or } I = \frac{V}{Z} \quad (7.51)$$

The impedance is the AC equivalent of resistance—it tells us how much the circuit opposes the flow of alternating current. Its SI unit is the ohm (Ω).

The **phase angle** ϕ between the source voltage and the current is given by:

$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R} \quad (7.52)$$

The phase angle can be positive, negative, or zero:

- **Inductive circuit** ($X_L > X_C$): $\phi > 0$ (voltage leads current)

For an ideal transformer, power input equals power output (conservation of energy):

$$V_p I_p = V_s I_s \quad (7.62)$$

Therefore:

$$\frac{I_s}{I_p} = \frac{N_p}{N_s} \quad (7.63)$$

Transformer Types:

- **Step-up transformer:** $N_s > N_p$, so $V_s > V_p$ and $I_s < I_p$
- **Step-down transformer:** $N_s < N_p$, so $V_s < V_p$ and $I_s > I_p$

Impedance Transformation

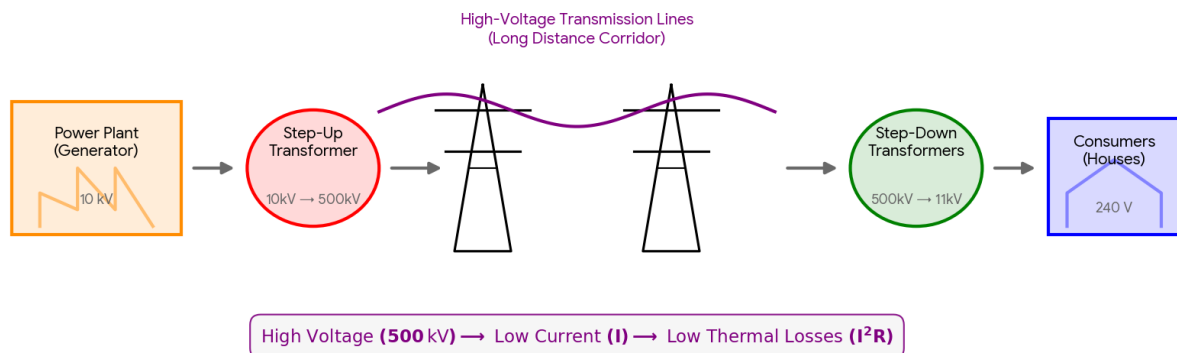
A transformer also transforms impedances. The impedance seen by the primary circuit is:

$$Z_p = \frac{V_p}{I_p} = \frac{(N_p/N_s)V_s}{(N_s/N_p)I_s} = \left(\frac{N_p}{N_s}\right)^2 \frac{V_s}{I_s} = \left(\frac{N_p}{N_s}\right)^2 Z_s \quad (7.64)$$

where Z_s is the load impedance connected to the secondary. This property is used for **impedance matching**—transforming a load impedance to match the source impedance for maximum power transfer.

Power Transmission and Energy Losses

Transformers are essential for efficient power transmission. Electrical power is generated at moderate voltages (10-20 kV). For transmission over long distances, step-up transformers raise the voltage to hundreds of kilovolts. The power transmitted is $P = VI$. For a given power, higher voltage means lower current. Since power loss in transmission lines is $P_{\text{loss}} = I^2 R$, reducing current dramatically reduces losses.



At the receiving end, step-down transformers reduce the voltage to safe levels for distribution (e.g., 120 V or 240 V for household use).

Example: A power plant generates 100 MW at 10 kV. For transmission at 500 kV, the current is reduced by a factor of 50. If the transmission line resistance is 10 Ω :

- Without step-up: $I = P/V = (100 \times 10^6)/(10 \times 10^3) = 10,000$ A, $P_{\text{loss}} = (10^4)^2 \times 10 = 10^9$ W (1000 MW—impossible!)
- With step-up: $I = (100 \times 10^6)/(500 \times 10^3) = 200$ A, $P_{\text{loss}} = (200)^2 \times 10 = 4 \times 10^5$ W = 0.4 MW (only 0.4% loss)

Real Transformer Losses:

- Copper losses ($I^2 R$ in windings)

- Eddy current losses (minimized by using laminated cores)
- Hysteresis losses (minimized by using soft iron cores)
- Flux leakage (minimized by proper core design)

Despite these losses, large power transformers can achieve efficiencies of 99% or higher.

Key Equations

Quantity	Equation	Variables
LC natural frequency	$\omega_0 = 1/\sqrt{LC}$	L = inductance, C = capacitance
Energy in LC circuit	$U = Q^2/2C = LI^2/2$	Q = max charge, I = max current
Damped frequency	$\omega' = \sqrt{\omega_0^2 - (R/2L)^2}$	R = resistance
RMS value (sinusoidal)	$I_{\text{rms}} = I_m/\sqrt{2}$, $V_{\text{rms}} = V_m/\sqrt{2}$	I_m, V_m = amplitudes
Inductive reactance	$X_L = \omega L = 2\pi fL$	ω = angular frequency
Capacitive reactance	$X_C = 1/(\omega C) = 1/(2\pi fC)$	
Impedance (series RLC)	$Z = \sqrt{R^2 + (X_L - X_C)^2}$	
Phase angle	$\tan \phi = (X_L - X_C)/R$	
Average power	$P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}} \cos \phi$	$\cos \phi = R/Z$
Resonance condition	$\omega_d = \omega_0 = 1/\sqrt{LC}$	
Transformer voltage ratio	$V_s/V_p = N_s/N_p$	N = number of turns
Transformer current ratio	$I_s/I_p = N_p/N_s$	
Impedance transformation	$Z_p = (N_p/N_s)^2 Z_s$	

Important Phase Relationships

- **Resistor:** Voltage and current in phase ($\phi = 0$)
- **Inductor:** Voltage leads current by 90° ($\phi = +90^\circ$)
- **Capacitor:** Voltage lags current by 90° ($\phi = -90^\circ$)

Summary of DC vs. AC Circuit Elements

Element	DC Behavior	AC Behavior
Resistor	$V = IR$	Same, in phase
Inductor	Short circuit (after transients)	$V = IX_L$, voltage leads
Capacitor	Open circuit (steady state)	$V = IX_C$, voltage lags

LET'S MAKE IT HAPPEN

13. Oscillation and Alternating Current

Oscillation and Alternating Current: One-Liners

LC Oscillations

1. An LC circuit consists of a capacitor and an inductor connected in series, storing energy in electric and magnetic fields respectively.
2. In an LC circuit, energy oscillates between the capacitor's electric field and the inductor's magnetic field.
3. The capacitor stores energy as $U_E = q^2/(2C)$, where q is the instantaneous charge.
4. The inductor stores energy as $U_B = \frac{1}{2}Li^2$, where i is the instantaneous current.
5. In an ideal LC circuit (no resistance), total energy is conserved: $U = q^2/(2C) + \frac{1}{2}Li^2 = \text{constant}$.
6. The differential equation for LC oscillations is $L d^2q/dt^2 + q/C = 0$, analogous to SHM.
7. The natural angular frequency of an LC circuit is $\omega_0 = 1/\sqrt{LC}$.
8. The natural frequency is $f = 1/(2\pi\sqrt{LC})$ and period is $T = 2\pi\sqrt{LC}$.
9. The charge as a function of time is $q(t) = Q\cos(\omega_0 t + \phi)$, where Q is the maximum charge.
10. The current is $i(t) = dq/dt = -\omega_0 Q\sin(\omega_0 t + \phi)$.
11. The current amplitude is $I = \omega_0 Q = Q/\sqrt{LC}$.
12. When the capacitor is fully charged, all energy is electric: $U_E = Q^2/(2C)$, $U_B = 0$.
13. When the capacitor is fully discharged, all energy is magnetic: $U_B = \frac{1}{2}LI^2$, $U_E = 0$.
14. The energy oscillates between electric and magnetic forms with frequency $2\omega_0$.
15. The electrical-mechanical analogy: charge $\leftrightarrow$ displacement, inductance $\leftrightarrow$ mass, reciprocal capacitance $\leftrightarrow$ spring constant.
16. The phase constant ϕ is determined by initial conditions: if $q(0) = Q$ and $i(0) = 0$, then $\phi = 0$.
17. If $q(0) = 0$ and $i(0) = I$, then $\phi = -\pi/2$ (or $3\pi/2$).

Damped RLC Circuits

18. In a real RLC circuit, resistance dissipates energy as heat, causing damped oscillations.
19. The differential equation for a damped RLC circuit is $L d^2q/dt^2 + R dq/dt + q/C = 0$.
20. This is analogous to a damped harmonic oscillator: $m d^2x/dt^2 + b dx/dt + kx = 0$.
21. For underdamped oscillations ($R < 2\sqrt{L/C}$): $q(t) = Qe^{-(R/2L)t}\cos(\omega't + \phi)$.
22. The damped angular frequency is $\omega' = \sqrt{\omega_0^2 - (R/2L)^2}$.
23. The amplitude decays exponentially with time constant $\tau = 2L/R$.
24. Critically damped occurs when $R = 2\sqrt{L/C}$, returning to equilibrium fastest without oscillation.
25. Overdamped occurs when $R > 2\sqrt{L/C}$, returning to equilibrium slowly without oscillation.

Alternating Current Fundamentals

26. Alternating current (AC) varies sinusoidally with time, reversing direction periodically.
27. The standard household frequency is 50 Hz in most countries (60 Hz in North America).
28. AC generator emf: $\mathcal{E}(t) = \mathcal{E}_m\sin(\omega_d t)$ or $v(t) = V_m\cos(\omega_d t)$.
29. RMS (root-mean-square) value: $I_{\text{rms}} = I_m/\sqrt{2}$, $V_{\text{rms}} = V_m/\sqrt{2}$.
30. RMS values give the equivalent DC value for power dissipation in a resistor.
31. For a sinusoidal AC, the average of the square is half the square of the amplitude.
32. The process of converting AC to DC is called rectification.

69. A step-up transformer has $N_s > N_p$, increasing voltage and decreasing current.
70. A step-down transformer has $N_s < N_p$, decreasing voltage and increasing current.
71. Transformers only work with AC because a changing magnetic field is required for induction.
72. Impedance transformation: $Z_p = (N_p/N_s)^2 Z_s$.
73. High-voltage transmission reduces $I^2 R$ losses in power lines.
74. Power transmission: step-up at generation, step-down at consumption.
75. Real transformer losses include copper losses, eddy current losses, hysteresis losses, and flux leakage.
76. Laminated cores reduce eddy current losses; soft iron cores reduce hysteresis losses.

Practice MCQs

1. In an LC circuit, energy is stored in:

- A) The resistor only
- B) The capacitor only
- C) The inductor only
- D) Both the capacitor and the inductor

Correct Answer: D

2. The energy stored in a capacitor is:

- A) $U_E = \frac{1}{2} LI^2$
- B) $U_E = q^2/(2C)$
- C) $U_E = \frac{1}{2} CV^2$
- D) Both B and C are correct

Correct Answer: D

3. The energy stored in an inductor is:

- A) $U_B = q^2/(2C)$
- B) $U_B = \frac{1}{2} LI^2$
- C) $U_B = \frac{1}{2} CV^2$
- D) $U_B = \frac{1}{2} LQ^2$

Correct Answer: B

4. In an ideal LC circuit, total energy is:

- A) $U = q^2/(2C) + \frac{1}{2} LI^2$
- B) $U = q^2/(2C) - \frac{1}{2} LI^2$
- C) $U = \frac{1}{2} LI^2 - q^2/(2C)$
- D) $U = q^2/C + LI^2$

Correct Answer: A

5. The differential equation for LC oscillations is:

- A) $L d^2q/dt^2 + R dq/dt + q/C = 0$
- B) $L d^2q/dt^2 + q/C = 0$

$$C) C d^2q/dt^2 + q/L = 0$$

$$D) L d^2q/dt^2 - q/C = 0$$

Correct Answer: B

6. The natural angular frequency of an LC circuit is:

- A) $\omega_0 = \sqrt{LC}$
- B) $\omega_0 = 1/\sqrt{LC}$
- C) $\omega_0 = LC$
- D) $\omega_0 = \sqrt{L/C}$

Correct Answer: B

7. The period of LC oscillations is:

- A) $T = 2\pi\sqrt{LC}$
- B) $T = 2\pi/\sqrt{LC}$
- C) $T = \sqrt{LC}$
- D) $T = 2\pi LC$

Correct Answer: A

8. The charge in an LC circuit as a function of time is:

- A) $q(t) = Q \cos(\omega_0 t + \phi)$
- B) $q(t) = Q \sin(\omega_0 t + \phi)$
- C) $q(t) = Q e^{-\omega_0 t}$
- D) $q(t) = Q(1 - e^{-\omega_0 t})$

Correct Answer: A

9. The current in an LC circuit is:

- A) $i(t) = \omega_0 Q \cos(\omega_0 t + \phi)$
- B) $i(t) = -\omega_0 Q \sin(\omega_0 t + \phi)$
- C) $i(t) = Q \sin(\omega_0 t + \phi)$
- D) $i(t) = -\omega_0 Q \cos(\omega_0 t + \phi)$

Correct Answer: B



Part V: Solid Sate and Electronics



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Chapter 14: Physics Of Solids

Introduction

Solids surround us in every aspect of our daily lives—from the steel beams that support buildings to the silicon chips that power our computers, from the magnets that hold notes on refrigerators to the superconducting wires inside medical imaging machines. What makes steel hard and lead soft? Why does copper conduct electricity while rubber insulates? What makes iron magnetic but not aluminum? The answers to these fundamental questions lie in understanding the internal structure of solids—the arrangement of atoms, the bonding between them, and the behavior of electrons within the material.

This chapter provides a comprehensive, unified treatment of the physical properties of solids. We shall explore three interconnected pillars of solid state physics: mechanical properties (how solids respond to forces), electrical properties (how solids conduct electricity), and magnetic properties (how solids respond to magnetic fields). At the heart of all these properties lies the **atomic structure**—the ordered or disordered arrangement of atoms that determines virtually every characteristic of a solid material.

The chapter is designed to be self-contained. We begin with the classification of solids based on atomic arrangement, then progress through mechanical deformation and elasticity, followed by the quantum-mechanical band theory that explains electrical conductivity, and finally the fascinating world of magnetic materials. Throughout, we emphasize conceptual understanding, mathematical formulation, and physical interpretation.

Classification Of Solids Based On Atomic Arrangement

The Fundamental Question: Why Do Solids Behave Differently?

The properties of any solid material—its hardness, conductivity, transparency, magnetic behavior—are ultimately determined by **how its constituent atoms, ions, or molecules are arranged** and **how they are bonded together**. By understanding structure at the atomic level, we can explain and even predict macroscopic behavior.

Solids can be broadly classified into three categories based on the degree of order in their atomic arrangement: **crystalline solids**, **amorphous (glassy) solids**, and **polycrystalline (polymeric) solids**.

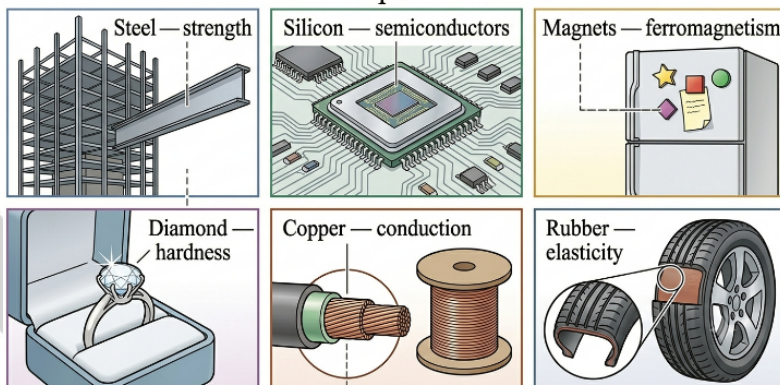
Crystalline Solids

Definition: A crystalline solid is one in which the constituent atoms, ions, or molecules are arranged in a regular, repeating, three-dimensional pattern known as a **crystal lattice**. This regular arrangement is called **long-range order** because the pattern extends throughout the entire crystal.

Key Characteristics:

- Atoms occupy well-defined positions in a periodic array
- The arrangement repeats identically over large distances (millions of atomic spacings)

Solids Shape Our World



Definition: Stress is the internal restoring force per unit area that develops within a deformed body in response to an applied external force.

$$\sigma = \frac{F}{A}$$

where:

- F = magnitude of the applied force (N)
- A = cross-sectional area over which the force acts (m^2)
- σ = stress in pascals ($\text{Pa} = \text{N/m}^2$)

Physical Interpretation: When you pull on a wire, the atoms at any cross-section experience forces from their neighbors. The stress is a measure of the intensity of these interatomic forces—the force per unit area transmitted across an imaginary plane inside the material.

Strain (ϵ)

Definition: Strain is the fractional change in a dimension (or shape) of a body due to applied stress. Because it is a ratio of lengths, strain is dimensionless.

For linear deformation:

$$\epsilon = \frac{\Delta L}{L_0}$$

where:

- ΔL = change in length (m)
- L_0 = original length (m)
- ϵ = strain (no units)

Types of Stress and Strain

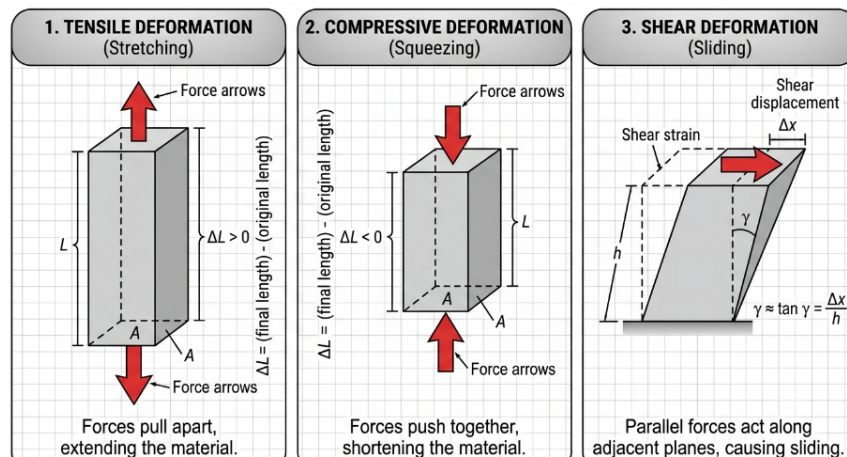
The way a solid deforms depends on how the force is applied. Three fundamental modes of deformation exist:

Tensile Stress and Strain

Tensile stress occurs when forces act perpendicular to a surface, pulling the material apart (stretching it). The stress is normal (perpendicular) to the surface.

$$\sigma_{\text{tensile}} = \frac{F}{A}$$

Tensile strain is the fractional increase in length:



Region B–C: Plastic Deformation

Beyond the yield point, the material undergoes **plastic deformation**—it does not return to its original shape when the load is removed. The atoms have moved to new equilibrium positions. The strain increases rapidly with little increase in stress.

Point C: Ultimate Tensile Strength (UTS)

The maximum stress the material can withstand is called the **ultimate tensile strength** (σ_{UTS}). This is often quoted as the "strength" of a material in engineering specifications.

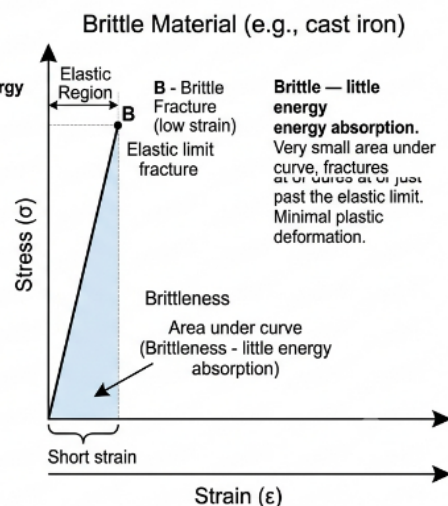
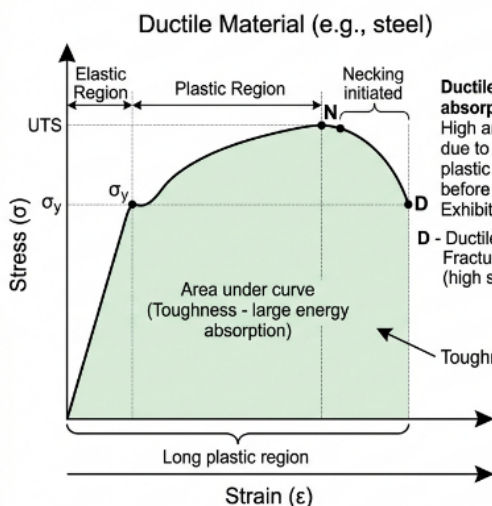
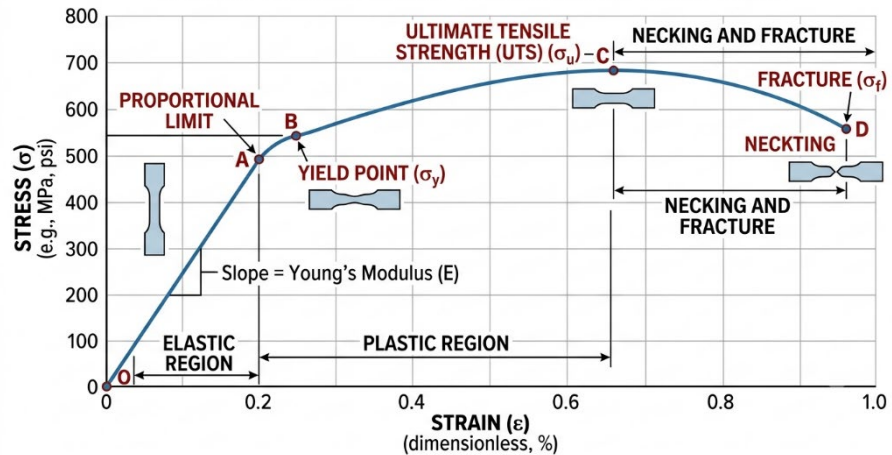
Region C–D: Necking and Fracture

After reaching the UTS, the specimen begins to **neck**—its cross-sectional area decreases locally. The engineering stress (calculated using the original area) appears to decrease, but the true stress (using the actual area) continues to increase. Eventually, the specimen fractures at point D.

Classification Based on Stress-Strain Behavior

- **Ductile materials** (e.g., copper, mild steel, gold): Undergo significant plastic deformation before fracture; exhibit necking; absorb large amounts of energy.
- **Brittle materials** (e.g., glass, cast iron, diamond, ceramics): Fracture shortly after the elastic limit with little or no plastic deformation; absorb little energy.
- **Elastomers** (e.g., rubber): Can undergo very large elastic strains (several hundred percent) before failure.

STRESS-STRAIN CURVE FOR A DUCTILE MATERIAL



Conduction Band: The next higher band above the valence band. Electrons in this band are free to move throughout the crystal and can conduct electric current.

Band Gap (E_g): The energy difference between the bottom of the conduction band and the top of the valence band. No electron states exist within this gap.

Fermi Energy (E_F): The energy of the highest occupied state at $T = 0$ K.

Fermi Level: The energy level at which the probability of occupation by an electron is exactly 0.5 at any temperature.

The Density of States

The **density of states** $N(E)$ is the number of available electron energy levels per unit volume per unit energy interval. For free electrons in three dimensions, quantum mechanical calculation yields:

$$N(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2}$$

where:

- $m = 9.109 \times 10^{-31}$ kg (electron mass)
- $h = 6.626 \times 10^{-34}$ J·s (Planck's constant)
- E = energy (in joules)

When E is expressed in electron-volts (eV), the equation becomes:

$$N(E) = \frac{8\sqrt{2}\pi m^{3/2} e^{3/2}}{h^3} E^{1/2} \text{ (in } m^{-3} eV^{-1} \text{)}$$

Physical Interpretation: The density of states tells us how many energy levels are available at each energy. It is like a "density of parking spaces" at each energy level—but it says nothing about whether those spaces are occupied.

Fermi-Dirac Statistics: The Occupancy Probability

At any temperature above absolute zero, electrons occupy available energy states according to the **Fermi-Dirac distribution function**. The probability $P(E)$ that a state at energy E is occupied by an electron is:

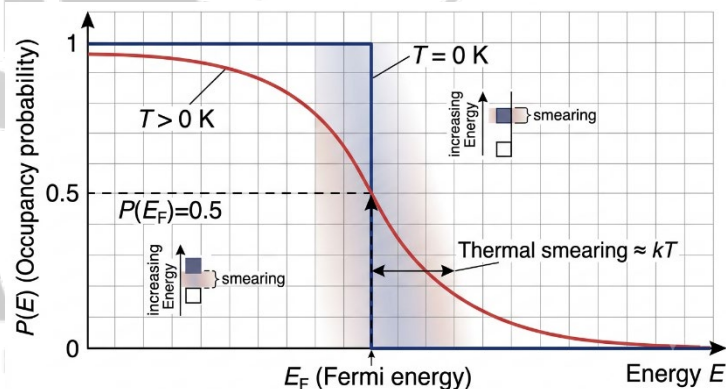
$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where:

- $k = 8.617 \times 10^{-5}$ eV/K (Boltzmann constant)
- T = absolute temperature (K)
- E_F = Fermi energy (eV)

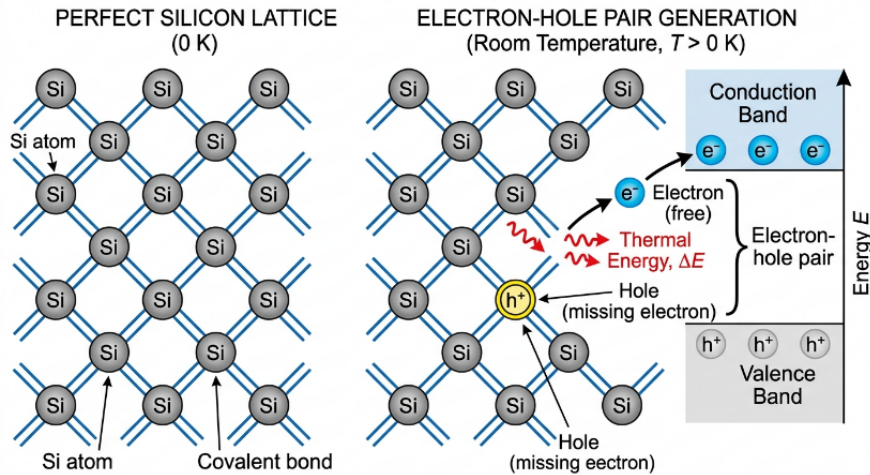
Key Properties:

- At $T = 0$ K: $P(E) = 1$ for $E < E_F$, $P(E) = 0$ for $E > E_F$
- At $E = E_F$: $P(E) = 0.5$ at any temperature
- Only electrons within about kT of E_F can be thermally excited to higher states



At room temperature, thermal energy breaks some covalent bonds, creating **electron-hole pairs**. The number of electrons in the conduction band (n) equals the number of holes in the valence band (p) in an intrinsic semiconductor:

$$n = p = n_i$$

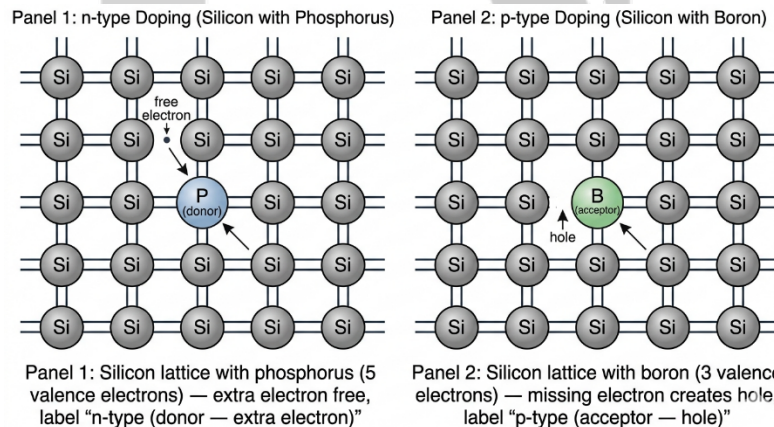


where n_i is the **intrinsic carrier concentration**. For silicon at 300 K, $n_i \approx 1.5 \times 10^{16} \text{ m}^{-3}$; for germanium, $n_i \approx 2.5 \times 10^{19} \text{ m}^{-3}$.

Extrinsic Semiconductors: Doping

The electrical conductivity of a semiconductor can be dramatically increased—by factors of a million or more—by introducing tiny amounts (typically 1 part in 10^7 to 10^8) of suitable impurity atoms. This process is called **doping**, and the resulting materials are called **extrinsic semiconductors**.

Doping creates two types of semiconductors: **n-type** (negative charge carriers dominate) and **p-type** (positive charge carriers dominate).



n-Type Semiconductors

How to make n-type silicon: Add a small amount of a **pentavalent** impurity—an element with five valence electrons, such as phosphorus (P), arsenic (As), or antimony (Sb)—to pure silicon.

What happens: Four of the five valence electrons of the impurity atom form covalent bonds with the four neighboring silicon atoms. The **fifth electron** is only weakly bound to the impurity ion core and can

Applications of p-n Junctions

Light-Emitting Diode (LED): When forward-biased, electrons and holes recombine across the junction. The energy released (approximately E_g) can be emitted as a photon of light. The wavelength is:

$$\lambda = \frac{hc}{E_g}$$

For a red LED with $E_g = 1.9$ eV: $\lambda = \frac{(6.63 \times 10^{-34})(3.0 \times 10^8)}{(1.9)(1.60 \times 10^{-19})} = 650$ nm

Photodiode: Incident light creates electron-hole pairs in the depletion region, producing a photocurrent. This allows detection of light.

Solar Cell: A large-area p-n junction converts sunlight directly into electrical power.

Transistor (MOSFET): A three-terminal device where the gate voltage controls the current between source and drain, enabling amplification and switching.

Superconductors

Discovery and Basic Phenomenon

In 1911, Heike Kamerlingh Onnes discovered that the electrical resistance of mercury drops abruptly to **zero** when cooled below 4.2 K. Materials that exhibit this behavior are called **superconductors**. The temperature at which resistance vanishes is the **critical temperature** (T_c).

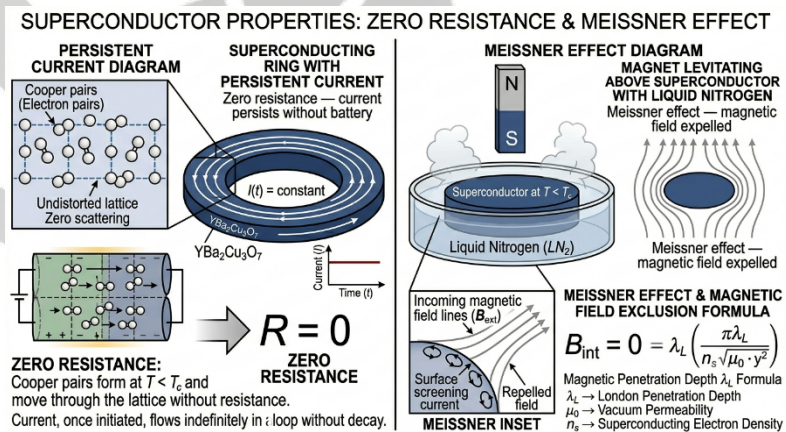
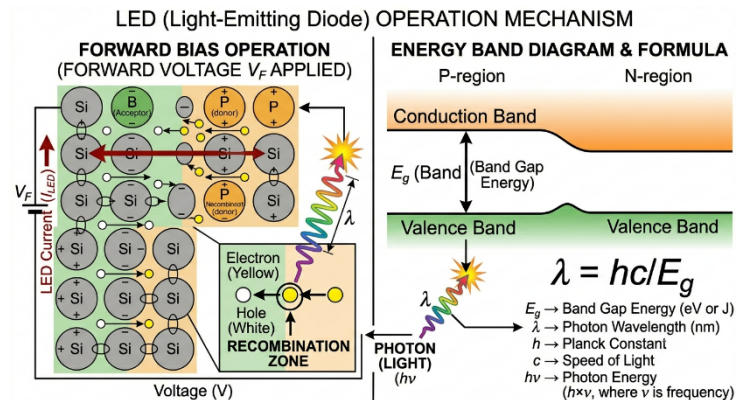
Key Properties of Superconductors:

- **Zero resistance:** Once a current is established in a superconducting loop, it persists indefinitely without any power source.
- **Meissner effect:** A superconductor expels all magnetic fields from its interior (perfect diamagnetism).
- **Critical field:** A sufficiently strong magnetic field destroys superconductivity.
- **Critical current:** A sufficiently large current destroys superconductivity.

Types of Superconductors

Low-temperature superconductors (Type I): Pure metals like mercury ($T_c = 4.2$ K), lead ($T_c = 7.2$ K), aluminum ($T_c = 1.2$ K). Require cooling with liquid helium (4.2 K), which is expensive.

High-temperature superconductors (Type II): Ceramic materials discovered in 1986, such as $\text{YBa}_2\text{Cu}_3\text{O}_7$ ($T_c = 92$ K) and thallium-based compounds ($T_c \approx 125$ K). These can be cooled with liquid nitrogen (77 K), which is much cheaper and more practical.



Origin: An applied magnetic field induces currents in the electron orbits (Lenz's law), which create a magnetic moment opposing the field. This effect occurs in all materials but is only observable when no stronger magnetic effect is present.

Key Characteristics:

- Susceptibility χ is negative and small ($\sim -10^{-5}$)
- Independent of temperature
- Materials are weakly repelled by magnets

Examples: Copper, bismuth, silver, gold, water, most organic compounds

Paramagnetism

Definition: Paramagnetic materials have permanent atomic magnetic moments, but these moments are randomly oriented at room temperature due to thermal agitation. When an external field is applied, the moments partially align with the field, producing a net magnetization in the direction of the field.

Origin: Atoms with unpaired electrons (e.g., transition metals, rare earths) possess net magnetic moments. In the absence of a field, thermal energy randomizes their orientations. An applied field does work to align them against thermal agitation.

Key Characteristics:

- Susceptibility χ is positive and small ($\sim 10^{-5}$ to 10^{-3})
- Follows Curie law: $\chi = C/T$ (decreases with increasing temperature)
- Materials are weakly attracted to magnets

Examples: Aluminum, platinum, manganese, liquid oxygen

Ferromagnetism

Definition: Ferromagnetic materials exhibit strong, permanent magnetization even in the absence of an external field. They are strongly attracted to magnets and can be made into permanent magnets.

Origin: In ferromagnetic materials (iron, cobalt, nickel, gadolinium, and their alloys), there is a strong quantum mechanical interaction called **exchange coupling** that aligns the atomic magnetic moments **parallel** to each other over large regions, even without an external field.

Magnetic Domains

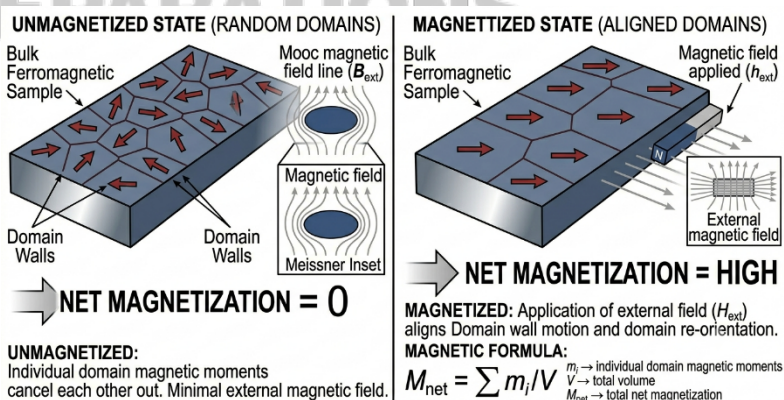
A key concept in understanding ferromagnetism is the **magnetic domain**.

Definition: A magnetic domain is a region within a ferromagnetic crystal (typically 10^{-6} to 10^{-3} m in size,

containing 10^{12} to 10^{16} atoms) in which all atomic magnetic moments are aligned in the same direction.

Each domain is spontaneously magnetized to saturation.

In an unmagnetized specimen: Domains are oriented randomly. The magnetic fields from different domains cancel, so the net macroscopic magnetization is zero.



In a magnetized specimen: When an external field is applied, domains aligned with the field grow at the expense of domains aligned opposite to the field. Domain walls move, and domains rotate. The specimen becomes magnetized.

At saturation: All domains are aligned with the field, and the specimen behaves like a single giant magnet.

Curie Temperature (T_c)

As temperature increases, thermal vibrations tend to disrupt the alignment of atomic magnetic moments. At a characteristic temperature called the **Curie temperature** (or Curie point), the thermal energy becomes sufficient to overcome the exchange coupling that maintains ferromagnetic order.

Above T_c : The material becomes **paramagnetic**—it no longer exhibits spontaneous magnetization but responds weakly to external fields.

Below T_c : The material is ferromagnetic.

Curie temperatures of common ferromagnets:

- Iron (Fe): 770°C (1043 K)
- Cobalt (Co): 1130°C (1403 K)
- Nickel (Ni): 358°C (631 K)
- Gadolinium (Gd): 20°C (293 K)

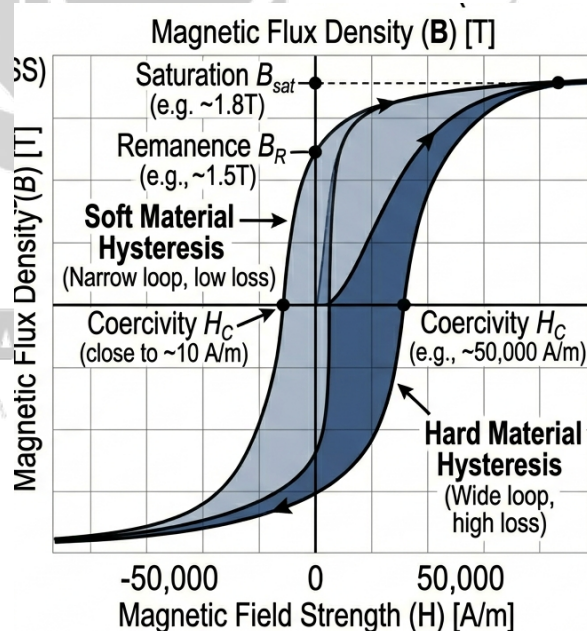
Practical significance: A permanent magnet heated above its Curie temperature loses all its magnetization permanently (unless remagnetized).

Hysteresis and the B-H Loop

When a ferromagnetic material is taken through a complete cycle of magnetization (magnetized in one direction, then reversed, then back), the magnetic flux density B lags behind the applied magnetic field strength H . This phenomenon is called **magnetic hysteresis** (from Greek *hysteresis* = lagging behind).

Key Parameters from the Hysteresis Loop:

1. **Saturation magnetization (B_{sat}):** The maximum flux density achievable when all domains are aligned.
2. **Remanence (or Retentivity) (B_R):** The flux density remaining when the applied field is reduced to zero. A high remanence is desirable for permanent magnets.
3. **Coercivity (or Coercive force) (H_C):** The reverse field required to reduce the flux density to zero. A high coercivity indicates a material that is difficult to demagnetize.
4. **Hysteresis loss:** The area enclosed by the loop represents the energy dissipated as heat per unit volume per magnetization cycle. This energy is used to overcome domain wall friction and is lost as heat.



Soft vs. Hard Magnetic Materials



Physics Of Solids: One-Liners

Classification of Solids

1. The properties of solids are determined by how their constituent atoms, ions, or molecules are arranged and bonded together.
2. Crystalline solids have a regular, repeating, three-dimensional pattern called a crystal lattice with long-range order.
3. Crystals have smooth faces, sharp edges, cleave along specific planes, exhibit anisotropy, and have sharp melting points.
4. The fundamental repeating unit of a crystal is the unit cell; stacking unit cells generates the complete crystal lattice.
5. There are seven basic crystal systems: cubic, tetragonal, orthorhombic, monoclinic, triclinic, trigonal, and hexagonal.
6. Simple cubic has atoms only at cube corners; body-centered cubic (BCC) has an additional atom at the center.
7. Face-centered cubic (FCC) has atoms at corners plus one at each face center (e.g., copper, aluminum).
8. Amorphous solids have no long-range order; they possess only short-range order like a frozen liquid.
9. Amorphous solids are isotropic, have no sharp melting point, and exhibit conchoidal fracture.
10. The glass transition temperature is the range over which a liquid becomes an amorphous solid upon cooling.
11. Polycrystalline solids are composed of numerous tiny crystals (grains) separated by grain boundaries.
12. Polymers are large molecules consisting of repeating monomers linked by covalent bonds.

Mechanical Properties: Stress and Strain

13. Stress is the internal restoring force per unit area: $\sigma = F/A$, with SI unit pascal (Pa).
14. Strain is the fractional change in dimension: $\epsilon = \Delta L/L_0$, dimensionless.
15. Tensile stress occurs when forces pull a material apart; compressive stress pushes inward.
16. Shear stress occurs when forces act parallel to a surface: $\tau = F_{\parallel}/A$.
17. Shear strain is $\gamma = \Delta x/h = \tan \theta \approx \theta$ for small angles.
18. Hooke's law: stress is proportional to strain within the elastic limit; modulus = stress/strain.
19. Young's modulus: $Y = (F/A)/(\Delta L/L_0) = FL_0/(A\Delta L)$, measures resistance to length change.
20. Shear modulus: $G = (F/A)/(\Delta x/h) = Fh/(A\Delta x)$, measures resistance to shape change.
21. Bulk modulus: $B = -\Delta P/(\Delta V/V_0)$, measures resistance to uniform compression.
22. Compressibility is the reciprocal of bulk modulus: $\kappa = 1/B$.
23. Diamond has the highest Young's modulus (~1120 GPa); rubber has the lowest (~0.01 GPa).
24. The proportional limit is the highest stress where Hooke's law is obeyed.
25. The elastic limit (yield point) is beyond which permanent deformation occurs.
26. The ultimate tensile strength is the maximum stress a material can withstand.
27. Ductile materials undergo significant plastic deformation before fracture; brittle materials fracture with little plastic deformation.
28. Elastomers (e.g., rubber) can undergo very large elastic strains before failure.

Practice MCQs

1. Crystalline solids are characterized by:

- A) Random atomic arrangement
- B) Regular, repeating three-dimensional pattern (long-range order)
- C) No sharp melting point
- D) Isotropic properties

Correct Answer: B

2. Amorphous solids are characterized by:

- A) Long-range order
- B) Short-range order only (frozen liquid structure)
- C) Anisotropic properties
- D) Sharp melting point

Correct Answer: B

3. The fundamental repeating unit of a crystal is called the:

- A) Grain boundary
- B) Unit cell
- C) Domain
- D) Monomer

Correct Answer: B

4. Body-centered cubic (BCC) has atoms at:

- A) Corners only
- B) Corners plus face centers
- C) Corners plus body center
- D) Face centers only

Correct Answer: C

5. Face-centered cubic (FCC) has atoms at:

- A) Corners only
- B) Corners plus face centers
- C) Corners plus body center
- D) Face centers only

Correct Answer: B

6. Amorphous solids exhibit:

- A) Anisotropy
- B) Isotropic properties
- C) Sharp melting points
- D) Cleavage planes

Correct Answer: B

7. Polycrystalline solids consist of:

- A) A single crystal

B) Numerous tiny crystals (grains) with grain boundaries

- C) Randomly arranged atoms throughout
- D) Only one grain

Correct Answer: B

8. Polymers are made of repeating units called:

- A) Domains
- B) Grains
- C) Monomers
- D) Unit cells

Correct Answer: C

9. Stress is defined as:

- A) Force per unit volume
- B) Force per unit area
- C) Change in length per unit length
- D) Force times distance

Correct Answer: B

10. Strain is defined as:

- A) Force per unit area
- B) Change in length per unit length
- C) Force per unit volume
- D) Energy per unit volume

Correct Answer: B

11. The SI unit of stress is the:

- A) Newton
- B) Joule
- C) Pascal
- D) Watt

Correct Answer: C

12. Strain is:

- A) Dimensionless
- B) Measured in pascals
- C) Measured in meters
- D) Measured in newtons

Correct Answer: A

13. Tensile stress occurs when forces:

- A) Push inward on a material
- B) Pull a material apart
- C) Act parallel to a surface

Chapter 15: Electronics

Introduction

Electronics is the branch of physics that deals with the emission, behaviour, and control of electrons (and other charge carriers) in semiconductors, vacuum, and gases. It is the foundation of modern civilization—powering computers, mobile phones, televisions, medical diagnostic equipment, automated teller machines, and countless other devices that define contemporary life.

The remarkable advancement of electronics over the past five decades stems primarily from our understanding and manipulation of **semiconductors**—materials whose electrical conductivity lies between that of

conductors (like copper) and insulators (like rubber). Semiconductor devices such as diodes, transistors, and integrated circuits (ICs) can control, amplify, and convert electrical signals with extraordinary efficiency, speed, and reliability.

This chapter provides a comprehensive, self-contained treatment of electronics. We begin with the fundamental physics of semiconductors, progress through the operation of p-n junctions and transistors, explore rectification and amplification, and conclude with digital electronics including logic gates and their applications. The material is developed step-by-step, with all mathematical expressions fully explained and all physical concepts interpreted in depth.

FUNDAMENTAL CONCEPTS

Classification of Materials by Electrical Conductivity

All solid materials can be classified into three categories based on their ability to conduct electric current:

Property	Conductors	Semiconductors	Insulators
Resistivity ($\Omega \cdot \text{m}$)	$10^{-8} - 10^{-6}$	$10^{-5} - 10^3$	$> 10^7$
Examples	Copper, Silver, Aluminum	Silicon, Germanium	Rubber, Glass, Wood
Free charge carriers	Abundant	Moderate	Negligible

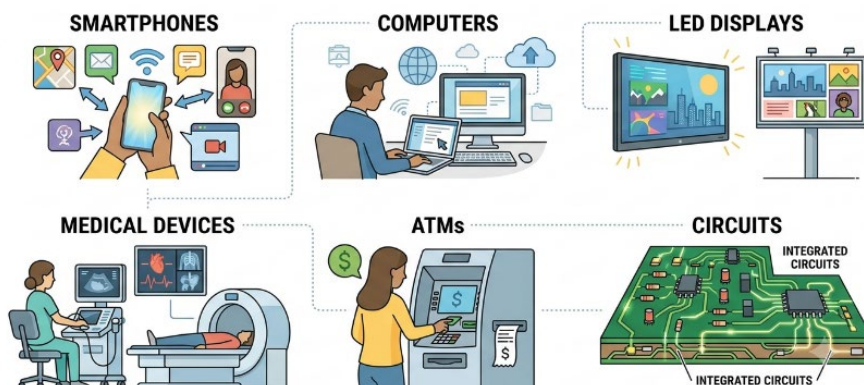
The Semiconductor Material: Silicon and Germanium

Silicon (Si) and germanium (Ge) are the most important elemental semiconductors. Both belong to Group IV of the periodic table and have **four valence electrons**. In a pure crystal at absolute zero temperature (0 K), these four electrons form **covalent bonds** with four neighbouring atoms, resulting in a perfect lattice with no free charge carriers.

Intrinsic Semiconductors

A **pure semiconductor crystal containing no impurities** is called an **intrinsic semiconductor**.

At temperatures above 0 K, thermal energy causes some covalent bonds to break. When a bond breaks:



has **only three valence electrons**. It can only form three covalent bonds with neighbouring silicon atoms. The fourth silicon atom cannot complete its bond, creating a **hole** (a missing electron position).

Terminology:

- The trivalent impurity is called an **acceptor** (it accepts an electron to fill the hole)
- **Majority carriers:** Holes (positive charge carriers)
- **Minority carriers:** Electrons

Concept Check: In a p-type semiconductor, holes are the majority carriers. However, the material is still electrically neutral because the acceptor atom, after accepting an electron, becomes a negative ion, balancing the positive charge of the holes.

Comparison of N-Type and P-Type Semiconductors

Property	N-Type	P-Type
Dopant	Pentavalent (Group V)	Trivalent (Group III)
Dopant examples	P, As, Sb	B, Al, In, Ga
Majority carriers	Electrons (negative)	Holes (positive)
Minority carriers	Holes	Electrons
Dopant function	Donor (gives e^-)	Acceptor (takes e^-)
Net charge	Zero	Zero

THE P-N JUNCTION (SEMICONDUCTOR DIODE)

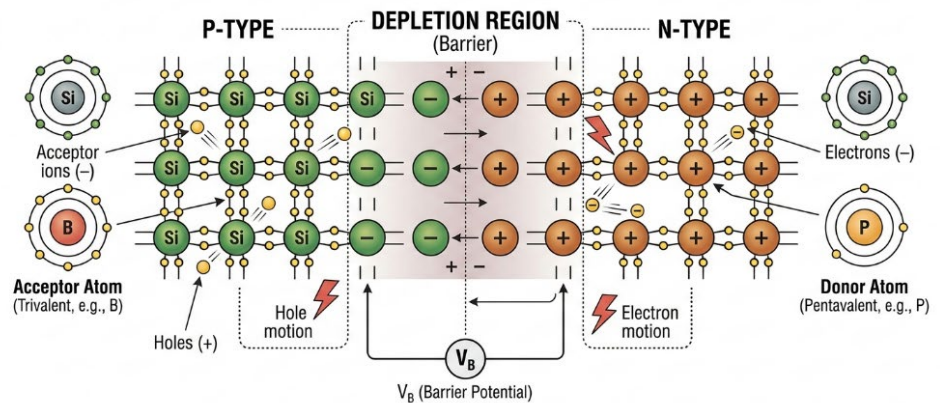
Formation of the P-N Junction

A **p-n junction** is formed when a single crystal of semiconductor is grown such that one half is doped p-type and the other half is doped n-type. The boundary between these two regions is called the **metallurgical junction**.

The Depletion Region and Barrier Potential

Immediately after junction formation, the following occurs:

1. **Diffusion:** Holes from the p-side diffuse into the n-side; electrons from the n-side diffuse into the p-side



2. **Recombination:** Holes and electrons meet and annihilate near the junction
3. **Ion formation:** As charge carriers recombine, they leave behind:
 - **Negative acceptor ions** on the p-side (since holes left)
 - **Positive donor ions** on the n-side (since electrons left)
4. **Depletion region:** A region devoid of mobile charge carriers forms around the junction
5. **Barrier potential:** The fixed ions create an electric field that opposes further diffusion

The Barrier Potential Values:

Diode Resistance:

- **Forward resistance:** $r_f = \Delta V_f / \Delta I_f$ (typically a few ohms)
- **Reverse resistance:** $r_r = \Delta V_r / \Delta I_r$ (typically several megaohms)

Physical Interpretation: The diode acts as a **unidirectional valve**—it conducts easily in the forward direction (low resistance) but blocks current in the reverse direction (high resistance). This property is the basis of rectification.

RECTIFICATION

Rectification is the process of converting alternating current (AC) into direct current (DC). Since most electronic devices require DC for operation, rectifiers are essential components in power supplies.

Half-Wave Rectification

In half-wave rectification, **only one half** (positive or negative) of the AC input cycle is allowed to pass to the output.

Operation:

Half-cycle	Terminal A	Diode State	Output
Positive ($0 \rightarrow T/2$)	Positive	Forward biased (ON)	Current flows
Negative ($T/2 \rightarrow T$)	Negative	Reverse biased (OFF)	No current

Peak Inverse Voltage (PIV): The maximum reverse voltage the diode must withstand. In half-wave rectification, $PIV = V_m$ (the peak input voltage).

Limitations of Half-Wave Rectification:

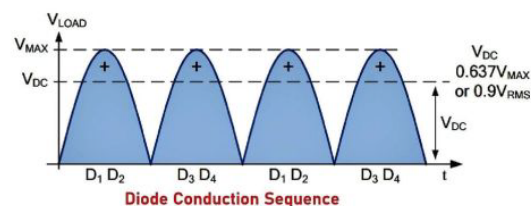
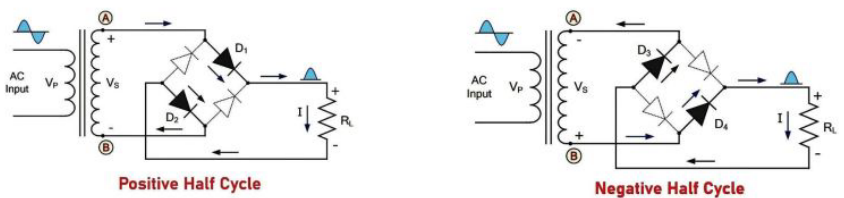
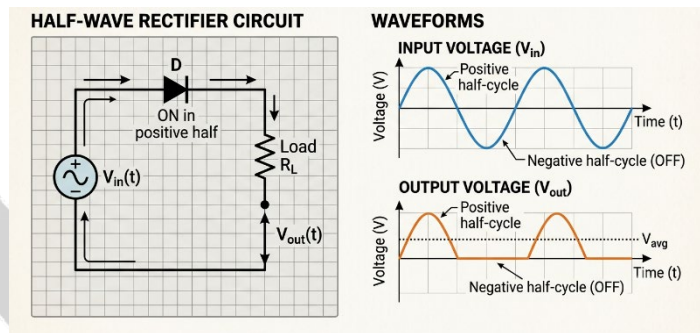
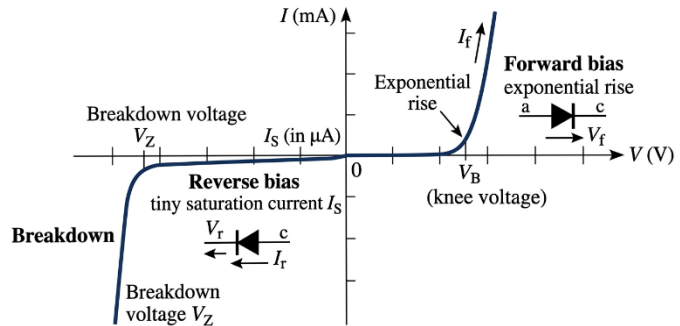
- Only 50% of input power is utilized
- Output is pulsating with large ripple
- Efficiency is low ($\approx 40.6\%$)

Full-Wave Rectification

Full-wave rectification uses **both halves** of the AC input cycle. The most common configuration is the **bridge rectifier** using four diodes.

Advantages of Full-Wave Rectification:

- Higher efficiency ($\approx 81.2\%$)

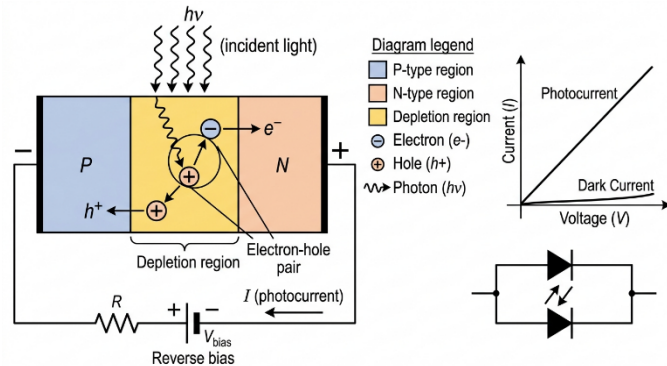


Photodiode

A **photodiode** is a p-n junction designed to detect light. It is operated in **reverse bias** and responds to incident light by producing an electric current.

Working Principle:

- In reverse bias, the depletion region is wide
- Incident photons with energy $\geq E_g$ break covalent bonds
- Electron-hole pairs are generated within the depletion region
- The electric field sweeps electrons to n-side and holes to p-side
- Reverse current increases proportionally to light intensity



Characteristics:

- Dark current: Very small reverse current with no light (μA or nA)
- Response time: Nanoseconds (extremely fast)
- Quantum efficiency: Typically 50–80%

Applications:

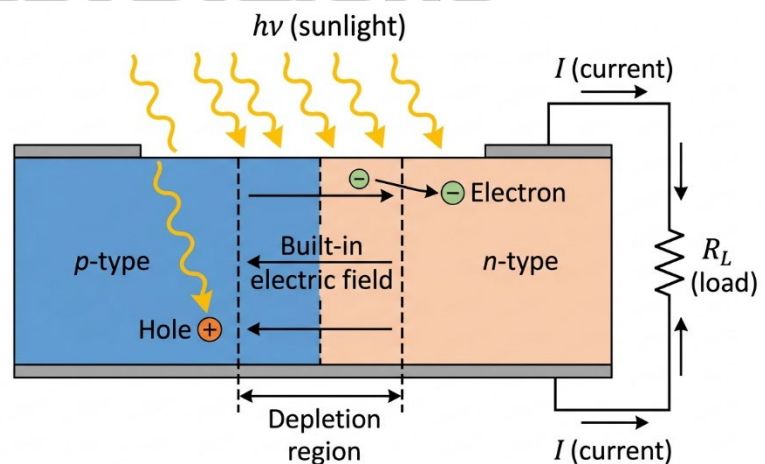
- Light detection in optical communication
- Automatic switching (street lights)
- Medical imaging
- Smoke detectors

Photovoltaic Cell (Solar Cell)

A **photovoltaic cell** converts light energy directly into electrical energy. Unlike a photodiode, it operates with **no external bias** and generates power rather than just detecting light.

Working Principle:

1. Light is absorbed in the depletion region
2. Electron-hole pairs are generated
3. The built-in electric field separates electrons and holes
4. Electrons flow to the n-side, holes to the p-side
5. Connecting an external load causes current to flow



Applications:

- Solar power generation (residential and utility-scale)
- Spacecraft power systems

Key Biasing Conditions:

1. $V_{BE} \approx 0.7 \text{ V}$ for Si (forward bias)
2. $V_{CE} > V_{BE}$ (collector more positive than emitter for NPN)
3. V_{CC} is typically much larger than V_{BB}

Current Relationships

Fundamental Current Equation:

$$I_E = I_B + I_C$$

where:

- I_E = Emitter current (largest)
- I_C = Collector current (approximately equal to I_E)
- I_B = Base current (smallest)

Physical Interpretation: The emitter injects carriers (electrons in NPN, holes in PNP). Most of these carriers are swept across the thin base into the collector. Only a tiny fraction recombine in the base and flow out as base current.

Current Gain Parameters:

Alpha (α) — Common-base current gain:

$$\alpha = \frac{I_C}{I_E} \text{ (typically 0.95 to 0.99)}$$

Since $I_C \approx I_E$, $\alpha \approx 1$.

Beta (β) — Common-emitter current gain:

$$\beta = \frac{I_C}{I_B} \text{ (typically 50 to 400)}$$

Relationship between α and β :

$$\beta = \frac{\alpha}{1 - \alpha} \text{ and } \alpha = \frac{\beta}{1 + \beta}$$

Derivation:

$$\beta = \frac{I_C}{I_B} = \frac{I_C}{I_E - I_C} = \frac{I_C/I_E}{1 - I_C/I_E} = \frac{\alpha}{1 - \alpha}$$

Example: If $\alpha = 0.98$, then

$$\beta = \frac{0.98}{1 - 0.98} = \frac{0.98}{0.02} = 49$$

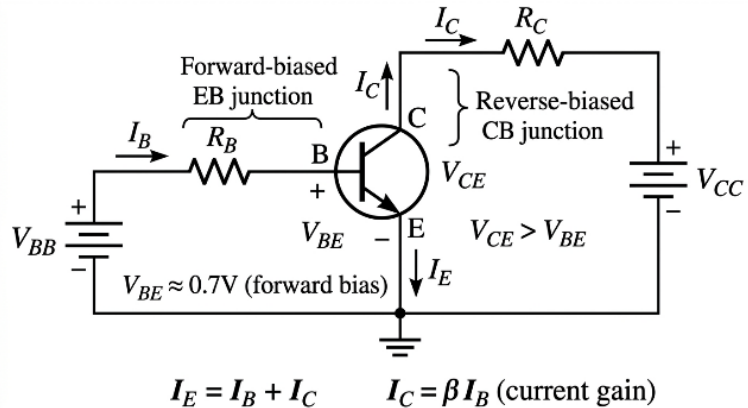
Physical Interpretation of β : A small change in base current produces a large change in collector current. This is the basis of **current amplification**.

Transistor as a Switch

In digital circuits, transistors operate in two distinct states: **cutoff** (OFF) and **saturation** (ON).

Two States:

State	Condition	Base Current	Collector Current	C-E Voltage	Equivalent
Cutoff (OFF)	$V_{BE} < 0.7 \text{ V}$	$I_B = 0$	$I_C = 0$	$V_{CE} \approx V_{CC}$	Open switch
Saturation (ON)	$V_{BE} \geq 0.7 \text{ V}$	$I_B > I_{B(\min)}$	$I_C = \beta I_B$	$V_{CE} \approx 0.1 \text{ V}$	Closed switch



Determining the Base Current Required for Saturation:

For a load requiring collector current $I_{C(sat)}$:

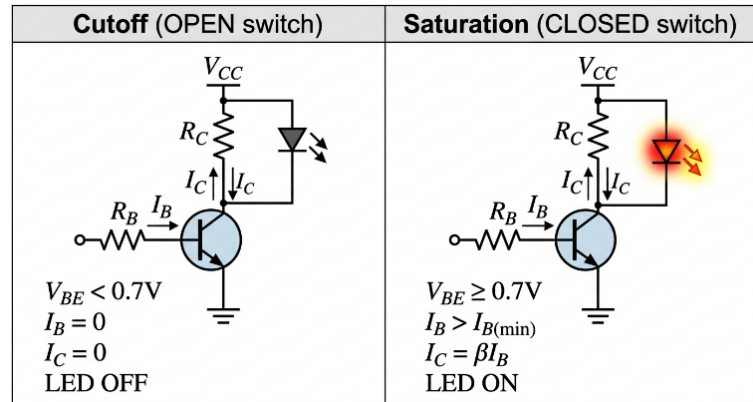
$$I_{B(min)} = \frac{I_{C(sat)}}{\beta}$$

In practice, use $I_B = 2 \times I_{B(min)}$ to ensure hard saturation.

Example Problem: A relay requires 50 mA to operate. The transistor has $\beta = 100$. Calculate the minimum base current.

Solution:

$$I_{B(min)} = \frac{50 \text{ mA}}{100} = 0.5 \text{ mA} = 500 \mu\text{A}$$



Transistor as an Amplifier

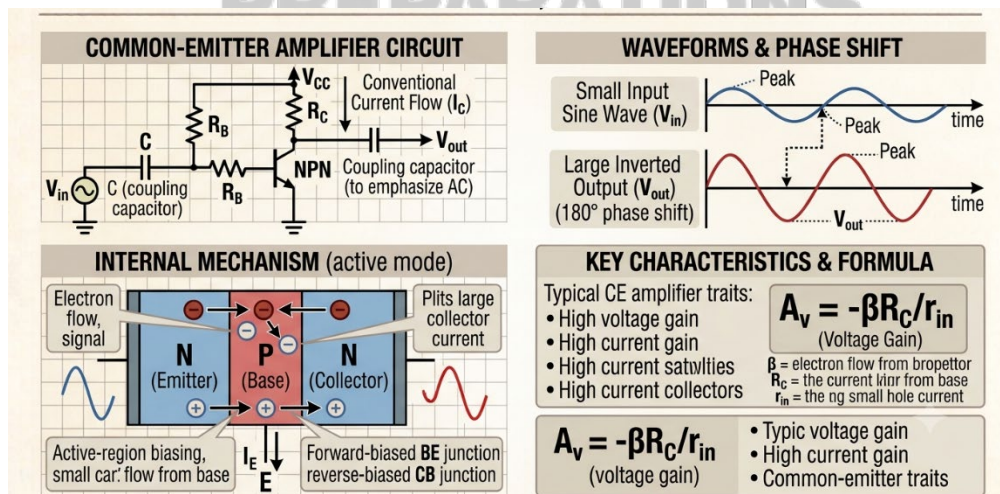
In analog circuits, transistors operate in the **active region** (between cutoff and saturation) to amplify small signals.

Voltage Gain Derivation:

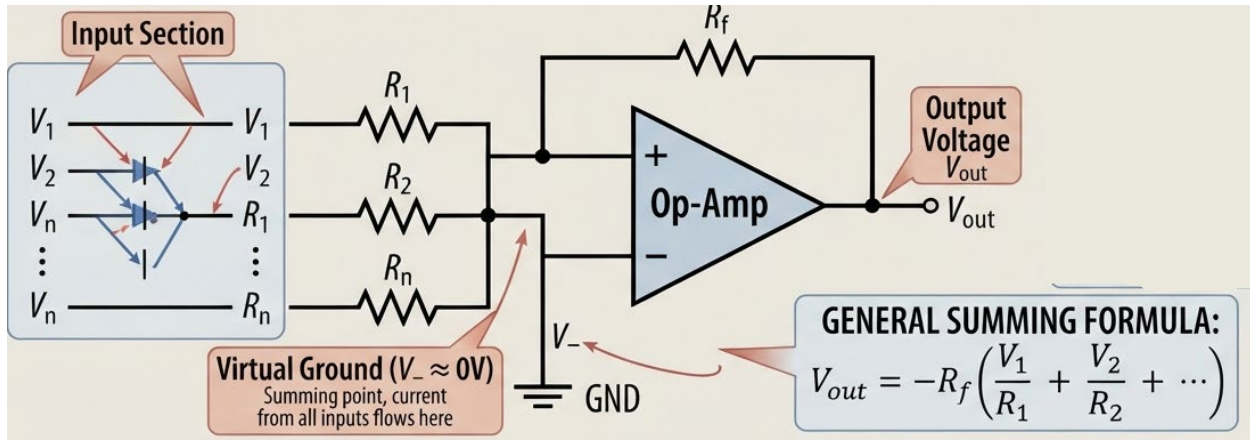
1. Input voltage causes base current change: $\Delta I_B = \Delta V_{in} / r_{in}$
2. Collector current change: $\Delta I_C = \beta \Delta I_B$
3. Output voltage change: $\Delta V_{out} = -\Delta I_C \times R_C = -\beta \Delta I_B R_C$
4. Voltage gain: $A_v = \frac{\Delta V_{out}}{\Delta V_{in}} = -\frac{\beta R_C}{r_{in}}$

Physical Interpretation:

- The negative sign indicates **180° phase inversion** between input and output
- The gain can be hundreds or thousands
- r_{in} is the input resistance of the transistor (typically 1–10 kΩ)



- Non-inverting terminal grounded



Output Derivation

By superposition (or KCL at the inverting node):

$$I_f = I_1 + I_2 + \dots + I_n$$

$$-\frac{V_{out}}{R_f} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots + \frac{V_n}{R_n}$$

$$V_{out} = -R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots + \frac{V_n}{R_n} \right)$$

If $R_1 = R_2 = \dots = R_n = R$:

$$V_{out} = -\frac{R_f}{R} (V_1 + V_2 + \dots + V_n)$$

Applications

- **Audio mixers:** Combine multiple audio signals
- **Digital-to-analog converters:** Sum binary-weighted currents
- **Mathematical operations:** Arithmetic addition

The Integrator

Circuit Configuration

- Capacitor C in feedback path
- Resistor R at input
- Non-inverting terminal grounded

Output Derivation

The current through the resistor:

$$I_R = \frac{V_{in}}{R}$$

This current flows into the capacitor (no current enters the op-amp input):

$$I_C = -I_R = -\frac{V_{in}}{R}$$

The voltage across the capacitor (output voltage):

$$V_{out} = \frac{1}{C} \int I_C dt = -\frac{1}{RC} \int V_{in} dt$$

Therefore:

Gain required: $A \geq 29$

LC Oscillators (Colpitts and Hartley)

Colpitts Oscillator

- Uses a parallel LC circuit in the feedback path
- Capacitive voltage divider provides feedback
- Frequency:

$$f = \frac{1}{2\pi\sqrt{LC_{eq}}}$$

where:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Hartley Oscillator

- Uses a parallel LC circuit
- Inductive voltage divider provides feedback
- Frequency:

$$f = \frac{1}{2\pi\sqrt{L_{eq}C}}$$

where:

$$L_{eq} = L_1 + L_2 + 2M$$

(M is mutual inductance)

Crystal Oscillators

- Use the piezoelectric property of quartz crystals
- Extremely stable frequency
- Used in watches, clocks, microprocessors

Key fact: Quartz crystals can achieve frequency stability of 10^{-6} or better.

TIMERS AND THE 555 TIMER

The 555 timer IC is one of the most popular and versatile integrated circuits ever designed. It can be configured as an astable multivibrator (oscillator), monostable multivibrator (one-shot), or bistable multivibrator (flip-flop).

Pin Configuration

Astable Mode (Free-Running Oscillator)

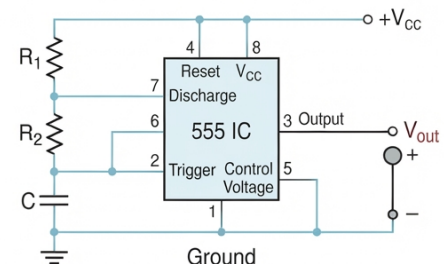
Circuit Description

- Resistors R_1 , R_2 and capacitor C determine frequency
- No external trigger needed
- Continuous square wave output

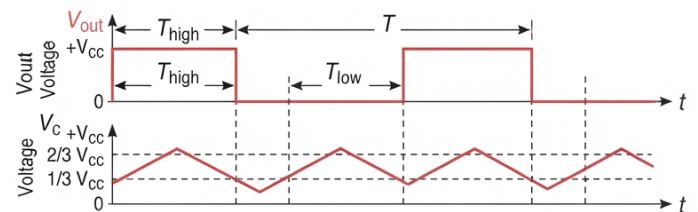
Frequency and Duty Cycle

$$T_{high} = 0.693(R_1 + R_2)C$$

555 Timer Astable Mode Circuit



Waveforms: Astable Mode

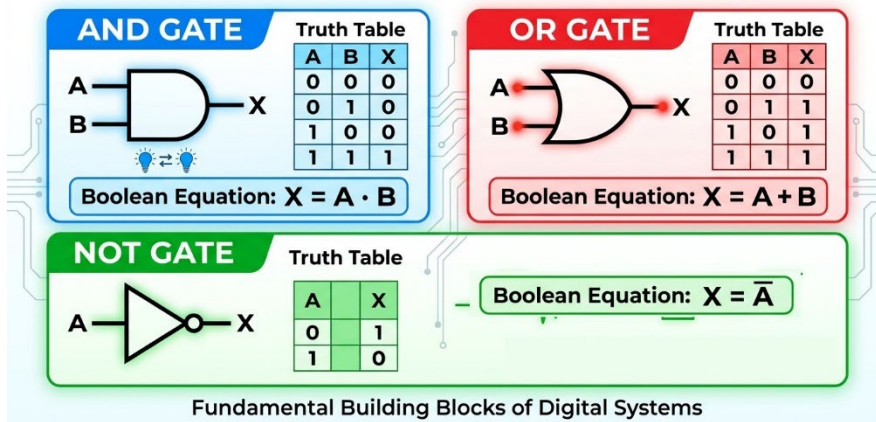


$$T_{high} = 0.693(R_1 + R_2)C$$

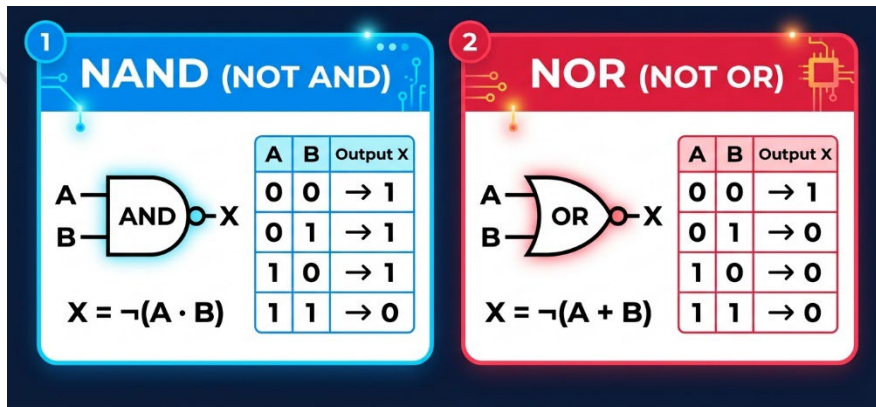
$$T_{low} = 0.693R_2C$$

$$f = 1.44/((R_1 + 2R_2)C)$$

Basic Logic Gates



Universal Gates

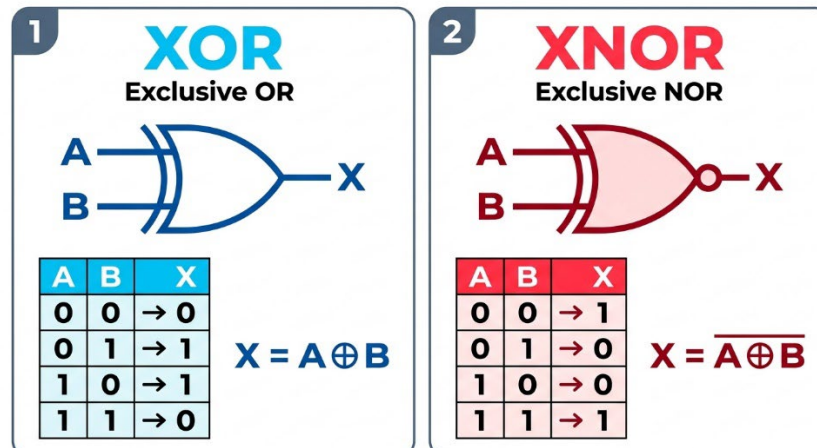


Why NAND and NOR are called "Universal Gates": Any logic function (AND, OR, NOT) can be constructed using only NAND gates or only NOR gates. This simplifies manufacturing since only one type of gate needs to be produced.

Derived Gates

XOR Gate (Exclusive OR)

XNOR Gate (Exclusive NOR) = XOR followed by NOT

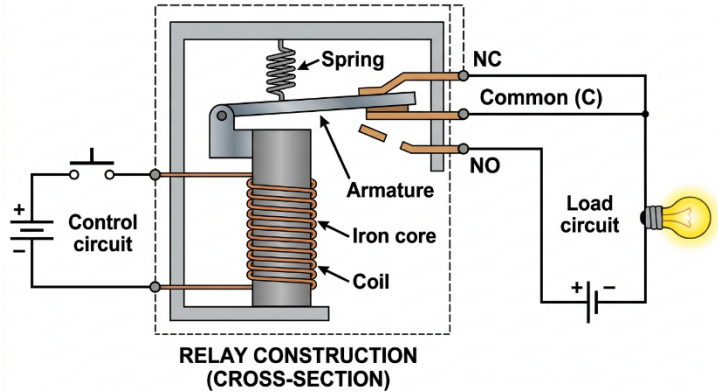


Solid State Relay (SSR)	No	Fast (μs)	Very high	Precision control, computers
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Solid State Relay Operation:

An SSR uses an **optocoupler** (LED + phototransistor) to provide isolation:

1. Control current turns on an internal LED
2. Light from the LED falls on a phototransistor or photodiode
3. The photodetector triggers a switching device (e.g., TRIAC or transistor)
4. The load circuit is switched without any moving parts



Applications of Relays:

- Turning on/off high-power devices (motors, heaters, lights) with low-power signals
- Protecting circuits from overload or faults
- Timer-controlled switching
- Automotive systems (headlights, horns, starter motors)
- Telephone exchanges and communication systems

QUANTUM COMPUTING (Emerging Technology)

Quantum computing represents a paradigm shift from classical computing. While not yet fully mature, it promises exponentially greater computational power for specific problems.

Key Concepts:

Classical Computer	Quantum Computer
Bit (0 or 1)	Qubit (superposition of 0 and 1)
Deterministic	Probabilistic
Linear speedup	Exponential speedup for certain problems
Room temperature operation	Requires near-absolute zero (~ 10 mK)
Well-established	Still in early development

CONCEPTUAL SUMMARY

Concept	Key Equation	Physical Meaning
Inverting amplifier	$A_v = -R_f/R_1$	Inverts and amplifies; gain set by resistor ratio
Non-inverting amplifier	$A_v = 1 + R_f/R_1$	Amplifies without inversion; gain ≥ 1
Voltage follower	$A_v = 1$	Buffer; high input, low output impedance
Summing amplifier	$V_{out} = -\frac{R_f}{R} \sum V_i$	Adds multiple input signals
Integrator	$V_{out} = -\frac{1}{RC} \int V_{in} dt$	Mathematical integration (low-pass)
Differentiator	$V_{out} = -RC \frac{dV_{in}}{dt}$	Mathematical differentiation (high-pass)
Comparator	$V_{out} = \pm V_{sat}$	Compares two voltages; digital output

Electronics: One-Liners

Fundamental Semiconductor Physics

1. Electronics is the branch of physics dealing with the emission, behavior, and control of electrons in semiconductors, vacuum, and gases.
2. Semiconductors have electrical conductivity between that of conductors and insulators.
3. Silicon and germanium are the most important elemental semiconductors, both having four valence electrons.
4. An intrinsic semiconductor is a pure crystal with no impurities; at $T = 0$ K, it has no free charge carriers.
5. At temperatures above 0 K, thermal energy breaks covalent bonds, creating free electrons and holes.
6. A hole behaves as a positively charged particle and moves through the crystal opposite to electron flow.
7. In an intrinsic semiconductor, the number density of electrons equals the number density of holes: $n = p = n_i$.
8. For silicon at 300 K, $n_i \approx 1.5 \times 10^{10} \text{ cm}^{-3}$.
9. Doping is the process of deliberately adding impurities to a semiconductor to increase conductivity.
10. n-type semiconductors are created by doping with pentavalent atoms (P, As, Sb); electrons are majority carriers.
11. Pentavalent impurities are called donors because they donate an extra electron.
12. p-type semiconductors are created by doping with trivalent atoms (B, Al, In, Ga); holes are majority carriers.
13. Trivalent impurities are called acceptors because they accept an electron, creating a hole.
14. Despite having majority carriers, both n-type and p-type semiconductors remain electrically neutral.
15. The law of mass action for semiconductors: $np = n_i^2$.

The p-n Junction (Semiconductor Diode)

16. A p-n junction is formed when a single crystal has one half doped p-type and the other half doped n-type.
17. Diffusion of carriers across the junction creates a depletion region devoid of mobile charge carriers.
18. The depletion region contains fixed positive donor ions on the n-side and negative acceptor ions on the p-side.
19. The built-in barrier potential is 0.7 V for silicon and 0.3 V for germanium at room temperature.
20. Forward bias connects positive terminal to p-side, negative to n-side; this reduces the barrier and allows current flow.
21. Reverse bias connects positive to n-side, negative to p-side; this increases the barrier and blocks current.
22. The reverse saturation current is due to minority carriers and is typically in microamperes.
23. A diode acts as a unidirectional valve—conducting easily in forward bias and blocking in reverse bias.

90. A solid-state relay uses an optocoupler (LED + phototransistor) for isolation with no moving parts.
91. Relays provide isolation between control and load circuits.

Quantum Computing

92. A classical bit is either 0 or 1; a qubit can be in superposition of 0 and 1.
93. Quantum computing offers exponential speedup for certain problems.
94. Qubits must be cooled to near absolute zero (≈ 10 mK) to prevent decoherence.
95. Applications include cryptography, drug discovery, and optimization problems.

Practice MCQs

1. Electronics is the branch of physics that deals with:

- A) The study of light
- B) The emission, behavior, and control of electrons
- C) The study of magnetic fields
- D) The study of heat

Correct Answer: B

2. Semiconductors have conductivity:

- A) Between conductors and insulators
- B) Higher than conductors
- C) Lower than insulators
- D) The same as conductors

Correct Answer: A

3. Silicon and germanium have how many valence electrons?

- A) 2
- B) 3
- C) 4
- D) 5

Correct Answer: C

4. An intrinsic semiconductor:

- A) Is heavily doped
- B) Is pure with no impurities
- C) Has only holes as carriers
- D) Has only electrons as carriers

Correct Answer: B

5. At $T = 0$ K, an intrinsic semiconductor has:

- A) Many free electrons
- B) No free charge carriers
- C) Only holes
- D) Only electrons

Correct Answer: B

6. A hole behaves as:

- A) A negatively charged particle
- B) A positively charged particle
- C) A neutral particle
- D) A photon

Correct Answer: B

7. In an intrinsic semiconductor:

- A) $n > p$
- B) $n < p$
- C) $n = p$
- D) $n = 0$

Correct Answer: C

8. The intrinsic carrier concentration of silicon at 300 K is approximately:

- A) $1.5 \times 10^{10} \text{ cm}^{-3}$
- B) $1.5 \times 10^{16} \text{ cm}^{-3}$
- C) $1.5 \times 10^{22} \text{ cm}^{-3}$
- D) $1.5 \times 10^{28} \text{ cm}^{-3}$

Correct Answer: A

9. Doping is the process of:

- A) Removing impurities
- B) Adding impurities to control conductivity
- C) Melting the semiconductor
- D) Creating a vacuum

Correct Answer: B

10. n-type semiconductors are created by doping with:

- A) Trivalent atoms
- B) Pentavalent atoms
- C) Noble gases
- D) Alkali metals

Correct Answer: B



Part VI: Modern Physics



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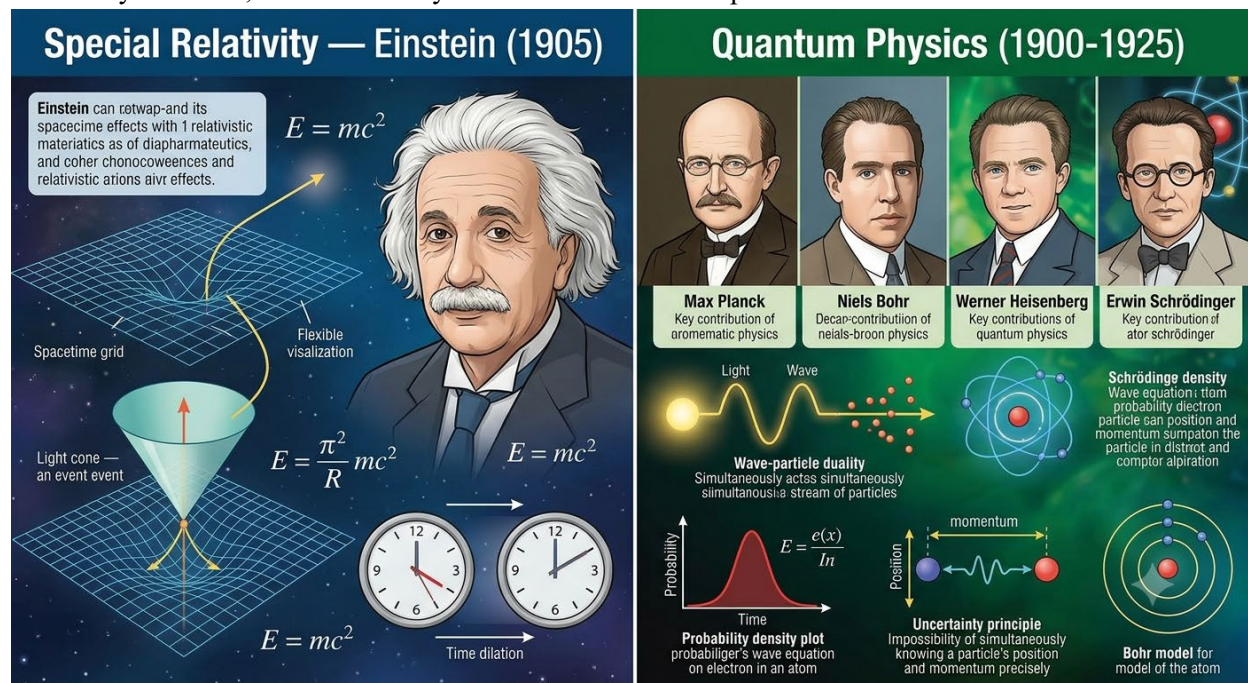


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Chapter 16: Modern Physics

Introduction

Physics, as a discipline, has undergone two profound revolutions in the twentieth century that fundamentally altered our understanding of space, time, and matter. The first of these revolutions—Einstein's theory of relativity—emerged from a careful examination of the concepts of simultaneity, time, and length when observers move relative to one another at speeds approaching the speed of light. The second revolution—quantum physics—arose from the realization that energy, momentum, and even matter itself are quantized, existing only in discrete packets rather than in continuous amounts. Prior to 1900, physics was dominated by Newtonian mechanics, Maxwell's electromagnetism, and thermodynamics—a collection of theories so successful that many believed the fundamental laws of nature had been largely discovered. Yet beneath this apparent completeness lay deep cracks: the constancy of the speed of light contradicted Newtonian velocity addition, the ultraviolet catastrophe plagued blackbody radiation, and the stability of atoms remained inexplicable.



Special Theory of Relativity

Frames of Reference: The Foundation of Relativity

Before we can understand relativity, we must first understand what it means to measure motion. A **frame of reference** is a coordinate system (typically specified by an origin and a set of axes) that allows us to determine the positions and velocities of objects. All measurements of motion are made relative to some frame of reference.

Consider two persons seated in a bus moving with constant velocity v along a straight road. To each other, they appear at rest. But to an observer standing on the roadside, both are moving with velocity v . The description of motion depends entirely on the chosen frame of reference.

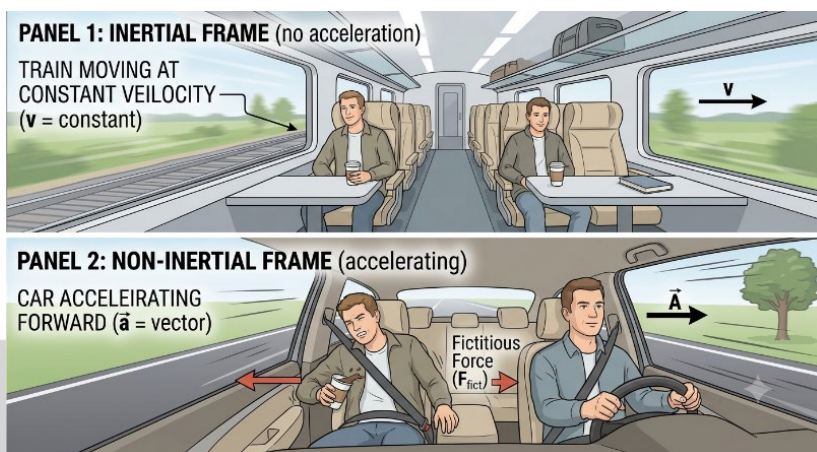
Inertial Frames of Reference

An **inertial frame of reference** is one in which Newton's first law holds: an object at rest remains at rest, and an object in motion continues in motion with constant velocity along a straight line, unless acted upon by an external force.

Definition: A frame of reference that is either at rest or moving with uniform velocity (zero acceleration) is called an inertial frame.

Properties of inertial frames:

- They do not accelerate
- Any frame moving with constant velocity relative to an inertial frame is itself inertial
- Newton's laws of motion are valid in all inertial frames
- The laws of physics take the same mathematical form in all inertial frames



Examples: A stationary laboratory on Earth's surface (approximately inertial), a spaceship drifting at constant velocity in deep space, a train moving at constant speed on straight tracks.

Non-Inertial Frames of Reference

A **non-inertial frame of reference** is one that is accelerating relative to an inertial frame. In such frames, objects appear to experience forces even when no physical forces act upon them—these are called fictitious forces.

Examples: A car accelerating forward (you feel pressed into your seat), a rotating merry-go-round (you experience centrifugal force), an elevator starting or stopping.

Key Insight: In non-inertial frames, Newton's laws do not hold in their simple form unless we introduce fictitious forces. The distinction between inertial and non-inertial frames is fundamental to understanding relativity, as Einstein's postulates apply specifically to inertial frames.

The Postulates of Special Relativity

In 1905, Albert Einstein published his special theory of relativity, based on two postulates that seem deceptively simple but have profound implications.

Postulate 1: The Principle of Relativity

The laws of physics are the same for all observers in any inertial frame of reference.

This means that no experiment performed entirely within an inertial frame can determine whether that frame is "at rest" or "moving uniformly." There is no preferred or absolute frame of reference. The laws of electromagnetism, mechanics, thermodynamics—all physical laws—take identical form in every inertial frame.

This postulate extends Galileo's principle of relativity (which applied only to mechanics) to all of physics, including electromagnetism and optics. It does **not** claim that all measured quantities are the same in different frames; rather, it asserts that the **laws** relating those measurements are identical.

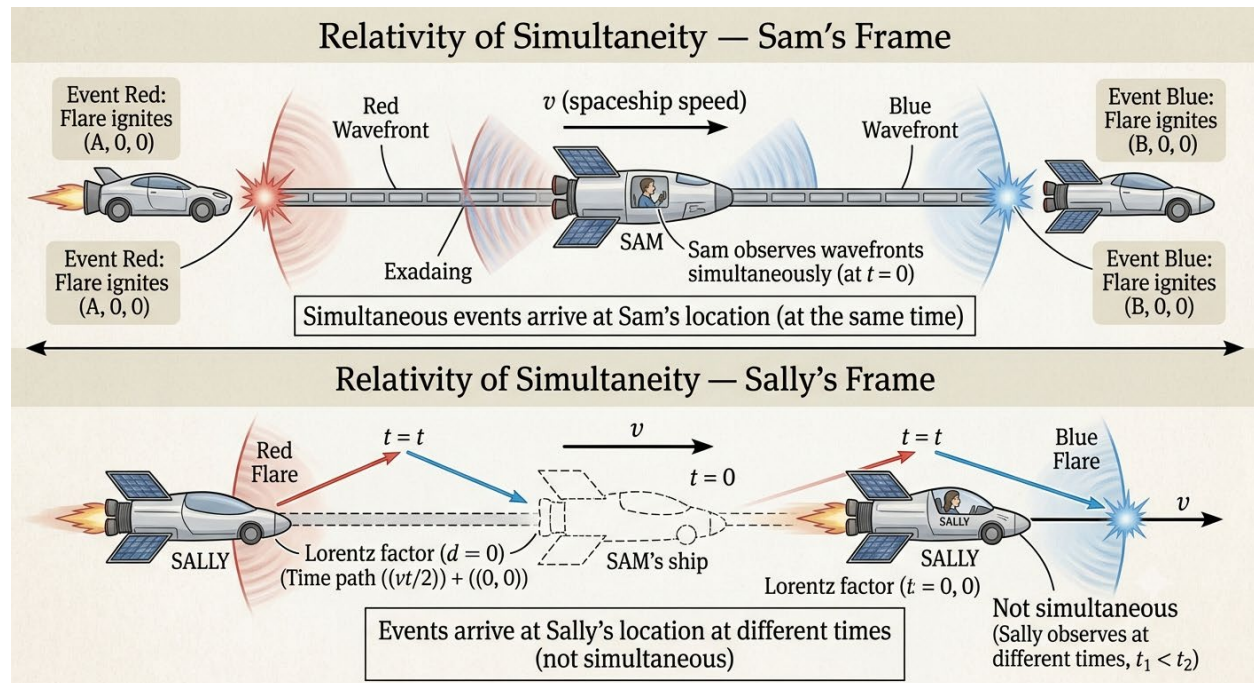
arrives. This procedure ensures that the time coordinate of an event is simply the reading on the clock nearest to the event at the instant it occurs.

The Relativity of Simultaneity

One of the most profound consequences of Einstein's postulates is that simultaneity is not absolute. Two events that are simultaneous in one inertial frame are generally **not** simultaneous in another frame moving relative to the first.

Consider the following thought experiment. Two long spaceships—the SS *Sally* and the SS *Sam*—move relative to each other with constant velocity $\vec{v}$. Each observer is stationed at the midpoint of her or his ship. Two meteorites strike the ships, one producing a red flare (event Red) and the other a blue flare (event Blue). The expanding wavefronts from these events travel outward at speed c .

From Sam's perspective (at rest relative to his ship), the wavefronts reach him at the same time, and he finds that he was exactly halfway between the two strike positions. He therefore concludes that the events were simultaneous.



From Sally's perspective (moving relative to Sam's ship), the wavefront from event Red reaches her **before** the wavefront from event Blue, yet she also finds that she was exactly halfway between the two strike positions on **her** ship. She therefore concludes that event Red occurred **before** event Blue. Both observers are correct. Simultaneity is relative—it depends on the motion of the observer. This is not a mere practical difficulty in measuring times; it is a fundamental property of the universe. For everyday speeds ($v \ll c$), the effect is too small to notice, which is why our intuition tells us that simultaneity is absolute.

Mathematical Formulation: Consider two events occurring at positions x_1 and x_2 at times $t_1 = t_2$ in frame S (simultaneous). In frame S' moving with velocity v relative to S , the time difference between the events is:

$$\Delta t' = t'_2 - t'_1 = \gamma \left(\Delta t - \frac{v \Delta x}{c^2} \right)$$

Properties of the Lorentz Factor:

Speed v	β	γ	Interpretation
0	0	1	No time dilation
0.1c	0.1	1.005	Negligible effect
0.5c	0.5	1.155	15.5% time dilation
0.9c	0.9	2.294	Time passes 2.29× slower
0.99c	0.99	7.089	7× slower
0.999c	0.999	22.37	22× slower
c	1	∞	Not attainable

Experimental Confirmation:

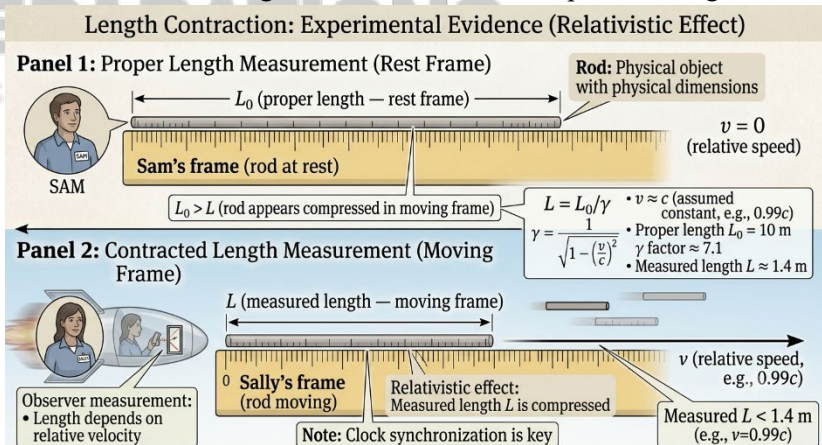
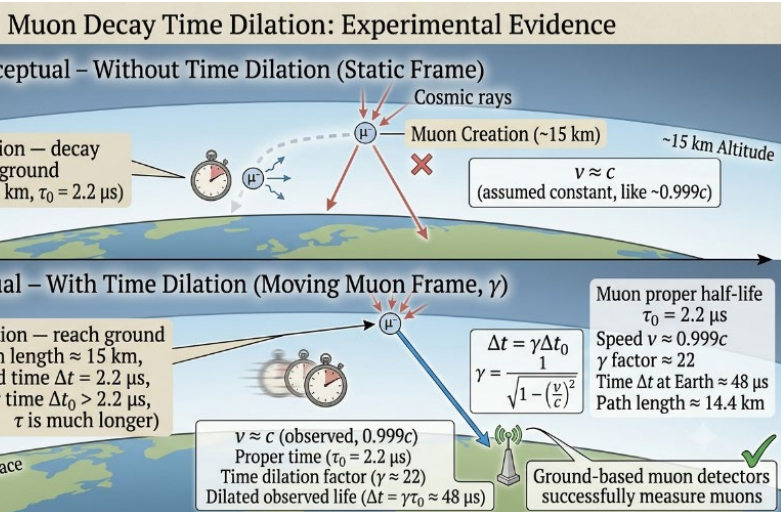
The time dilation effect has been confirmed with extraordinary precision. Muons produced in the upper atmosphere have a proper lifetime of about $2.2 \mu\text{s}$. Even traveling at nearly the speed of light, classical physics predicts they would decay before reaching Earth's surface.

However, due to time dilation, their lifetime in Earth's frame is extended by the factor γ , allowing many to reach the surface—exactly as observed.

Length Contraction

Just as time intervals are affected by relative motion, so are lengths. Consider a rod at rest in frame S' with proper length L_0 . An observer in frame S moving relative to the rod with speed v along the direction of the rod will measure a **shorter** length L . This is **length contraction**.

To derive this, consider a platform of proper length L_0 at rest in Sam's frame. Sally, moving at speed v relative to Sam, measures the length L of the platform as it passes her. In Sam's frame, the time for Sally to travel the length of the platform is $\Delta t = L_0/v$. In Sally's frame, the platform moves past her, and she measures the time as $\Delta t_0 = L/v$. Because $\Delta t = \gamma \Delta t_0$ (time dilation), we have:



$$\frac{L_0}{v} = \gamma \frac{L}{v} \Rightarrow L = \frac{L_0}{\gamma}$$

Thus:

$$L = L_0 \sqrt{1 - \beta^2} = \frac{L_0}{\gamma}$$

Important aspects of length contraction:

- Contraction occurs only along the direction of relative motion
- Lengths perpendicular to the motion are unchanged
- The proper length L_0 is the **maximum** possible measured length
- For a moving object, simultaneous measurement of its endpoints in the observer's frame is essential; otherwise, the measurement does not represent a true length

The Lorentz Transformation

The Galilean transformation, which relates coordinates in different inertial frames:

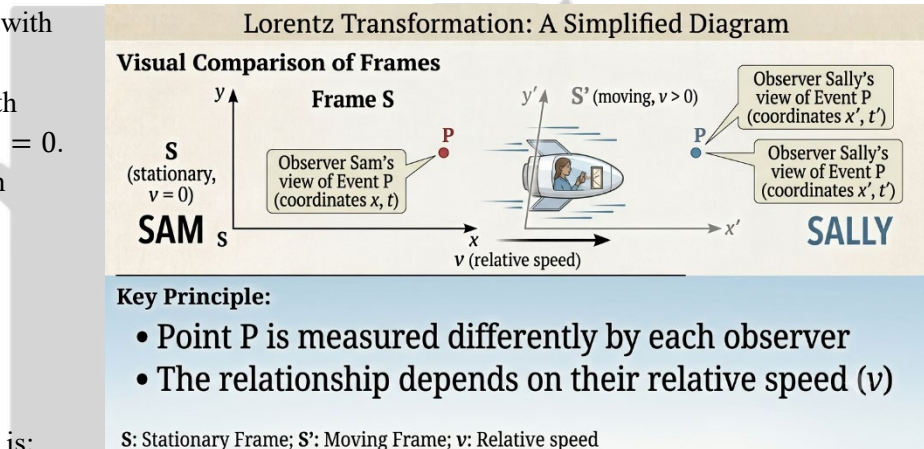
$$x' = x - vt, t' = t$$

works well for $v \ll c$ but fails at relativistic speeds. The correct transformation, derived from Einstein's postulates, is the **Lorentz transformation**.

Consider frame S' moving with velocity v along the common x and x' axes, with origins coinciding at $t = t' = 0$. The Lorentz transformation equations are:

$$\begin{aligned} x' &= \gamma(x - vt), \\ y' &= y, \\ z' &= z, \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right) \end{aligned}$$

The inverse transformation is:



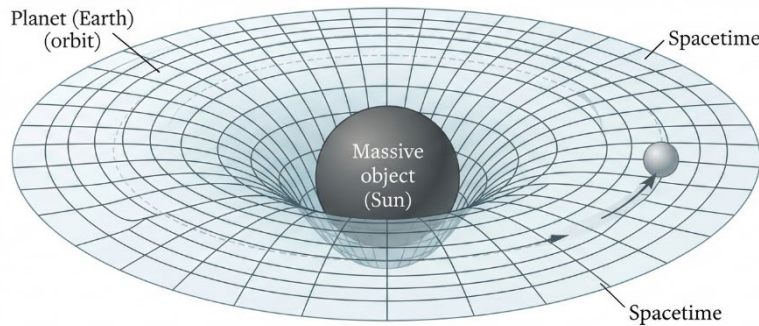
$$\begin{aligned} x &= \gamma(x' + vt'), \\ y &= y', \\ z &= z', \\ t &= \gamma\left(t' + \frac{vx'}{c^2}\right) \end{aligned}$$

Notice that space and time are **entangled**: the transformation for x' depends on t , and the transformation for t' depends on x . This entanglement is the mathematical expression of the relativity of simultaneity.

For pairs of events, the Lorentz transformation takes the difference form:

$$\begin{aligned} \Delta x' &= \gamma(\Delta x - v\Delta t), \\ \Delta t' &= \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right), \\ \Delta x &= \gamma(\Delta x' + v\Delta t'), \\ \Delta t &= \gamma\left(\Delta t' + \frac{v\Delta x'}{c^2}\right) \end{aligned}$$

From these, we can recover time dilation (by setting $\Delta x' = 0$) and length contraction (by setting $\Delta t = 0$).



Gravity = curvature of spacetime

The Geodesic

In General Relativity, objects move along **geodesics**—the shortest paths in curved spacetime.

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$$

where $\Gamma_{\alpha\beta}^\mu$ are the **Christoffel symbols** (connection coefficients) that describe how spacetime is curved.

Physical Interpretation: A freely falling object follows a geodesic—it is not "pulled" by a force but simply follows the curvature of spacetime.

The Schwarzschild Solution

The first exact solution to Einstein's field equations was found by Karl Schwarzschild in 1916, just months after Einstein published General Relativity. It describes the spacetime around a spherically symmetric, non-rotating mass.

The Schwarzschild Metric

For a non-rotating, spherically symmetric mass M :

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{2GM}{c^2 r}} + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

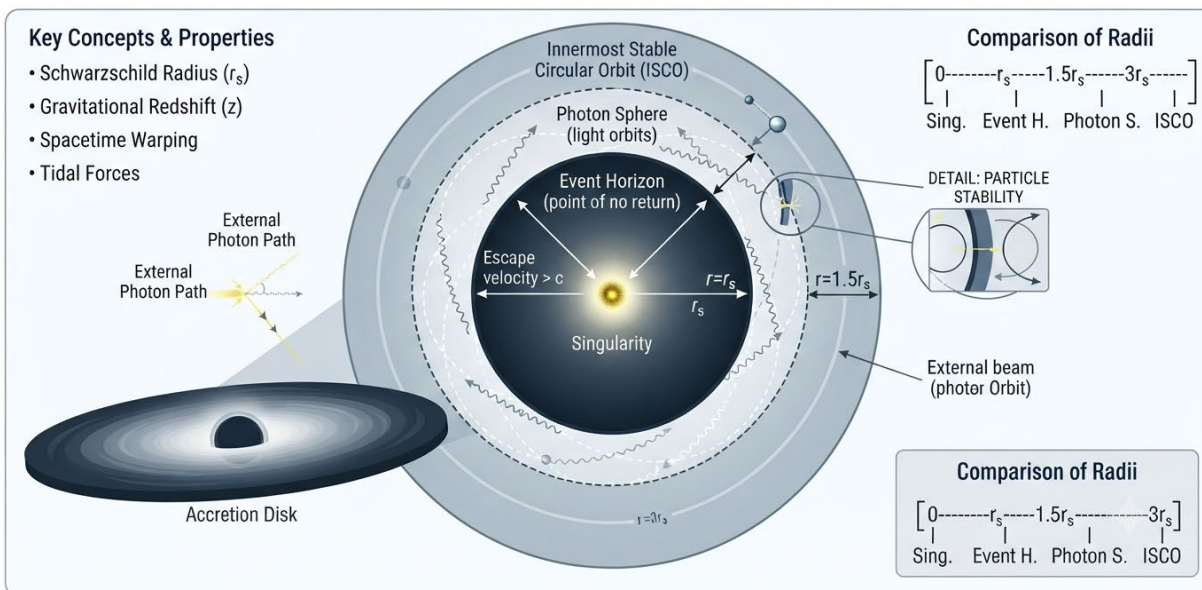
Key Features

Feature	Expression	Physical Meaning
Schwarzschild radius	$r_s = \frac{2GM}{c^2}$	Radius at which spacetime becomes singular
Gravitational time dilation	$\Delta t = \frac{\Delta t_0}{\sqrt{1 - r_s/r}}$	Clocks run slower near massive objects
Gravitational redshift	$\frac{\Delta\lambda}{\lambda} = \frac{GM}{c^2 r}$	Light loses energy climbing out of a gravitational field
Perihelion precession	$\Delta\theta = \frac{6\pi GM}{c^2 a(1 - e^2)}$	Mercury's orbit precesses due to spacetime curvature

The Schwarzschild Radius

The Schwarzschild radius r_s is the radius at which the escape velocity equals the speed of light:

BLACK HOLE ANATOMY: CROSS-SECTION AND KEY REGIONS



Types of Black Holes

Type	Mass Range	Formation
Stellar-mass	$3 - 100M_{\odot}$	Collapse of massive stars (supernovae)
Intermediate-mass	$100 - 10^5M_{\odot}$	Mergers of smaller black holes
Supermassive	$10^5 - 10^{10}M_{\odot}$	Found at centers of galaxies (e.g., Sagittarius A*)
Primordial	Any	Formed in the early universe

Black Hole Properties (No-Hair Theorem)

A black hole is completely described by only three properties:

- Mass (M)**
- Charge (Q)**
- Angular momentum (J)**

"Black holes have no hair"—John Wheeler

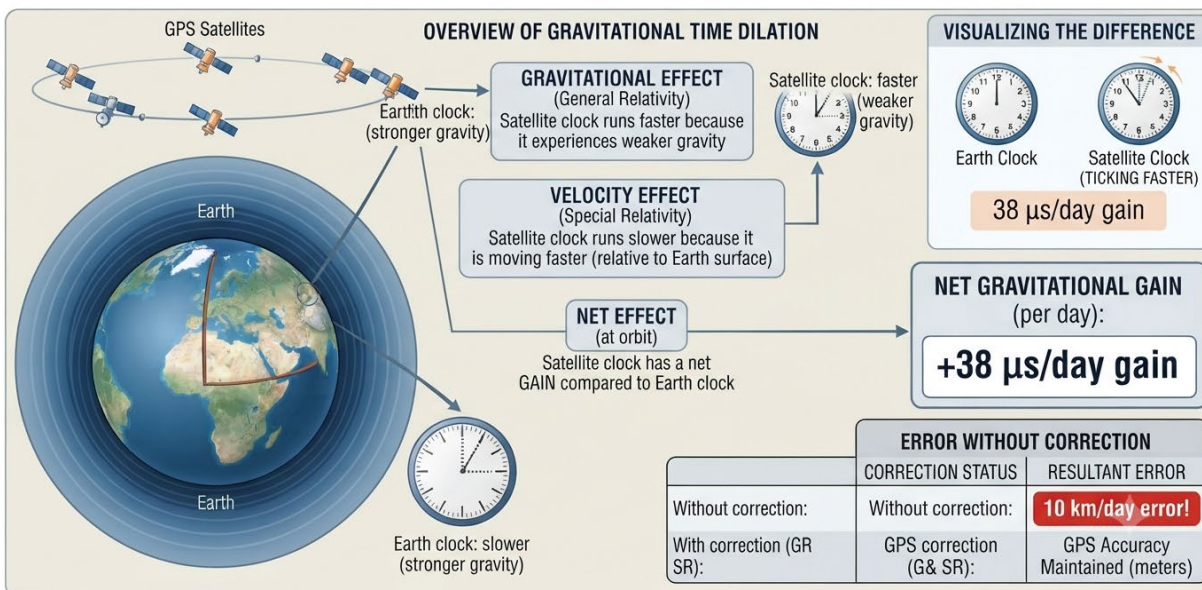
Gravitational Time Dilation

Clocks run slower in stronger gravitational fields.

Formula

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{2GM}{c^2r}}}$$

GPS & GENERAL RELATIVITY: GRAVITATIONAL TIME DILATION



Position	Time Dilation (relative to Earth)
Earth's surface	Reference
GPS satellites	Clocks run faster by 38 $\mu\text{s/day}$ (due to weaker gravity)
Near a black hole	Clocks run extremely slow (or stop at event horizon)

Example: GPS Satellites

Given Data:

- GPS satellite altitude: 20,200 km
- Earth's mass: 5.97×10^{24} kg
- Earth's radius: 6.37×10^6 m

Solution:

Time dilation factor at satellite:

$$\sqrt{1 - \frac{2GM}{c^2 r}} \approx 1 - 4.4 \times 10^{-10}$$

Satellite clocks gain about 38 μs per day compared to Earth clocks.

Practical Significance: GPS systems must correct for relativistic time dilation—without correction, GPS would be inaccurate by about 10 km per day!

Gravitational Redshift

Light loses energy as it climbs out of a gravitational field, causing its wavelength to increase (redshift).

Formula

For light emitted from radius r to infinity:

$$\frac{\Delta\lambda}{\lambda} = \frac{GM}{c^2 r}$$

Einstein's Photon Theory: Einstein proposed that light consists of particles called **photons**, each with energy $E = hf$. The photoelectric effect occurs when a single photon transfers its entire energy to a single electron in the metal.

Work Function (Φ): The minimum energy required to liberate an electron from the metal surface. It is a property of the metal.

Einstein's Photoelectric Equation:

$$hf = \Phi + K_{\max}$$

where $K_{\max}$ is the maximum kinetic energy of emitted electrons.

Threshold Frequency (f_0): The minimum frequency required for photoemission, when $K_{\max} = 0$:

$$hf_0 = \Phi \Rightarrow f_0 = \frac{\Phi}{h}$$

Threshold Wavelength (λ_0):

$$\lambda_0 = \frac{c}{f_0} = \frac{hc}{\Phi}$$

Stopping Potential (V_0): The retarding potential that just stops the most energetic photoelectrons:

$$eV_0 = K_{\max} = hf - \Phi$$

Graphical Analysis: A plot of stopping potential V_0 versus frequency f yields a straight line:

$$V_0 = \frac{h}{e}f - \frac{\Phi}{e}$$

Slope = h/e — allows determination of Planck's constant

- Intercept (on frequency axis) = f_0 — threshold frequency
- Intercept (on voltage axis) = $-\Phi/e$ — work function

Solved Example: A metal has work function 4.2 eV and is irradiated with light of wavelength 2000 Å. Find the maximum kinetic energy of emitted electrons.

Solution:

$$\lambda = 2000 \text{ Å} = 2 \times 10^{-7} \text{ m}$$

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{2 \times 10^{-7}} = 1.5 \times 10^{15} \text{ Hz}$$

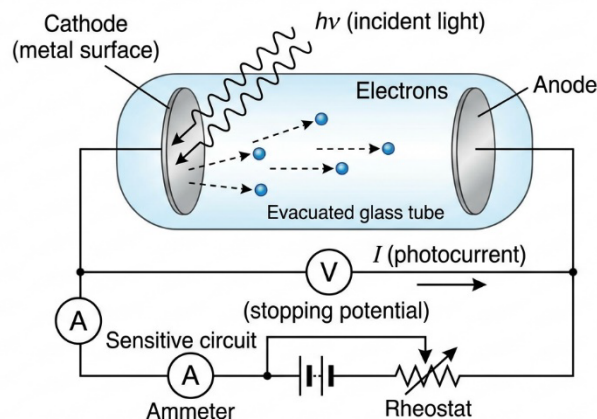
$$E_{\text{photon}} = hf = (6.63 \times 10^{-34})(1.5 \times 10^{15}) = 9.945 \times 10^{-19} \text{ J} = 6.21 \text{ eV}$$

$$K_{\max} = E_{\text{photon}} - \Phi = 6.21 - 4.2 = 2.01 \text{ eV}$$

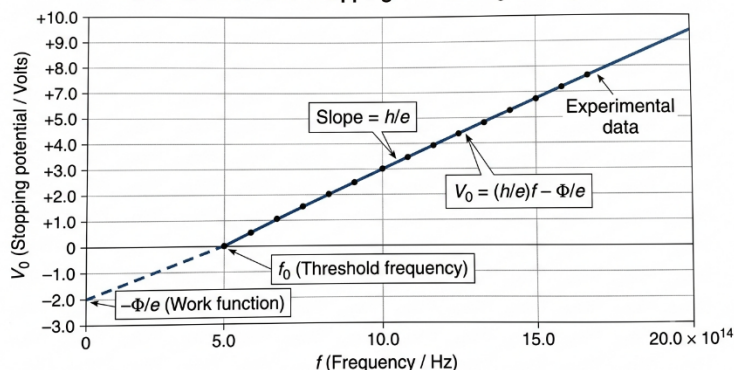
Photon Momentum and Compton Scattering

Although photons have no mass, they do carry momentum. From relativity, a photon (massless particle) has momentum:

Photoelectric Effect Apparatus

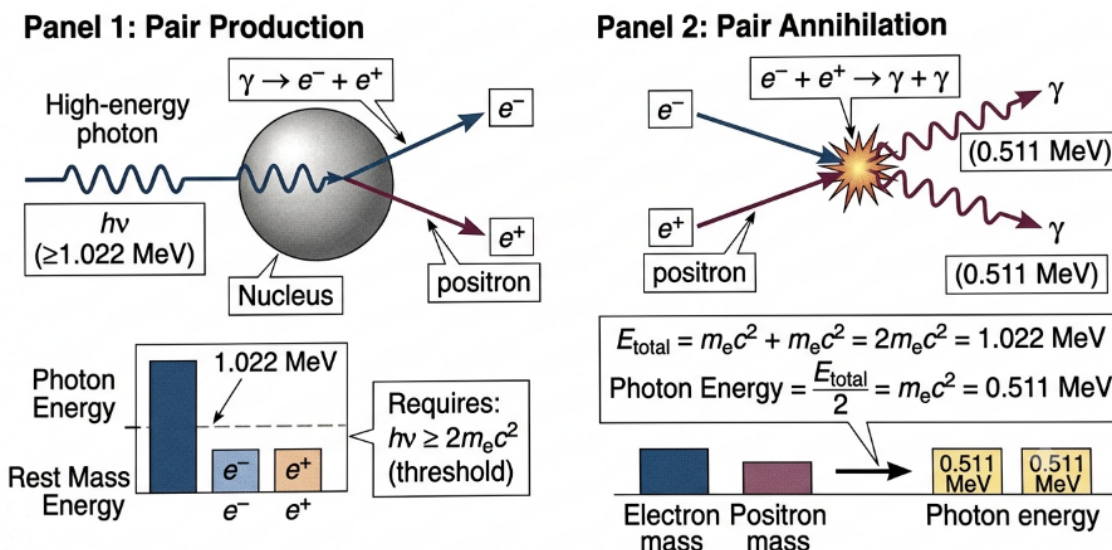


Photoelectric Effect: Stopping Potential V_0 vs Frequency f



Condition for pair production: The photon energy must exceed the combined rest energy of the electron and positron:

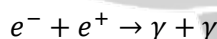
$$E_{\text{photon}} \geq 2m_0c^2 = 2 \times 0.511 \text{ MeV} = 1.022 \text{ MeV}$$



Conservation Laws in Pair Production:

- **Charge:** Photon (0 charge) $\rightarrow$ electron ($-e$) + positron ($+e$) $\rightarrow$ net charge 0
- **Energy:** $hf = (K_e + m_0c^2) + (K_{e^+} + m_0c^2)$
- **Momentum:** Requires presence of nucleus to conserve momentum

Pair Annihilation: The reverse process: an electron and positron collide and annihilate, producing two gamma-ray photons.



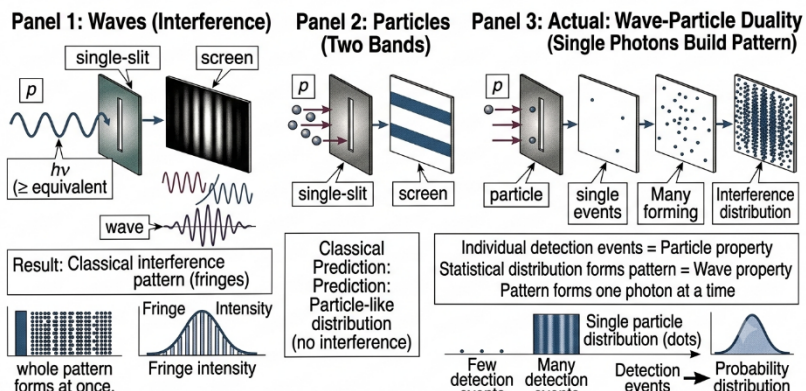
Each photon typically has energy $m_0c^2 = 0.511 \text{ MeV}$ (if initial kinetic energy is negligible). The two photons travel in opposite directions to conserve momentum.

Applications: Positron Emission Tomography (PET) scans use pair annihilation to create images of metabolic processes in the human body.

Light as a Probability Wave

The double-slit experiment reveals the strange nature of light in quantum physics. When light passes through two slits, an interference pattern forms—evidence of wave behavior.

However, if we reduce the intensity so that only one photon at a time reaches the slits, the pattern still builds up over time. Each photon seems to interfere with **itself**, as if it travels as a wave that explores both slits simultaneously.



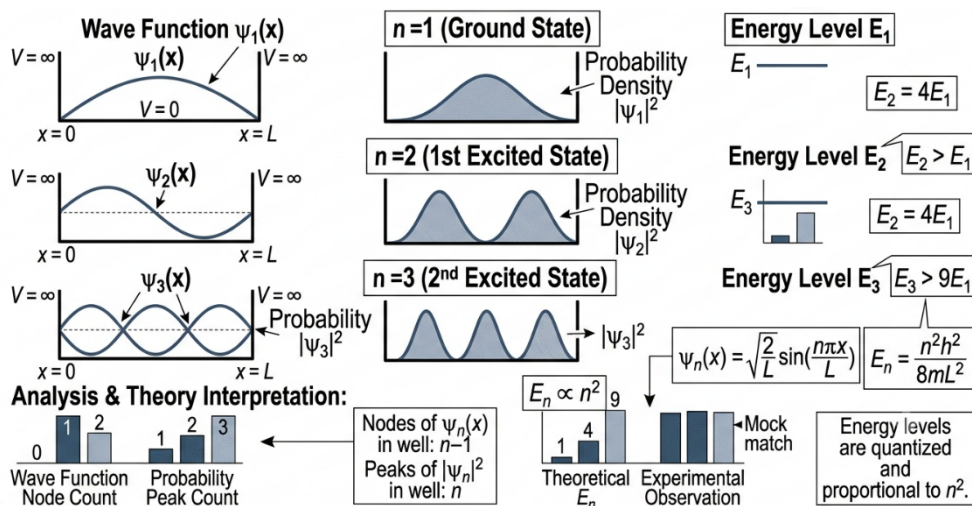
$$U(x) = \begin{cases} 0, & 0 < x < L, \\ \infty, & x \leq 0 \text{ or } x \geq L \end{cases}$$

The electron cannot exist outside the well, so the wave function must be zero at $x = 0$ and $x = L$. The solutions to Schrödinger's equation inside the well are:

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), n = 1, 2, 3, \dots$$

These are standing matter waves. The corresponding energies are:

$$E_n = \frac{h^2}{8mL^2} n^2$$



Key points:

- The quantum number n labels the energy levels
- The ground state ($n = 1$) has non-zero energy—the **zero-point energy**
- Energy levels are spaced farther apart as n increases ($E_n \propto n^2$)
- As L increases, energies decrease; as $L \rightarrow \infty$, the energy spectrum becomes continuous (free particle)

Transitions Between States

An electron in a quantum well can jump from one energy level to another by absorbing or emitting a photon. The photon energy must exactly match the energy difference:

$$hf = \frac{hc}{\lambda} = \Delta E = E_{\text{high}} - E_{\text{low}}$$

For an infinite well, the allowed transition energies are:

$$\Delta E_{mn} = \frac{h^2}{8mL^2} (m^2 - n^2)$$

Each transition produces a spectral line at a specific wavelength. This quantization of emission and absorption is the key to understanding atomic spectra.

m_ℓ	Magnetic quantum number	$-\ell, -\ell + 1, \dots, +\ell$
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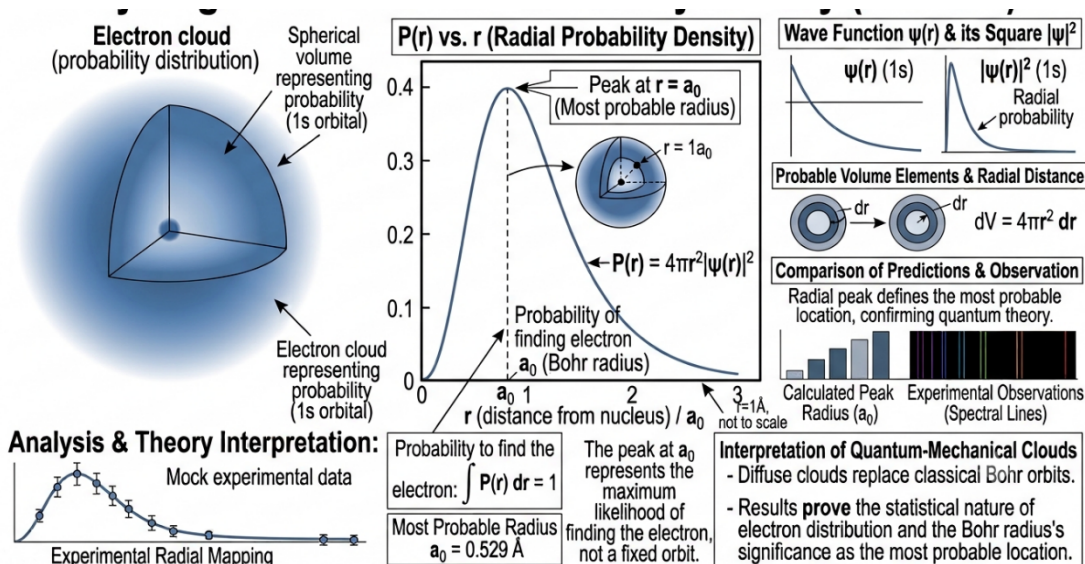
The ground state ($n = 1$) has $\ell = 0$ and $m_\ell = 0$. Its wave function is spherically symmetric:

$$\psi_{100}(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

The **radial probability density** $P(r)$, which gives the probability of finding the electron between r and $r + dr$, is:

$$P(r) = 4\pi r^2 |\psi|^2 = \frac{4}{a_0^3} r^2 e^{-2r/a_0}$$

The maximum probability occurs at $r = a_0$, the Bohr radius. The electron is most likely found at this distance from the nucleus, but there is no sharply defined orbit.



Hydrogen Spectrum

Transitions between energy levels produce spectral lines at wavelengths given by the Rydberg formula:

$$\frac{1}{\lambda} = R \left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2} \right)$$

where the Rydberg constant is:

$$R = \frac{me^4}{8\epsilon_0^2 h^3 c} = 1.097 \times 10^7 \text{ m}^{-1}$$

Spectral Series of Hydrogen:

Series	n_f	n_i	Region
Lyman	1	2, 3, 4, ...	Ultraviolet
Balmer	2	3, 4, 5, ...	Visible
Paschen	3	4, 5, 6, ...	Infrared

Elementary Particles and the Standard Model

Classification of Particles

Over the past century, hundreds of subatomic particles have been discovered. They can be classified according to several criteria:

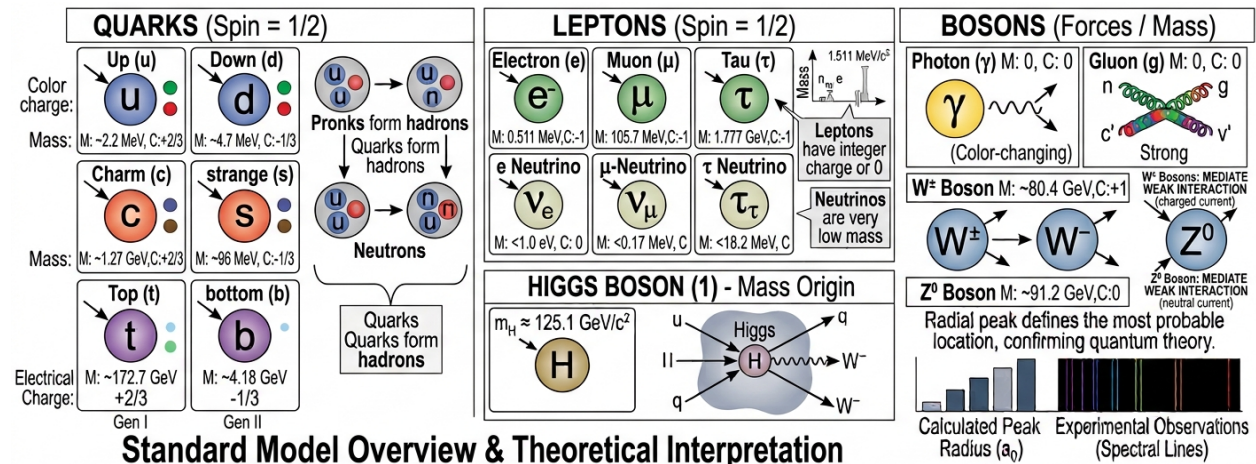
By spin (intrinsic angular momentum):

- **Fermions:** half-integer spin ($s = 1/2, 3/2, \dots$); obey Pauli exclusion principle
- **Bosons:** integer spin ($s = 0, 1, 2, \dots$); do not obey Pauli exclusion principle

By interaction with the strong force:

- **Hadrons:** particles that feel the strong force (protons, neutrons, pions, etc.)
 - **Baryons:** hadrons with half-integer spin (protons, neutrons, lambdas, sigmas, etc.)
 - **Mesons:** hadrons with integer spin (pions, kaons, etc.)
- **Leptons:** particles that do not feel the strong force (electrons, muons, neutrinos, etc.)

Particle vs. Antiparticle: Every particle has a corresponding antiparticle with the same mass and spin but opposite charge and other quantum numbers. When a particle meets its antiparticle, they can **annihilate**, converting their mass energy into photons.



Fundamental Particles (Fermions)

The Standard Model of particle physics classifies all known elementary particles into fermions (matter particles) and bosons (force carriers).

Quarks (6 flavors, each with 3 colors):

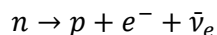
Generation	Flavor	Charge	Mass (MeV/c^2)
1st	Up (u)	$+2/3e$	2.3
1st	Down (d)	$-1/3e$	4.8
2nd	Charm (c)	$+2/3e$	1275
2nd	Strange (s)	$-1/3e$	95
3rd	Top (t)	$+2/3e$	173,000
3rd	Bottom (b)	$-1/3e$	4180

Leptons (6 flavors, no color charge):

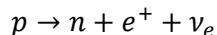
Generation	Flavor	Charge	Mass (MeV/c^2)
1st	Electron (e)	$-1e$	0.511
1st	Electron neutrino (ν_e)	0	$< 1 \times 10^{-6}$
2nd	Muon (μ)	$-1e$	105.7

Beta Decay and Neutrinos

Beta-minus decay (β^-): A neutron converts to a proton, emitting an electron and an electron antineutrino:



Beta-plus decay (β^+): A proton converts to a neutron, emitting a positron and an electron neutrino:



Quark Composition Changes in Beta Decay:

- Beta-minus decay (down $\rightarrow$ up): $d \rightarrow u + e^- + \bar{\nu}_e$
- Beta-plus decay (up $\rightarrow$ down): $u \rightarrow d + e^+ + \nu_e$

The Neutrino: Predicted by Wolfgang Pauli in 1930 to conserve energy and momentum in beta decay. Discovered experimentally in 1956. Neutrinos have extremely small mass (nonzero, as confirmed by neutrino oscillations) and interact only via the weak force.

Cosmology and the Big Bang

Hubble's Law

Observations of distant galaxies reveal that they are all moving away from us. The recessional speed v is proportional to the distance r :

$$v = H_0 r$$

where H_0 is the Hubble constant:

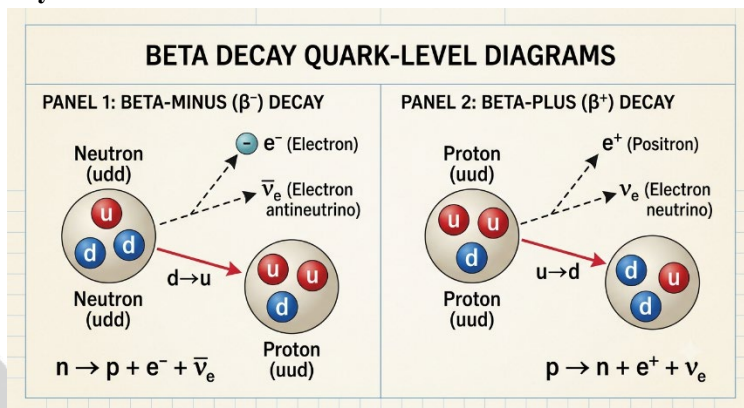
$$H_0 = 71.0 \text{ km/s} \cdot \text{Mpc} \\ = 21.8 \text{ mm/s} \cdot \text{ly}$$

This expansion implies that the universe was once much smaller and denser—the **Big Bang**.

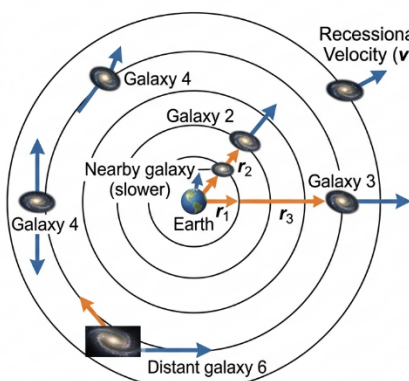
Assuming constant expansion rate, the age of the universe is approximately $1/H_0 \approx 13.8 \times 10^9$ years.

Cosmic Background Radiation

In 1965, Penzias and Wilson discovered a faint microwave background radiation coming uniformly from all directions. This radiation has a blackbody spectrum corresponding to a temperature of 2.7 K. It is the remnant of the hot, dense early universe, released when the universe became transparent to light about 379,000 years after the Big Bang.

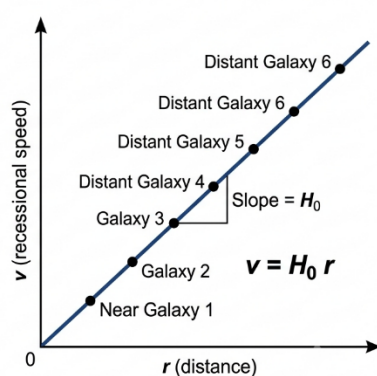


Hubble's Law: The Expanding Universe



Hubble's Law: The Expanding Universe

Hubble's Law: v vs. r Graph

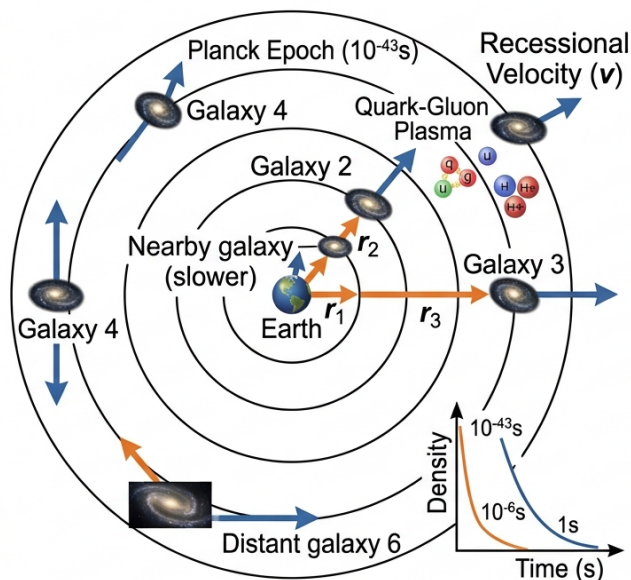


Hubble's Law: v vs. r Graph

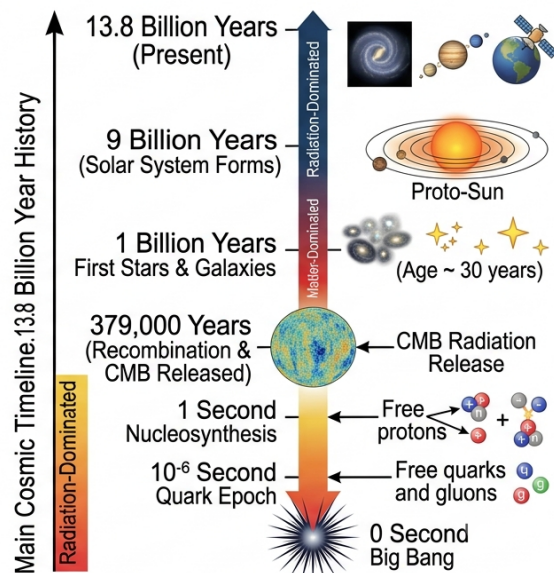
Dark Matter and Dark Energy

Observations of galactic rotation curves show that visible matter accounts for only about 4% of the universe's energy density. An additional 23% is **dark matter**—nonluminous matter that interacts gravitationally but not electromagnetically. The remaining 73% is **dark energy**, a mysterious form of energy causing the expansion of the universe to accelerate.

A. Big Bang & Early Evolution (First Second)



B. Temperature & Density Evolution



Main Cosmic Timeline: 13.8 Billion Year History

Composition of the Universe:

- Dark Energy: 73%
- Dark Matter: 23%
- Baryonic Matter (ordinary matter): 4%

Cosmology

Cosmology is the study of the universe as a whole—its origin, evolution, structure, and ultimate fate.

The Expanding Universe

Hubble's Law

$$v = H_0 r$$

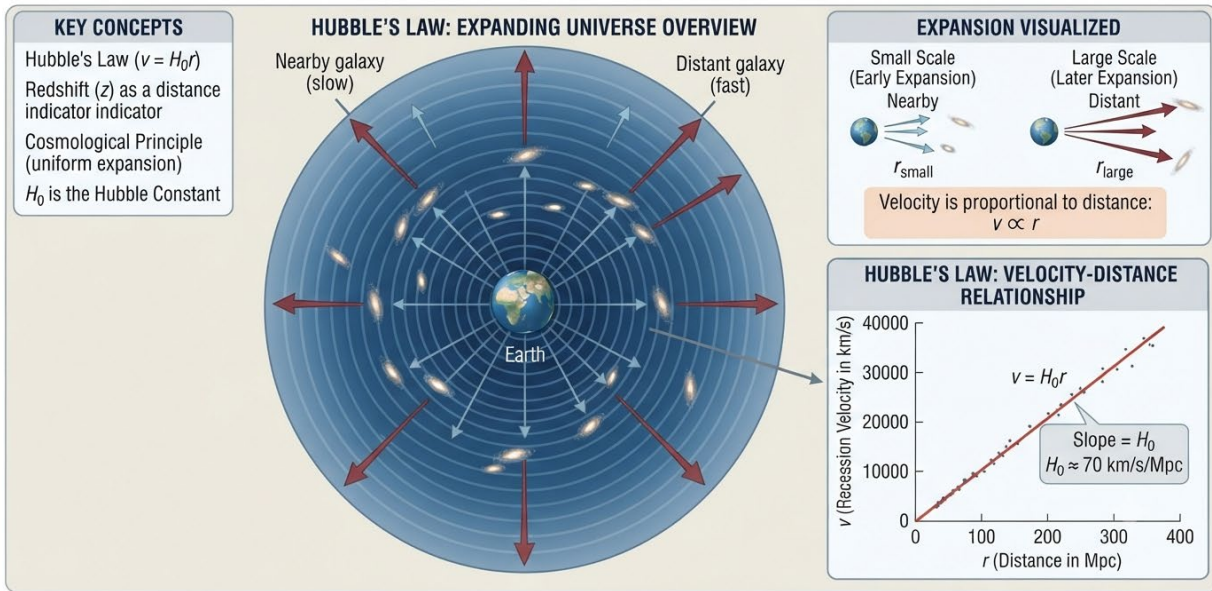
Where:

- v = recessional velocity of a galaxy
- r = distance to the galaxy
- H_0 = Hubble constant (≈ 70 km/s/Mpc)

Hubble Constant

Measurement Method	Value (km/s/Mpc)	Uncertainty
Planck (CMB)	67.4 ± 0.5	Very precise
SH0ES (Cepheids)	73.0 ± 1.0	Discrepancy (Hubble tension)

HUBBLE'S LAW: EXPANDING UNIVERSE OVERVIEW



The Hubble Tension

The Hubble tension is the discrepancy between the value of H_0 measured from the early universe (CMB) and from the late universe (local distance scale).

The Big Bang Theory

Timeline of the Universe

Time After Big Bang	Event	Temperature
10^{-43} s	Planck epoch (quantum gravity)	10^{32} K
10^{-36} s	Inflation begins	10^{29} K
10^{-32} s	Inflation ends	10^{28} K
10^{-12} s	Electroweak unification broken	10^{15} K
10^{-6} s	Quarks form hadrons (QCD transition)	10^{13} K
1 s	Neutrinos decouple	10^{10} K
3 min	Big Bang nucleosynthesis (H, He)	10^9 K
380,000 yr	Recombination (CMB released)	3000 K
200 Myr	First stars form	100 K
1 Gyr	First galaxies form	50 K
13.8 Gyr	Present day	2.7 K

Cosmic Microwave Background (CMB)

- Remnant of the hot early universe
- Released at recombination (380,000 years after Big Bang)
- Current temperature: 2.725 K
- Blackbody spectrum: perfect Planck distribution

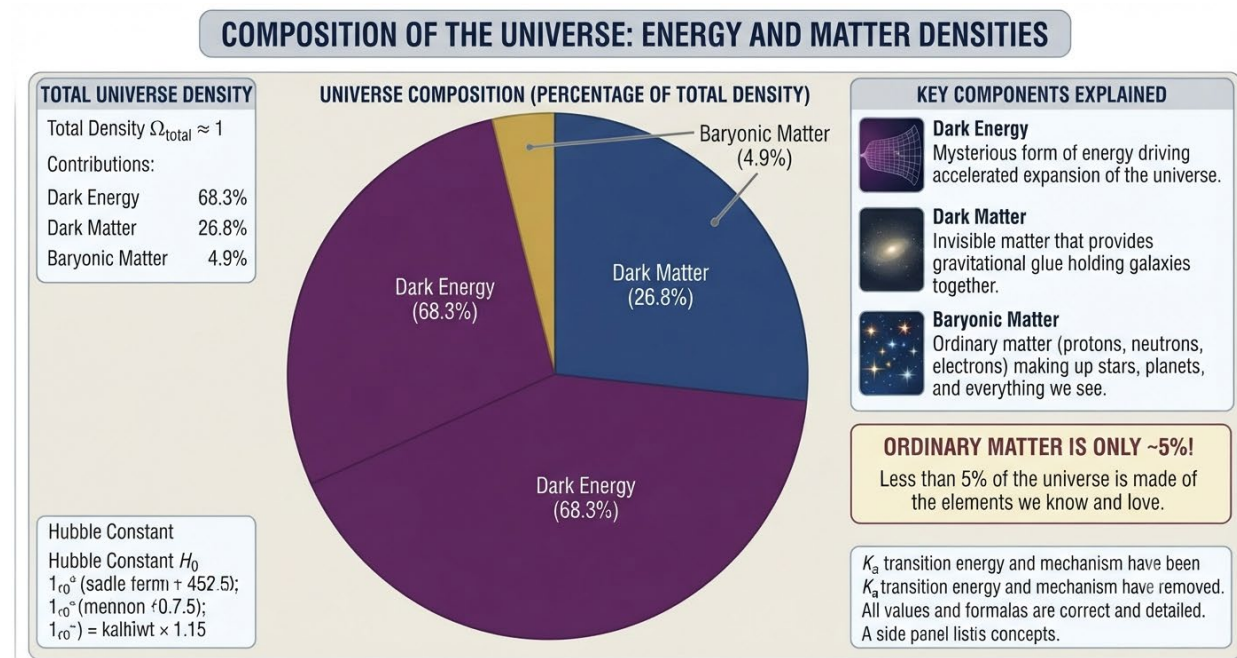
Dark Energy

Dark energy is the mysterious force causing the expansion of the universe to accelerate.

Evidence for Dark Energy

Observation	Evidence
Type Ia supernovae	Distant supernovae appear too dim → accelerated expansion
CMB	Flatness + matter density → missing energy
Baryon acoustic oscillations	Large-scale structure measurements

Composition of the Universe



Candidates for Dark Energy

Candidate	Description	Status
Cosmological constant	Λ (Einstein's "biggest blunder")	Best fit to data
Quintessence	Dynamical scalar field	Alternatives explored
Modified gravity	$f(R)$ gravity, MOND	Some models fit data

The Fate of the Universe

The ultimate fate depends on the total density parameter Ω .

Scenario	Ω	Fate
Big Crunch	$\Omega > 1$	Expansion stops → contraction → big crunch
Big Freeze	$\Omega < 1$ (with dark energy)	Expansion accelerates → heat death
Big Rip	Dark energy grows	Universe tears itself apart
Big Bounce	Cyclic universe	Universe undergoes cycles of expansion/contraction

Current Best Fit: Big Freeze (Heat Death)

- Expansion accelerates due to dark energy
- Galaxies move beyond cosmic horizon
- Stars burn out

- Universe becomes cold, dark, and dilute

The Cosmological Principle

The **cosmological principle** states that the universe is homogeneous (uniform) and isotropic (same in all directions) on large scales.

Implications

Principle	Implication
Homogeneity	No preferred location in the universe
Isotropy	No preferred direction in the universe
Cosmological Principle	The universe looks the same from any vantage point

The FLRW Metric

The Friedmann-Lemaître-Robertson-Walker (FLRW) metric describes an expanding universe:

$$ds^2 = -c^2 dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$$

where:

- $a(t)$ = scale factor (size of the universe)
- $k = +1, 0, -1$ = curvature (closed, flat, open)

The Friedmann Equations

The Friedmann equations describe the evolution of the universe.

First Friedmann Equation (Hubble Parameter)

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2}$$

Second Friedmann Equation (Acceleration)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3P}{c^2} \right)$$

Friedmann Equation with Cosmological Constant

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

where Λ is the cosmological constant.

Key Formulas to Memorize

Concept	Formula
Lorentz factor	$\gamma = 1/\sqrt{1 - v^2/c^2}$
Time dilation	$\Delta t = \gamma \Delta t_0$
Length contraction	$L = L_0/\gamma$
Lorentz transformation	$x' = \gamma(x - vt), t' = \gamma(t - vx/c^2)$
Velocity addition	$u = (u' + v)/(1 + u'v/c^2)$
Relativistic momentum	$p = \gamma mv$
Total energy	$E = \gamma mc^2$
Kinetic energy	$K = (\gamma - 1)mc^2$
Energy-momentum relation	$E^2 = (pc)^2 + (mc^2)^2$
Photon energy	$E = hf = hc/\lambda$
Photoelectric effect	$hf = K_{\max} + \Phi$
Compton shift	$\Delta\lambda = (h/mc)(1 - \cos\phi)$
de Broglie wavelength	$\lambda = h/p$

Fundamental Forces	Mediated by messenger particles: photon (EM), W/Z (weak), gluons (strong).
Big Bang	The universe began in a hot, dense state and has been expanding ever since.
Dark Matter/Energy	Most of the universe is not ordinary matter; dark energy causes accelerated expansion.

Modern Physics: One-Liners

Special Theory of Relativity

1. An inertial frame of reference is one in which Newton's first law holds; it is either at rest or moving with constant velocity.
2. Any frame moving with constant velocity relative to an inertial frame is itself inertial.
3. A non-inertial frame is one that accelerates relative to an inertial frame.
4. In non-inertial frames, fictitious forces appear (e.g., centrifugal force).
5. The First Postulate of Special Relativity: The laws of physics are the same for all observers in any inertial frame of reference.
6. The Second Postulate of Special Relativity: The speed of light in vacuum has the same value c in all inertial frames, regardless of the motion of the source or observer.
7. The speed of light is $c = 299,792,458 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s}$.
8. An event is anything that happens at a specific location and time, described by four spacetime coordinates (x, y, z, t) .
9. Simultaneity is relative; two events simultaneous in one inertial frame are generally not simultaneous in another.
10. The Lorentz factor is $\gamma = 1/\sqrt{1 - v^2/c^2} = 1/\sqrt{1 - \beta^2}$, where $\beta = v/c$.
11. $\gamma \geq 1$, with equality only when $v = 0$; as $v \rightarrow c$, $\gamma \rightarrow \infty$.
12. Time dilation: $\Delta t = \gamma \Delta t_0$, where Δt_0 is the proper time (measured in the frame where events occur at the same location).
13. Proper time is the smallest possible measured time between two events.
14. Length contraction: $L = L_0/\gamma$, where L_0 is the proper length (measured in the frame where the object is at rest).
15. Length contraction occurs only along the direction of relative motion; perpendicular lengths are unchanged.
16. Proper length is the maximum possible measured length.
17. The Lorentz transformation: $x' = \gamma(x - vt)$, $t' = \gamma(t - vx/c^2)$, with $y' = y$, $z' = z$.
18. The inverse Lorentz transformation: $x = \gamma(x' + vt')$, $t = \gamma(t' + vx'/c^2)$.
19. Relativistic velocity addition: $u = (u' + v)/(1 + u'v/c^2)$.
20. If $u' = c$, then $u = c$ regardless of v , consistent with the speed of light postulate.
21. Relativistic momentum: $\vec{p} = \gamma m \vec{v}$.
22. As $v \rightarrow c$, $p \rightarrow \infty$, explaining why massive particles cannot reach the speed of light.
23. Rest energy: $E_0 = mc^2$.
24. Total energy: $E = \gamma mc^2 = E_0 + K$.
25. Kinetic energy: $K = (\gamma - 1)mc^2$.

121. The cosmic microwave background (CMB) is the remnant of the hot early universe, with temperature 2.725 K.
122. The CMB was discovered by Penzias and Wilson in 1965.
123. The CMB is a perfect blackbody spectrum.
124. Dark matter (~23% of the universe) interacts gravitationally but not electromagnetically.
125. Dark energy (~73% of the universe) causes accelerated expansion.
126. Ordinary (baryonic) matter makes up only ~4% of the universe.
127. Galactic rotation curves provide evidence for dark matter.
128. Type Ia supernovae provide evidence for dark energy (accelerated expansion).
129. The cosmological principle: the universe is homogeneous and isotropic on large scales.
130. The FLRW metric describes an expanding universe: $ds^2 = -c^2 dt^2 + a(t)^2(dr^2/(1 - kr^2) + r^2 d\Omega^2)$.
131. The scale factor $a(t)$ describes the size of the universe.
132. The Friedmann equations describe the evolution of the universe.
133. The cosmological constant Λ drives accelerated expansion.
134. The fate of the universe: Big Freeze (heat death) is the current best fit.
135. Inflation (exponential expansion in the first 10^{-32} s) solves the horizon, flatness, and monopole problems.

Practice MCQs

1. An inertial frame of reference is one in which:

- A) Newton's first law does not hold
- B) Newton's first law holds
- C) Objects experience fictitious forces
- D) The frame is accelerating

Correct Answer: B

2. A non-inertial frame is one that:

- A) Moves with constant velocity
- B) Accelerates relative to an inertial frame
- C) Is at rest
- D) Has no forces

Correct Answer: B

3. The First Postulate of Special Relativity states that:

- A) The speed of light is constant in all frames
- B) The laws of physics are the same in all inertial frames
- C) Time is absolute
- D) Length is absolute

Correct Answer: B

4. The Second Postulate of Special Relativity states that:

- A) The laws of physics are the same in all inertial frames
- B) The speed of light is constant in all inertial frames
- C) Time dilation occurs
- D) Length contraction occurs

Correct Answer: B

5. The speed of light in vacuum is approximately:

- A) 3.00×10^6 m/s
- B) 3.00×10^8 m/s
- C) 3.00×10^{10} m/s
- D) 3.00×10^4 m/s

Correct Answer: B

6. The Lorentz factor γ is:

- A) $\gamma = \sqrt{1 - v^2/c^2}$
- B) $\gamma = 1/\sqrt{1 - v^2/c^2}$
- C) $\gamma = 1 - v^2/c^2$
- D) $\gamma = 1 + v^2/c^2$

Correct Answer: B

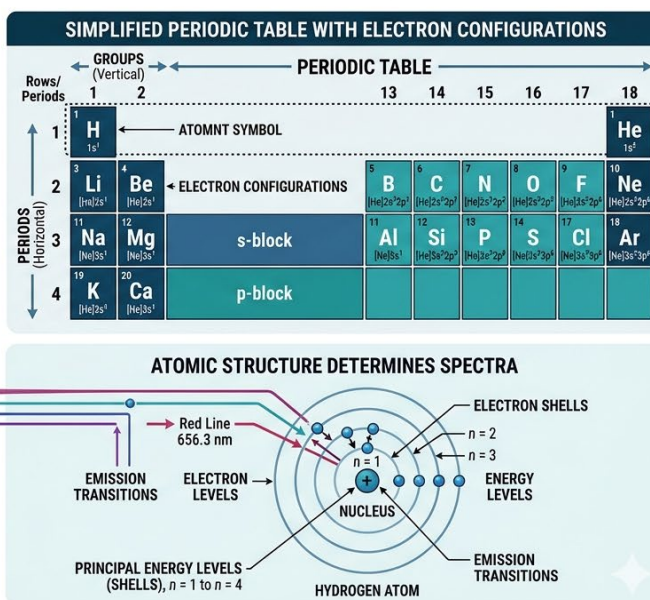
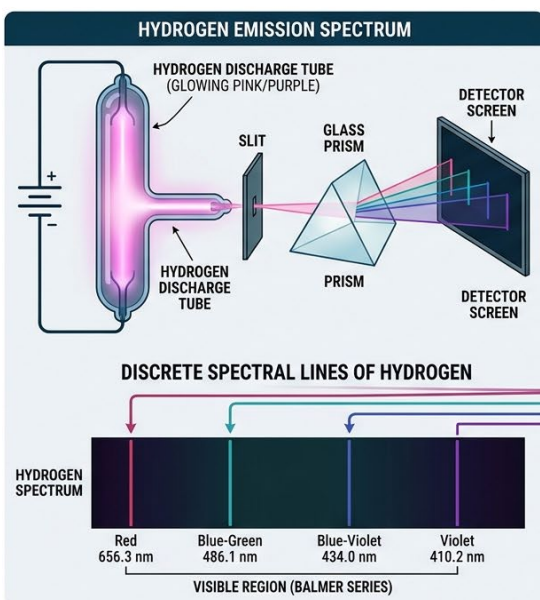
7. As $v \rightarrow c$, the Lorentz factor γ tends to:

- A) 1

Chapter 17: Atomic Spectra

Introduction

The investigation of atomic structure represents one of the most profound intellectual achievements in the history of physics. At the turn of the 20th century, physicists grappled with a fundamental question: If atoms are the basic building blocks of matter, how are they constructed, and why do they emit and absorb light only at specific wavelengths? The answers to these questions would eventually require the abandonment of classical physics and the development of quantum mechanics.



FUNDAMENTAL QUANTUM PROPERTIES OF ATOMS

Stability and Systematic Organization of Atoms

Atoms are remarkably stable. Essentially all the atoms that form our tangible world have existed without change for billions of years. This stability, along with the ability of atoms to combine into molecules and stack into rigid solids, finds its explanation only in quantum physics.

The periodic table of the elements displays a systematic organization that reflects the underlying quantum structure of atoms. Figure 1 shows a plot of ionization energies—the energy required to remove the most loosely bound electron from a neutral atom—as a function of atomic number Z . The periodic repetition of properties through the six complete horizontal periods demonstrates that atoms are constructed according to systematic rules. The numbers of elements in these periods—2, 8, 8, 18, 18, and 32—are precisely predicted by quantum physics.

Quantum Numbers of the Atomic Electron

Every electron in an atom is characterized by a set of quantum numbers that determine its allowed states. Table 1 summarizes these quantum numbers, their allowed values, and their physical significance.

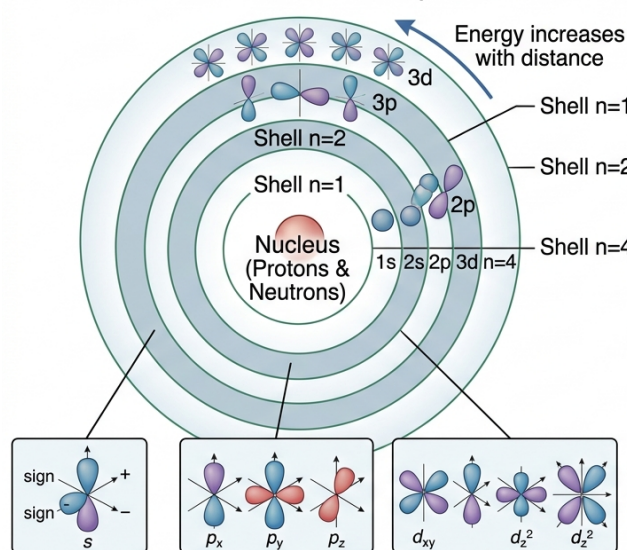
Electron Quantum Numbers

Quantum Number	Symbol	Allowed Values	Physical Significance
Principal	n	1, 2, 3, ...	Distance from nucleus; energy level (in hydrogen)
Orbital (azimuthal)	ℓ	0, 1, 2, ..., $(n - 1)$	Orbital angular momentum magnitude
Orbital magnetic	m_ℓ	0, ± 1 , ± 2 , ..., $\pm \ell$	z-component of orbital angular momentum
Spin	s	$1/2$ (always for electron)	Intrinsic spin angular momentum magnitude
Spin magnetic	m_s	$\pm 1/2$	z-component of spin angular momentum

Important Note: For a free electron (not bound in an atom), only the spin quantum numbers s and m_s are relevant. For an electron trapped in an atom, all quantum numbers n , ℓ , m_ℓ , s , and m_s apply.

Atomic Shell Structure Diagram

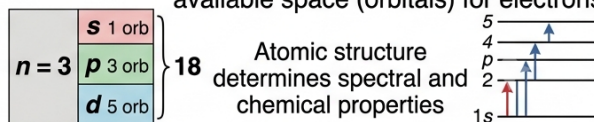
Atomic Shell and Subshell Spatial Visual



Electron Subshell Configuration and Quantum Number Table

Shell (n)	Subshell (ℓ)	Orbitals ($2\ell+1$)	Max electrons $2(2\ell+1)$	Max ev. e^-
1	s ($\ell = 0$)	$(2 \cdot 0 + 1) = 1$	$2(1) = 2$	2e ⁻
2	s ($\ell = 0$)	1	2	8e ⁻
	p ($\ell = 1$)	$(2 \cdot 1 + 1) = 3$	$2(3) = 6$	
3	s ($\ell = 0$)	1	2	18e ⁻
	p ($\ell = 1$)	3	6	
	d ($\ell = 2$)	$(2 \cdot 2 + 1) = 5$	$2(5) = 10$	
Total electrons in shell $n = \sum_{\ell=0}^{n-1} 2(2\ell+1) = 2n^2$				2e ⁻

Quantum numbers (n , ℓ) dictate the available space (orbitals) for electrons



Definitions of Shells and Subshells:

- Shell:** All states with the same principal quantum number n
- Subshell:** All states with the same values of n and ℓ

For a given ℓ , there are $(2\ell+1)$ possible values of m_ℓ , and for each m_ℓ there are two possible values of m_s (spin up and spin down). Therefore, a subshell contains:

$$\text{Number of states in subshell} = 2(2\ell + 1)$$

The total number of states in a shell with quantum number n is:

$$\text{Number of states in shell} = 2n^2$$

Spectroscopic Notation for Subshells

For convenience, the values of ℓ are represented by letters according to the following scheme:

$$\ell = 012345 \dots$$

$$\text{letter} = s p d f g h \dots$$

$$\mu_B = \frac{e\hbar}{2m} = 9.274 \times 10^{-24} \text{ J/T} = 5.788 \times 10^{-5} \text{ eV/T}$$

Spin Angular Momentum

Every electron—whether in an atom or free—possesses an intrinsic angular momentum called **spin**. The name is misleading because the electron is not actually spinning; there is nothing rotating, yet the electron has angular momentum. This is a purely quantum mechanical property with no classical counterpart.

The magnitude of spin angular momentum is:

$$S = \sqrt{s(s+1)} \hbar, \text{ for } s = \frac{1}{2}$$

The electron is said to be a **spin-1/2 particle** (protons and neutrons are also spin-1/2 particles).

The component along the z-axis is:

$$S_z = m_s \hbar, \text{ for } m_s = \pm \frac{1}{2}$$

The two states are conventionally called **spin up** ($m_s = +1/2$) and **spin down** ($m_s = -1/2$).

Spin Magnetic Dipole Moment

Associated with the spin angular momentum is a spin magnetic dipole moment:

$$\vec{\mu}_s = -\frac{e}{m} \vec{S}$$

The magnitude is:

$$\mu_s = \frac{e}{m} \sqrt{s(s+1)} \hbar$$

and the z-component is:

$$\mu_{s,z} = -2m_s \mu_B = \mp \mu_B \text{ for } m_s = \pm \frac{1}{2}$$

Note the factor of 2: The spin magnetic moment is twice what one would expect from the orbital formula. This factor, called the **g-factor** (gyromagnetic ratio), is approximately 2 for the electron and arises from relativistic quantum mechanics (the Dirac equation).

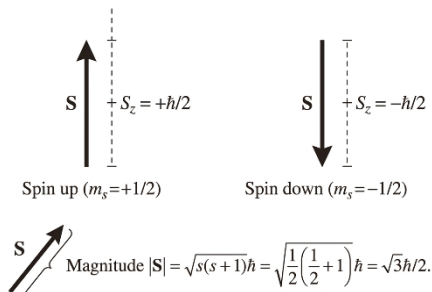
Total Angular Momentum of Multielectron Atoms

For an atom containing more than one electron, we define a total angular momentum $\vec{J}$ as the vector sum of the angular momenta of all individual electrons—both orbital and spin:

$$\vec{J} = (\vec{L}_1 + \vec{L}_2 + \cdots + \vec{L}_Z) + (\vec{S}_1 + \vec{S}_2 + \cdots + \vec{S}_Z)$$

Similarly, the total magnetic dipole moment is the vector sum of the individual magnetic moments. However, because of the factor of 2 in the spin magnetic moment, the resultant magnetic dipole moment does not point in the direction of $-\vec{J}$. Instead, we define an **effective magnetic dipole moment** $\vec{\mu}_{\text{eff}}$ as the component of the vector sum in the direction of $-\vec{J}$.

In typical atoms, the orbital and spin angular momenta of most electrons cancel vectorially to zero. Then $\vec{J}$ and $\vec{\mu}_{\text{eff}}$ are due to a relatively small number of electrons—often only a single valence electron.



THE STERN-GERLACH EXPERIMENT

Experimental Setup and Results

In 1922, Otto Stern and Walther Gerlach performed a groundbreaking experiment that provided direct experimental proof that atomic magnetic moments are quantized. Their apparatus (Figure 3) consisted of:

1. An oven that vaporized silver atoms
2. A collimator (narrow slit) to produce a narrow beam of atoms
3. An electromagnet with specially shaped poles to produce a nonuniform magnetic field with a large vertical gradient dB/dz
4. A glass detector plate to collect the silver deposit

Classical prediction: The magnetic dipole moment components μ_z should range continuously from $-\mu$ to $+\mu$. Thus, the force $F_z = \mu_z(dB/dz)$ should vary continuously, producing a continuous vertical spread of the silver deposit on the plate.

Experimental result: The atoms formed **two distinct spots** on the glass plate—one above and one below the undeflected position. This demonstrated that μ_z can have only two values, not a continuous range.

Explanation of the Two-Spot Result

We now understand that a silver atom consists of many electrons, but all orbital and spin magnetic moments cancel except for a single electron. Moreover, the orbital dipole moment of that electron is zero ($\ell = 0$). Therefore, the combined dipole moment of a silver atom is simply the spin magnetic dipole moment of that single electron.

From the spin magnetic moment equation:

$$\mu_{s,z} = -2m_s\mu_B$$

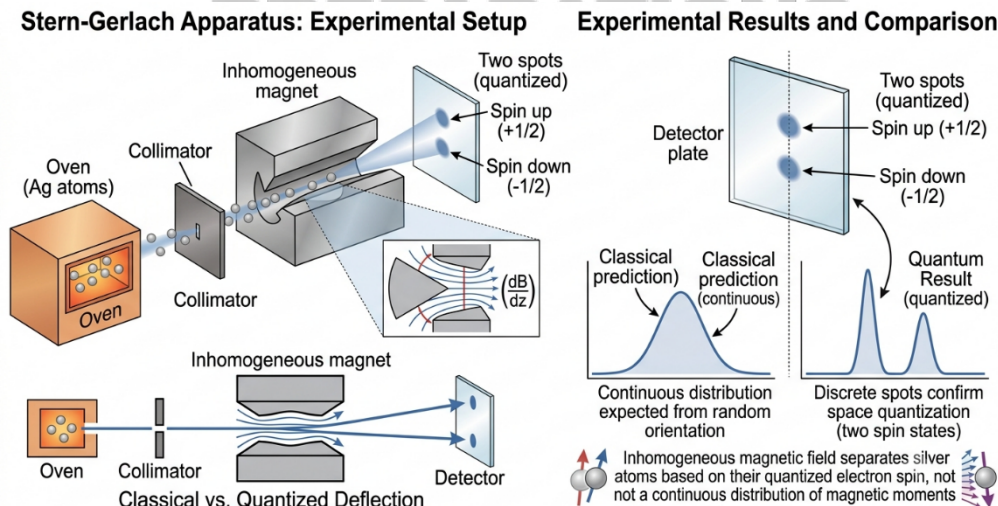
with $m_s = \pm 1/2$, we obtain:

$$\mu_{s,z} = -\mu_B \text{ (for } m_s = +1/2, \text{ spin up) and } \mu_{s,z} = +\mu_B \text{ (for } m_s = -1/2, \text{ spin down)}$$

The force on the atoms is therefore:

$$F_z = \mu_z \frac{dB}{dz} = \pm \mu_B \frac{dB}{dz}$$

producing exactly two deflections and hence two spots.



Stern-Gerlach Experiment: Discovery of Space Quantization and Electron Spin

Historical significance: Although no one knew about electron spin at the time, the Stern-Gerlach results were actually the first experimental evidence for electron spin.

Mathematical Analysis of Beam Deflection

Sample Problem: In a Stern-Gerlach experiment, the magnetic field gradient is $dB/dz = 1.4 \text{ T/mm} = 1.4 \times 10^3 \text{ T/m}$. The magnet region has length $w = 3.5 \text{ cm} = 0.035 \text{ m}$ in the direction of the beam. The silver atoms have speed $v = 750 \text{ m/s}$ and mass $M = 1.8 \times 10^{-25} \text{ kg}$. Find the deflection distance.

Solution: The force on an atom is $F_z = \mu_B (dB/dz)$ (taking the positive deflection case). The acceleration is:

$$a_z = \frac{F_z}{M} = \frac{\mu_B (dB/dz)}{M}$$

The time to traverse the magnet region is $t = w/v$. Since the initial vertical velocity is zero, the deflection is:

$$d = \frac{1}{2} a_z t^2 = \frac{1}{2} \left(\frac{\mu_B (dB/dz)}{M} \right) \left(\frac{w}{v} \right)^2 = \frac{\mu_B (dB/dz) w^2}{2 M v^2}$$

Substituting values:

$$d = \frac{(10^{-24} \text{ J/T})(1.4 \times 10^3 \text{ T/m})(0.035 \text{ m})^2}{2(1.8 \times 10^{-25} \text{ kg})(750 \text{ m/s})^2} = 7.85 \times 10^{-5} \text{ m} \approx 0.08 \text{ mm}$$

The separation between the two subbeams is $2d = 0.16 \text{ mm}$.

THE PAULI EXCLUSION PRINCIPLE

Statement of the Principle

The Pauli exclusion principle, formulated by Wolfgang Pauli in 1925, governs any particle whose spin quantum number s is half-integer (i.e., fermions). For electrons, it states:

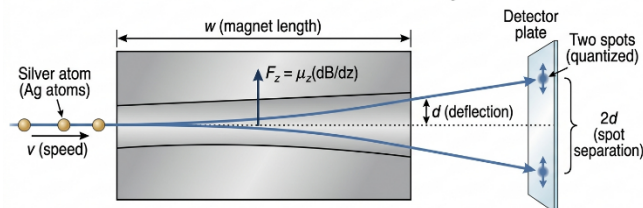
No two electrons in a trap (such as an atom) can have the same set of quantum numbers n, ℓ, m_ℓ, m_s .

Since all electrons have $s = 1/2$, this means that any two electrons in an atom must differ in at least one of the quantum numbers n, ℓ, m_ℓ , or m_s . Were this not true, atoms would collapse, and chemistry as we know it would not exist.

Application to Multiple Electrons in Traps

To understand how the exclusion principle determines electron configurations, consider progressively more confining traps:

Stern-Gerlach Beam Deflection Geometry and Calculation



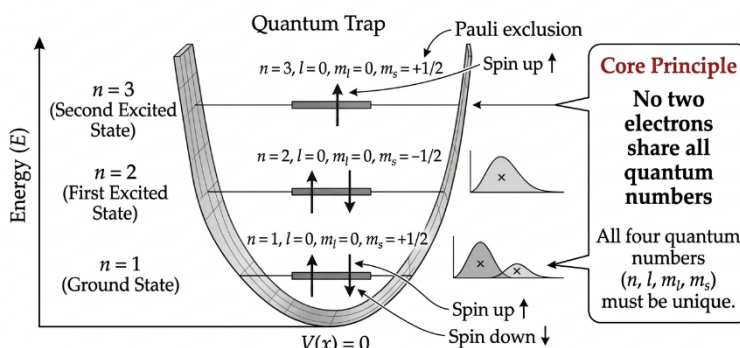
Analytical Breakdown:

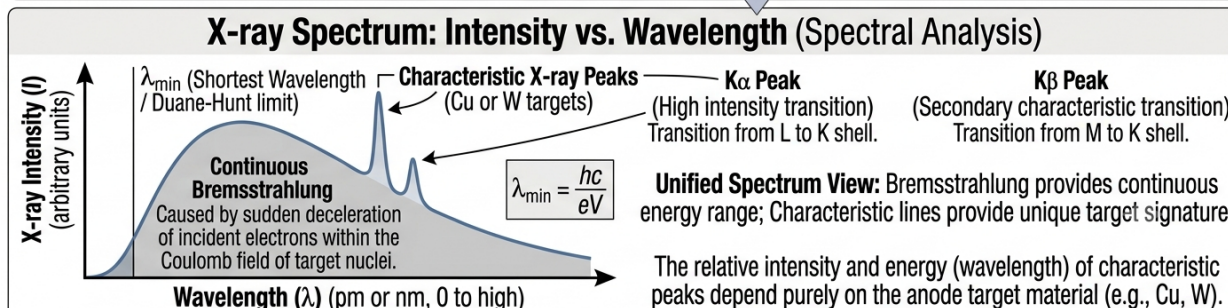
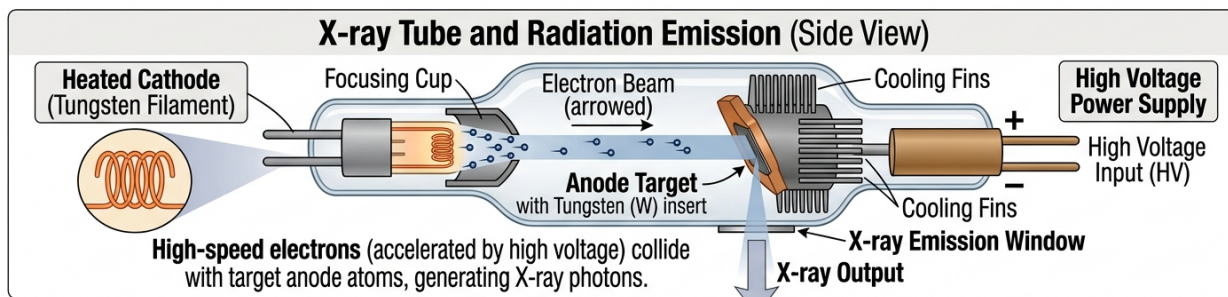
μ_B = silver atom beam
 dB/dz = deflect. length (length)
 M = magnetic moment
 v = (speed), v daton

$$d = \frac{\mu_B \frac{dB}{dz} \cdot w^2}{2 \cdot M \cdot v^2}$$

Deflection is proportional to magnet (w^2) and magnetic moment (μ_B), and inversely proportional to atom mass (M) and speed squared (v^2)

Pauli exclusion principle





The **cutoff wavelength** $\lambda_{\min}$ is a prominent feature—no X-rays are emitted with wavelength shorter than this value. It corresponds to a collision in which an incident electron loses **all** its initial kinetic energy K_0 in a single head-on collision. Conservation of energy gives:

$$K_0 = hf = \frac{hc}{\lambda_{\min}}$$

Thus:

$$\lambda_{\min} = \frac{hc}{K_0}$$

The cutoff wavelength is independent of the target material; it depends only on the accelerating voltage V (since $K_0 = eV$):

$$\lambda_{\min}(\text{in pm}) = \frac{1240}{V(\text{in kV})}$$

The Characteristic X-Ray Spectrum

Characteristic X-rays arise from a two-step process:

1. **Ejection of an inner-shell electron:** An energetic incident electron knocks out a deep-lying electron (e.g., from the K shell, $n = 1$), creating a vacancy or "hole."
2. **Filling the hole:** An electron from a higher-energy shell (e.g., L shell, $n = 2$ or M shell, $n = 3$) jumps down to fill the hole, emitting a photon whose energy equals the difference between the two energy levels.

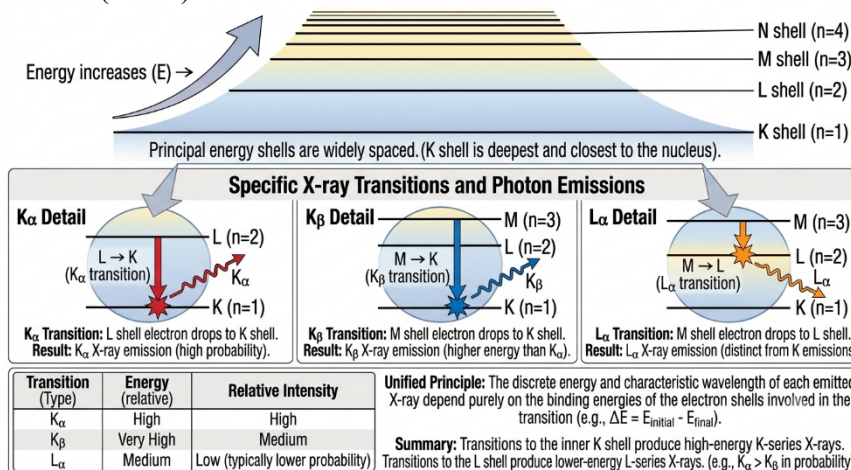
Nomenclature:

- **K series:** transitions to the *K* shell ($n = 1$)

- K_α : transition from *L* shell ($n = 2$) to *K* shell
- K_β : transition from *M* shell ($n = 3$) to *K* shell

- **L series:** transitions to the *L* shell ($n = 2$)

- L_α : transition from *M* shell to *L* shell
- etc.

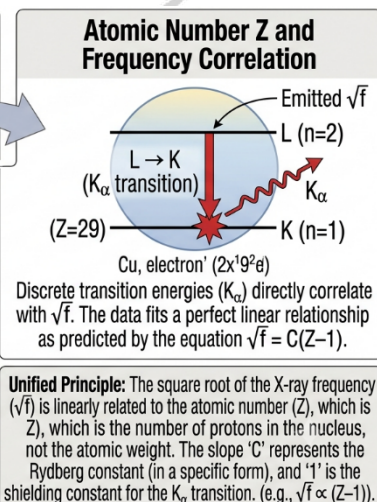
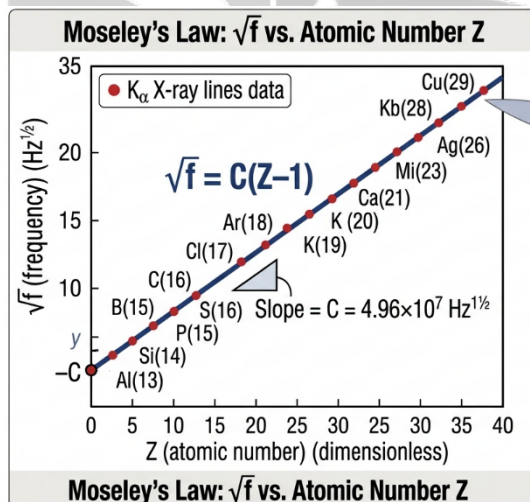


Moseley's Law and the Ordering of the Elements

In 1913, Henry Moseley systematically studied characteristic X-rays from 38 elements. He discovered that plotting the square root of the frequency $\sqrt{f}$ of the K_α line against the atomic number Z yielded a straight line.

Theoretical

derivation: For the innermost electrons in a multielectron atom, an electron in the *K* shell "sees" an effective nuclear charge of approximately $(Z - 1)e$ because the other *K* shell electron screens one unit of nuclear charge. Using the Bohr energy formula:



$$E_n = -\frac{(13.60 \text{ eV})(Z-1)^2}{n^2}$$

For the K_α transition ($n = 2 \rightarrow n = 1$):

$$\Delta E = E_2 - E_1 = -\frac{(13.60 \text{ eV})(Z-1)^2}{4} - \left(-\frac{(13.60 \text{ eV})(Z-1)^2}{1} \right) = (10.2 \text{ eV})(Z-1)^2$$

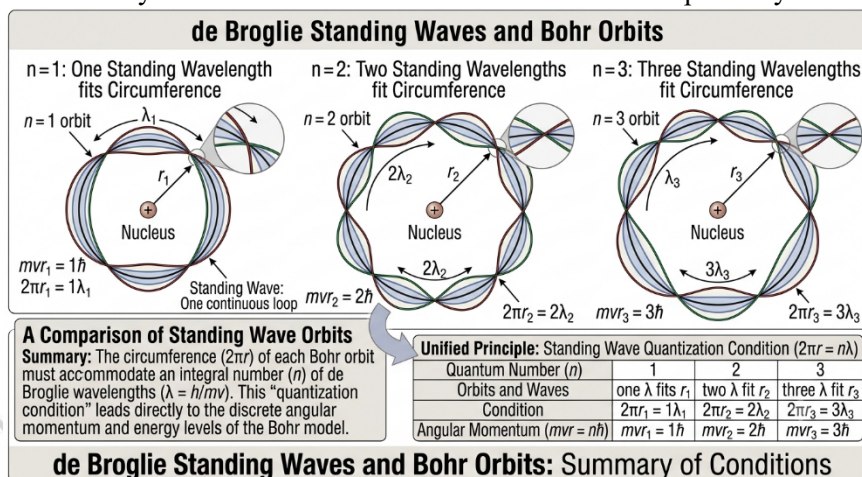
The frequency is:

$$f = \frac{\Delta E}{h} = (2.46 \times 10^{15} \text{ Hz})(Z-1)^2$$

Using de Broglie's relation $\lambda = h/mv$:

$$2\pi r = n \frac{h}{mv} \Rightarrow mvr = n \frac{h}{2\pi}$$

This is precisely Bohr's quantization condition. Thus, Bohr orbits correspond to standing de Broglie waves—an electron can only exist in orbits where its wave function "fits" perfectly around the nucleus.



MOMENTUM, IMPULSE, AND COLLISIONS IN ATOMIC PHYSICS

Momentum and Its Conservation

The **linear momentum** of a particle is a vector quantity:

$$\vec{p} = m\vec{v}$$

Newton's second law in terms of momentum states:

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt}$$

The **impulse** of a force is:

$$\vec{J} = \int_{t_1}^{t_2} \Sigma \vec{F} dt = \vec{p}_2 - \vec{p}_1$$

This is the **impulse-momentum theorem**: the change in momentum equals the impulse applied.

Conservation of Momentum

For a system of particles, the total momentum is:

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \dots = m_1\vec{v}_1 + m_2\vec{v}_2 + \dots$$

Law of conservation of momentum: If the vector sum of the external forces on a system is zero, the total momentum of the system is constant:

$$\Sigma \vec{F}_{\text{ext}} = 0 \Rightarrow \vec{P} = \text{constant}$$

Elastic and Inelastic Collisions

In atomic and nuclear physics, collisions between particles are fundamental to understanding processes such as neutron moderation in nuclear reactors.

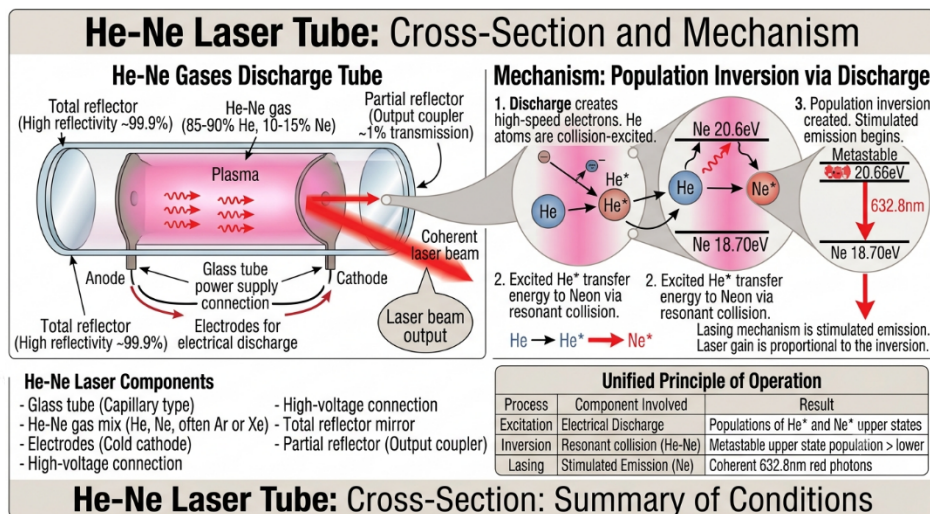
Collision Type	Momentum Conserved?	Kinetic Energy Conserved?
Elastic	Yes	Yes
Inelastic	Yes	No
Completely inelastic	Yes	No (maximum loss)

For a **completely inelastic collision** where objects stick together:

6. One mirror is partially transparent, allowing the laser beam to escape.
The photon energy for the lasing transition is:

$$\Delta E = 20.66 \text{ eV} - 18.70 \text{ eV} = 1.96 \text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{1.96 \text{ eV}} = 633 \text{ nm}$$



Properties of Laser Light

Property	Laser Light	Ordinary Light
Monochromaticity	Extremely high ($\Delta\lambda/\lambda$ up to 10^{-15})	Low (broad spectrum)
Coherence	Very high (coherence length km)	Low (coherence length < m)
Directionality	Very low divergence ($\sim 0.001^\circ$)	High divergence
Intensity	Can be extremely high (10^{17} W/cm^2)	Low (10^3 W/cm^2 for oxyacetylene)

CONCEPTUAL SUMMARY

Concept	Key Equation	Physical Meaning
Orbital angular momentum	$L = \sqrt{\ell(\ell + 1)}\hbar$	Quantized magnitude; $\ell = 0, 1, 2, \dots, (n - 1)$
Orbital magnetic moment	$\mu_{\text{orb}} = \frac{e}{2m}L$	Opposite direction to $\vec{L}$ for electron
Spin angular momentum	$S = \sqrt{s(s + 1)}\hbar, s = 1/2$	Intrinsic; no classical analog
Spin magnetic moment	$\mu_s = \frac{e}{m}S$	Twice the orbital value ($g \approx 2$)
Bohr magneton	$\mu_B = e\hbar/2m = 9.27 \times 10^{-24} \text{ J/T}$	Fundamental unit of magnetic moment
Pauli exclusion	No two electrons share same n, ℓ, m_ℓ, m_s	Determines electron configurations
Photon energy	$hf = E_{\text{high}} - E_{\text{low}}$	Energy conservation in atomic transitions
Rydberg formula	$1/\lambda = R_H(1/p^2 - 1/n^2)$	Hydrogen spectral lines
Cutoff wavelength	$\lambda_{\text{min}} = hc/K_0$	Shortest X-ray wavelength
Moseley's law	$\sqrt{f} = C(Z - 1)$	Determines atomic number ordering
Stimulated emission	$hf = E_2 - E_1$	Basis of laser operation



97. Spontaneous emission: an atom decays from an excited state, emitting a photon in a random direction with random phase.
98. Stimulated emission: an excited atom is induced to decay by a photon; emitted photon is identical to the stimulating photon.
99. Stimulated emission produces photons with the same energy, phase, polarization, and direction.
100. Population inversion ($N_2 > N_1$) is required for laser action.
101. Under thermal equilibrium, $N_2/N_1 = e^{-(E_2-E_1)/kT} < 1$.
102. Metastable states have unusually long lifetimes (up to 10^{-3} s vs. 10^{-8} s).
103. The helium-neon laser emits red light at 632.8 nm.
104. Helium atoms excite neon atoms via collision energy transfer.
105. Laser light properties: high monochromaticity, high coherence, high directionality, high intensity.
106. Laser light has coherence lengths of kilometers; ordinary light has coherence lengths less than a meter.
107. Laser divergence is typically $\sim 0.001^\circ$; ordinary light diverges rapidly.

Practice MCQs

1. The periodic table displays systematic organization because of:

- A) Classical physics
- B) Quantum structure of atoms
- C) Newtonian mechanics
- D) Maxwell's equations

Correct Answer: B

2. Ionization energy is the energy required to:

- A) Add an electron to an atom
- B) Remove the most loosely bound electron from a neutral atom
- C) Remove a proton from the nucleus
- D) Excite an electron to a higher state

Correct Answer: B

3. The principal quantum number n determines:

- A) Orbital shape
- B) Energy level and distance from nucleus
- C) Spin orientation
- D) Magnetic moment

Correct Answer: B

4. The orbital quantum number ℓ has allowed values:

- A) $0, 1, 2, \dots, n$
- B) $0, 1, 2, \dots, n - 1$

C) $1, 2, \dots, n$

D) $-n, \dots, +n$

Correct Answer: B

5. The number of states in a subshell with quantum number ℓ is:

A) $2\ell + 1$

B) $2(2\ell + 1)$

C) $2n^2$

D) n^2

Correct Answer: B

6. The total number of states in a shell with quantum number n is:

A) $2n$

B) $2n^2$

C) n^2

D) $2(2n + 1)$

Correct Answer: B

7. Spectroscopic notation $\ell = 2$ corresponds to:

A) s subshell

B) p subshell

C) d subshell

D) f subshell

Correct Answer: C

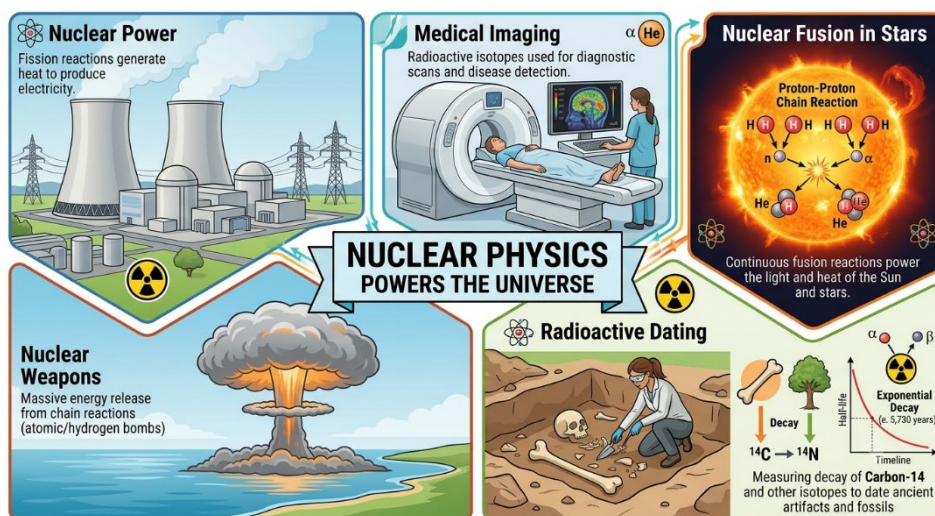
Chapter 18: Nuclear Physics

Introduction

What Is Nuclear Physics?

Nuclear physics is the branch of physics that deals with the atomic nucleus—its structure, properties, stability, and the transformations it undergoes. While atomic physics concerns itself with the electrons that surround the nucleus and determine the chemical behavior of matter, nuclear physics delves into the heart of the atom, where protons and neutrons are bound together by the strongest force in nature.

The nucleus, though it occupies only about 10^{-12} of the volume of the atom, contains more than 99.9% of its mass. Understanding the nucleus is essential not only for explaining the existence of matter as we know it but also for harnessing the immense energy locked within it—energy that powers the stars, sustains life on Earth through sunlight, and, through fission and fusion, holds the promise and peril of nuclear technology.



Discovering the Nucleus: The Rutherford Scattering Experiment

The Plum Pudding Model and Its Demise

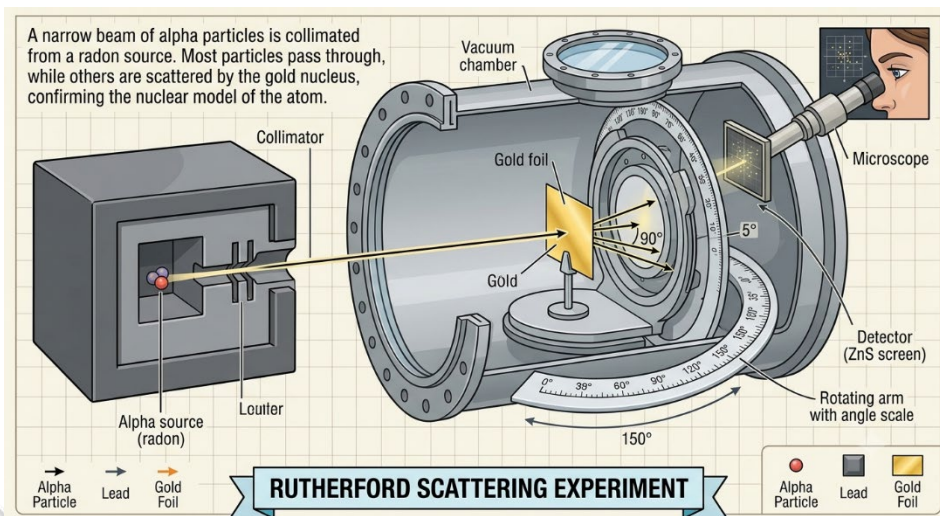
By the beginning of the twentieth century, physicists knew that atoms contained electrons—negatively charged particles discovered by J.J. Thomson in 1897. However, the arrangement of positive charge and mass within the atom remained a mystery. The prevailing model, advanced by Thomson himself, was the so-called "plum pudding" model: the positive charge was thought to be spread uniformly throughout the atom's volume, like a pudding, with electrons embedded within it like plums.

The Rutherford, Geiger, and Marsden Experiment

In 1910, Ernest Rutherford conceived an experiment to probe the structure of the atom by directing a beam of alpha particles (α -particles)—helium nuclei consisting of two protons and two neutrons, with a charge of $+2e$ and a mass approximately 7300 times that of an electron—at a thin gold foil. The experimental arrangement, shown schematically in **Figure**, consisted of:

1. A source of alpha particles (radon gas, a decay product of radium)
2. A thin gold foil target

- A movable detector (a zinc sulfide screen that emitted a flash of light when struck by an alpha particle)



The Astonishing Results

According to the plum pudding model, alpha particles passing through the diffuse positive charge of an atom should experience only tiny deflections, at most a fraction of a degree. The electrons, being extremely light, would be brushed aside like gnats by a thrown stone.

The experimental results, however, were profoundly different:

- Most alpha particles** passed straight through with little or no deflection.
- A small fraction** were deflected through large angles.
- A very few** (about 1 in 8000) were scattered backward at angles approaching 180° .

Rutherford famously remarked: *"It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you."*

Rutherford's Conclusions

From these observations, Rutherford deduced the following:

- The atom is mostly empty space.** The fact that most alpha particles passed through undeflected indicated that the atom consists primarily of empty space.
- The positive charge and most of the mass are concentrated in a tiny central region called the nucleus.** The large-angle deflections could only be explained if the positive charge was concentrated in a small, massive nucleus.

- **Nuclide:** A specific nuclear species characterized by its values of Z and A . Represented as A_ZX or, more commonly, AX , where X is the chemical symbol. For example, ${}^{197}\text{Au}$ represents gold with $A = 197$, $Z = 79$, and $N = 118$.
- **Isotopes:** Nuclides with the same Z (same element) but different A (different number of neutrons). For example, ${}^1\text{H}$, ${}^2\text{H}$ (deuterium), and ${}^3\text{H}$ (tritium) are isotopes of hydrogen.
- **Isobars:** Nuclides with the same A but different Z . For example, ${}^{14}\text{C}$ and ${}^{14}\text{N}$.
- **Isotones:** Nuclides with the same N but different Z .

The Nuclidic Chart (Segrè Chart)

The nuclidic chart plots Z versus N (or N versus Z). Stable nuclides form a narrow band called the **valley of stability**, as shown in **Figure**. For light nuclei, stable nuclides lie along the line $N = Z$. As Z increases, the neutron-proton ratio N/Z increases to about 1.6 for the heaviest stable nuclei, due to the increasing repulsive Coulomb force between protons, which requires extra neutrons to provide additional nuclear attraction without increasing Coulomb repulsion.

Important features:

- No stable nuclides exist for $Z > 83$ (bismuth).
- Magic numbers (2, 8, 20, 28, 50, 82, 126) correspond to particularly stable configurations (filled nuclear shells).
- Doubly magic nuclides (both Z and N magic)

include ${}^4\text{He}$ (2,2), ${}^{16}\text{O}$ (8,8), ${}^{40}\text{Ca}$ (20,20), and ${}^{208}\text{Pb}$ (82,126).

Nuclear Size and Density

Nuclear Radius

Scattering experiments with high-energy electrons (wavelengths small enough to probe nuclear structure) show that the nucleus is approximately spherical, with a radius given by:

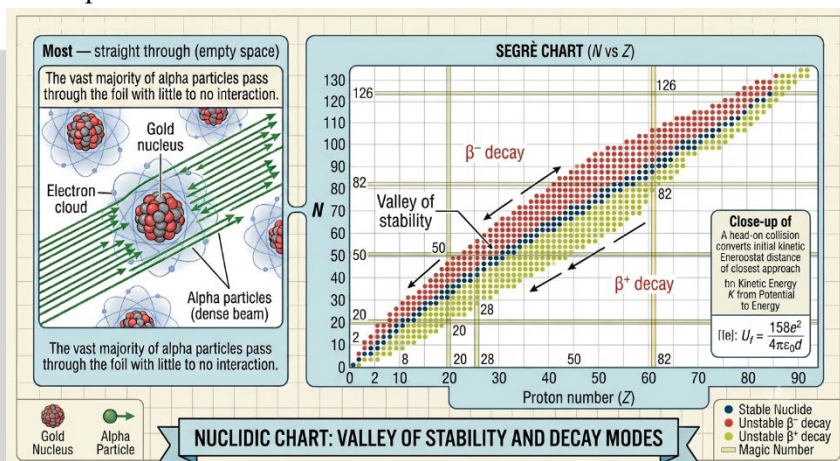
$$R = R_0 A^{1/3}$$

where:

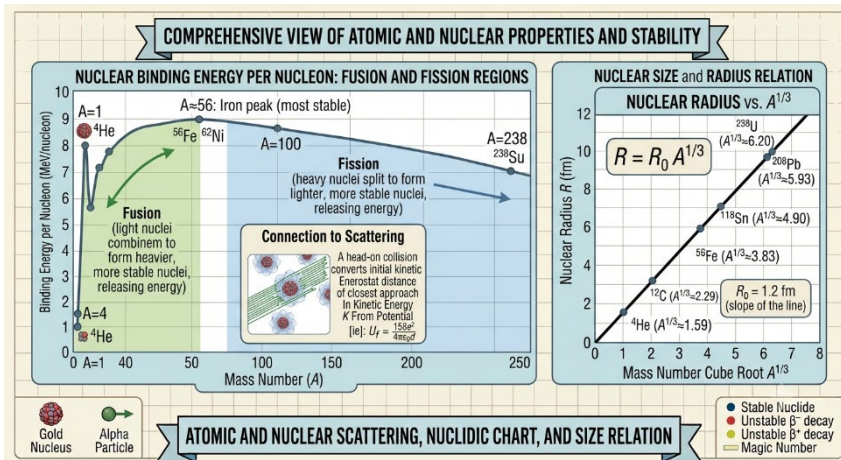
- $R_0 = 1.2 \times 10^{-15} \text{ m} = 1.2 \text{ fm}$ (femtometers or fermis)
- A is the mass number

This relationship implies that the nuclear volume is proportional to A :

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$$



where M_H is the mass of a neutral hydrogen atom and M_{atom} is the mass of the neutral atom containing the nucleus. This approach automatically accounts for the electron masses, which cancel out.



Example: For the ^{62}Ni nucleus ($Z = 28$, $A = 62$, $N = 34$), which has the highest binding energy per nucleon of any nuclide:

$$\Delta m = 28(1.007825 \text{ u}) + 34(1.008665 \text{ u}) - 61.928345 \text{ u} = 0.585365 \text{ u}$$

$$E_B = 0.585365 \text{ u} \times 931.5 \text{ MeV/u} = 545.3 \text{ MeV}$$

The binding energy per nucleon is:

$$\frac{E_B}{A} = \frac{545.3 \text{ MeV}}{62} = 8.795 \text{ MeV/nucleon}$$

The Binding Energy Curve

Key features of the curve:

- Sharp peak at $A = 4$ (^4He):** The alpha particle is exceptionally stable due to its double-magic nature.
- Broad maximum around $A = 56$ (iron and nickel):** These are the most tightly bound nuclei. Nuclides near this region are the most stable.
- Gradual decrease for $A > 56$:** This is due to the increasing Coulomb repulsion between protons, which reduces the binding per nucleon.
- Rapid decrease for $A < 10$:** Surface effects (nucleons on the surface are less tightly bound) reduce the binding per nucleon.

Semi-Empirical Mass Formula (Liquid Drop Model)

The liquid-drop model treats the nucleus as a charged liquid drop. The binding energy can be expressed as the sum of five terms:

$$E_B = C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(A-2Z)^2}{A} + C_5 \delta$$

where:

- Volume term ($C_1 A$):** Each nucleon interacts with nearest neighbors (saturation).
- Surface term ($-C_2 A^{2/3}$):** Nucleons on the surface are less tightly bound.
- Coulomb term ($-C_3 Z(Z-1)/A^{1/3}$):** Repulsive electric force between protons.
- Asymmetry term ($-C_4 (A-2Z)^2/A$):** $N = Z$ is most favorable for light nuclei.

- **Deflection in E field:** Slight deflection toward negative plate
- **Source:** Heavy nuclei (typically $Z > 83$)

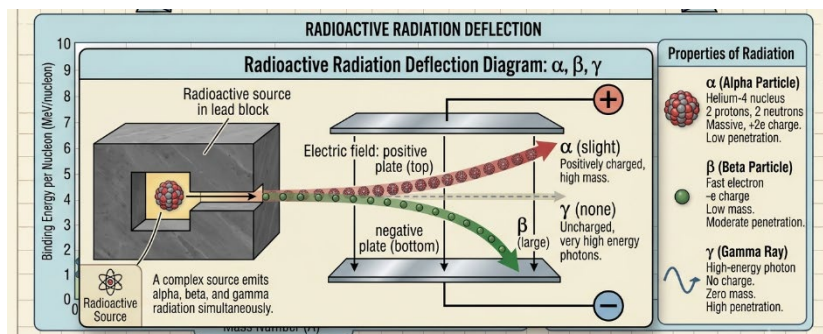
Beta (β) Radiation

There are two types:

1. **Beta-minus (β^-) radiation:** Electrons emitted from the nucleus
2. **Beta-plus (β^+) radiation:** Positrons emitted from the nucleus
 - **Charge:** $\beta^-: -e, \beta^+: +e$
 - **Mass:** 0.0005486 u (same as electron)
 - **Speed:** Up to 0.9995c
 - **Penetrating power:** Moderate (stopped by a few millimeters of aluminum)
 - **Ionizing ability:** Moderate
 - **Deflection in E field:** Large deflection (toward positive plate for β^- , toward negative for β^+)

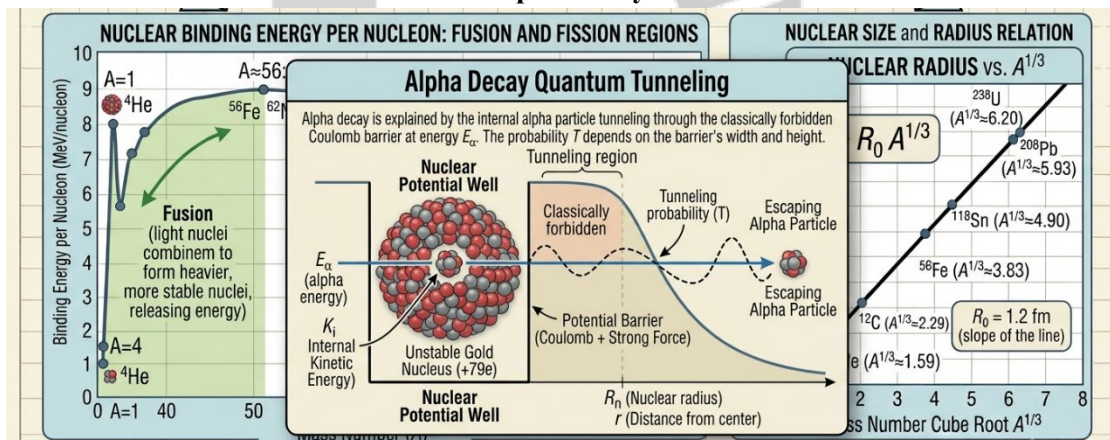
Gamma (γ) Radiation

- **Nature:** High-energy electromagnetic radiation (photons)
- **Charge:** 0
- **Mass:** 0
- **Speed:** c
- **Penetrating power:** Very high (stopped only by several centimeters of lead)
- **Ionizing ability:** Low
- **Deflection in E field:** None

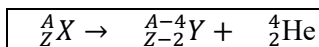


18. Nuclear Physics

Alpha Decay

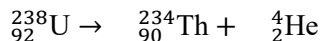


In alpha decay, a nucleus emits an alpha particle (${}^4_2\text{He}$). The general decay equation is:



The daughter nucleus has Z decreased by 2 and A decreased by 4.

Example:



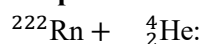
Alpha decay occurs primarily in heavy nuclei ($Z > 83$) where the nucleus reduces its size and neutron-proton ratio to reach stability.

Q-Value and Energy Release

The energy released in alpha decay (the Q -value) is:

$$Q = [m({}_Z^AX) - m({}_{Z-2}^{A-4}Y) - m({}_2^4\text{He})]c^2$$

Example: For ${}^{226}\text{Ra} \rightarrow$



$$\Delta m = 226.025410 \text{ u}$$

$$- (222.017578 \text{ u}$$

$$+ 4.002603 \text{ u})$$

$$= 0.005229 \text{ u}$$

$$Q = 0.005229 \text{ u} \times 931.5 \text{ MeV/u}$$

$$= 4.871 \text{ MeV}$$

Alpha Decay and Quantum Tunneling

Alpha decay is inhibited by a potential energy barrier. The alpha particle is trapped in the nucleus by the strong force but must tunnel through the Coulomb barrier to escape, as shown in **Figure**.

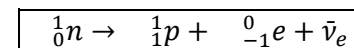
The probability of tunneling is extremely sensitive to the energy and width of the barrier. This explains why alpha decay half-lives vary by many orders of magnitude.

Beta Decay

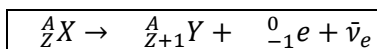
Beta decay involves the transformation of a nucleon within the nucleus, accompanied by the emission of an electron or positron and a neutrino/antineutrino.

Beta-Minus (β^-) Decay

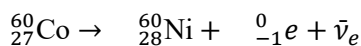
A neutron transforms into a proton:



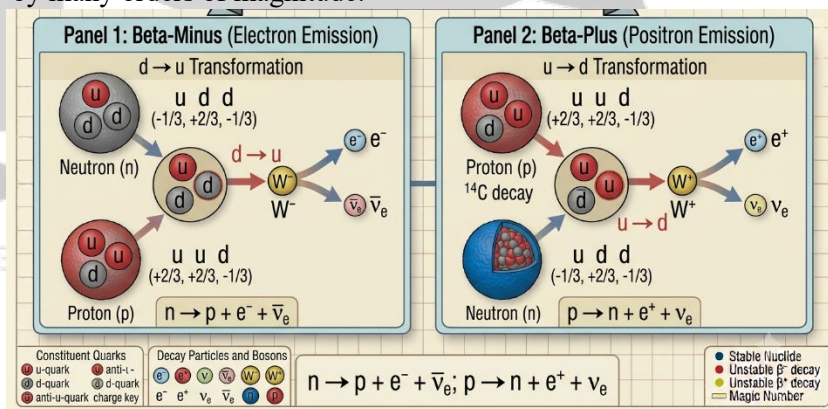
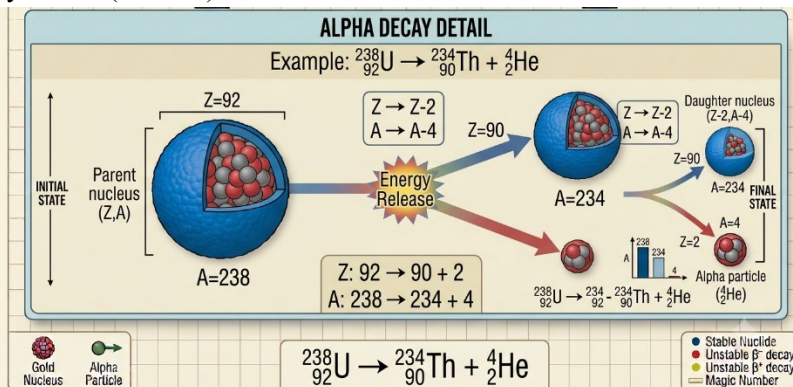
For a nucleus:



Example:

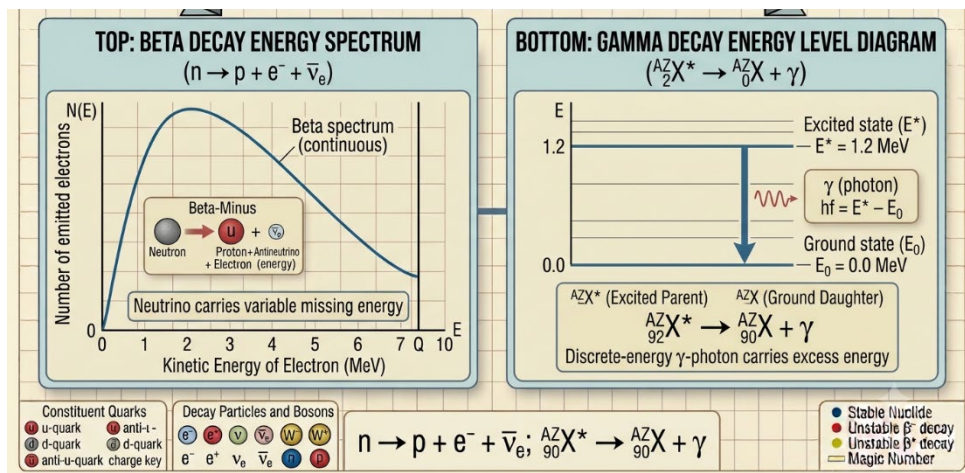
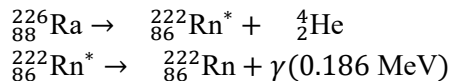


Beta-minus decay occurs in neutron-rich nuclides (those above the valley of stability), where the neutron-proton ratio is too high.



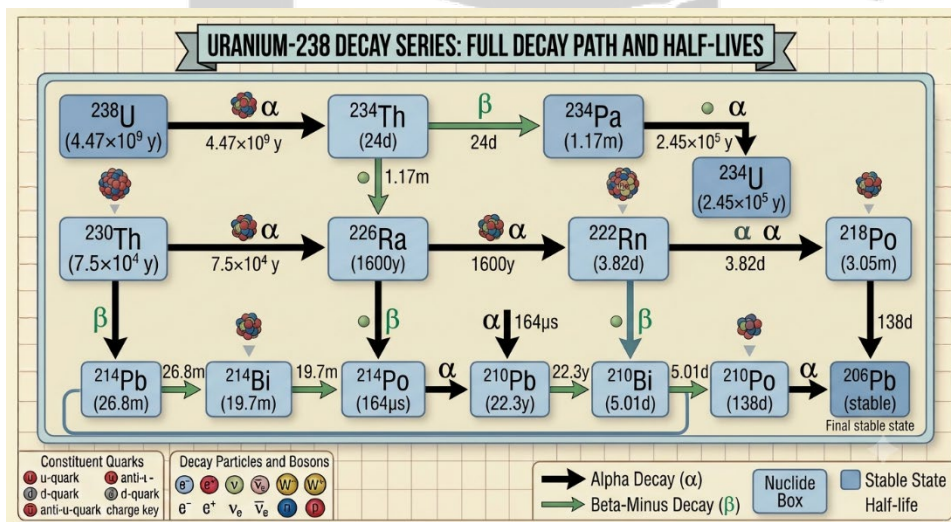
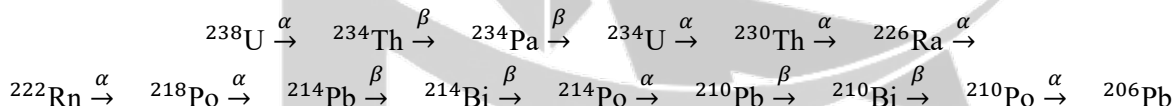
- Gamma decay often follows alpha or beta decay when the daughter nucleus is left in an excited state.

Example: In the alpha decay of $^{226}_{88}\text{Ra}$:



Radioactive Decay Series

A parent nucleus often decays to a daughter that is itself radioactive, leading to a sequence of decays until a stable nucleus is reached. The ^{238}U decay series (shown in **Figure**) ends at stable ^{206}Pb :



Conservation Laws in Nuclear Decay

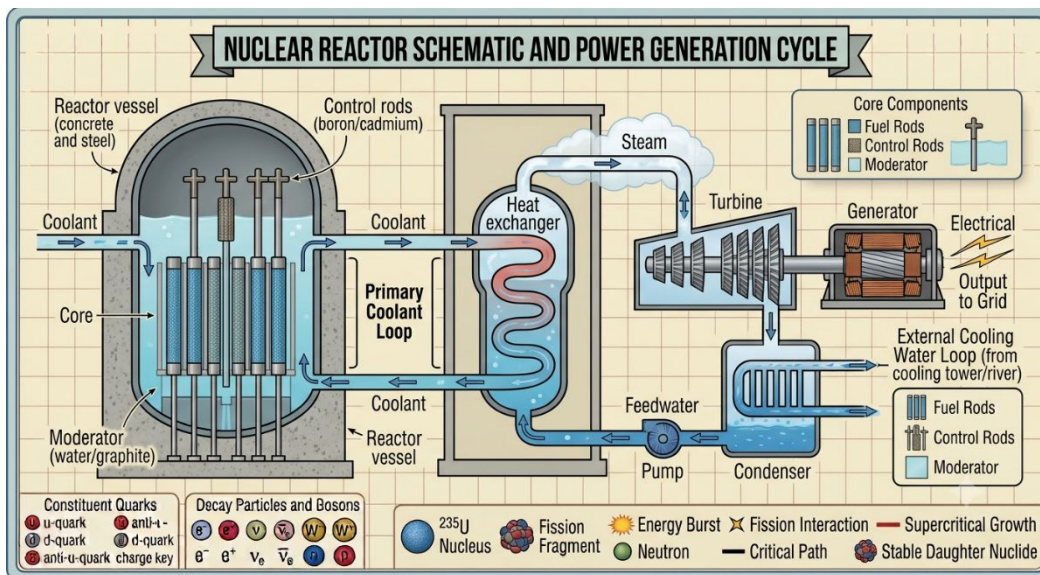
All nuclear processes obey the following conservation laws:

- Conservation of charge:** The sum of charges (in units of e) before and after the process must be equal.
- Conservation of nucleon number:** The total number of nucleons (A) is conserved.

- **Typical fission rate:** For a 3400 MW thermal reactor:

$$R = \frac{3400 \times 10^6 \text{ W}}{200 \text{ MeV/fission}} \times \frac{1 \text{ MeV}}{1.60 \times 10^{-13} \text{ J}} = 1.06 \times 10^{20} \text{ fissions/s}$$

Fuel consumption: A 3400 MW reactor consumes about 4.5 kg of ^{235}U per day.



Nuclear Fusion

The Basic Process

Nuclear fusion is the combining of two light nuclei to form a heavier nucleus, with the release of energy. Fusion releases energy because the binding energy per nucleon increases as we move from very light nuclei toward the iron peak.

Example: The deuterium-tritium (d-t) reaction:



The Coulomb Barrier

Fusion is inhibited by the Coulomb repulsion between positively charged nuclei. To fuse, nuclei must have sufficient kinetic energy to overcome the barrier. The barrier height for two protons is about 400 keV. This corresponds to a temperature of:

$$T = \frac{2E}{3k} = \frac{2 \times 400 \text{ keV}}{3 \times 8.62 \times 10^{-11} \text{ MeV/K}} \approx 3 \times 10^9 \text{ K}$$

Thermonuclear Fusion and Tunneling

Fusion can occur at lower temperatures through quantum tunneling and because the Maxwell-Boltzmann distribution has a high-energy tail of particles with energies much greater than the average. The fusion rate is the product of:

- $n(E)$: The number of particles with energy E
- $p(E)$: The probability of barrier penetration (tunneling)

These factors combine to give a peak in the fusion rate at an energy where both are significant.

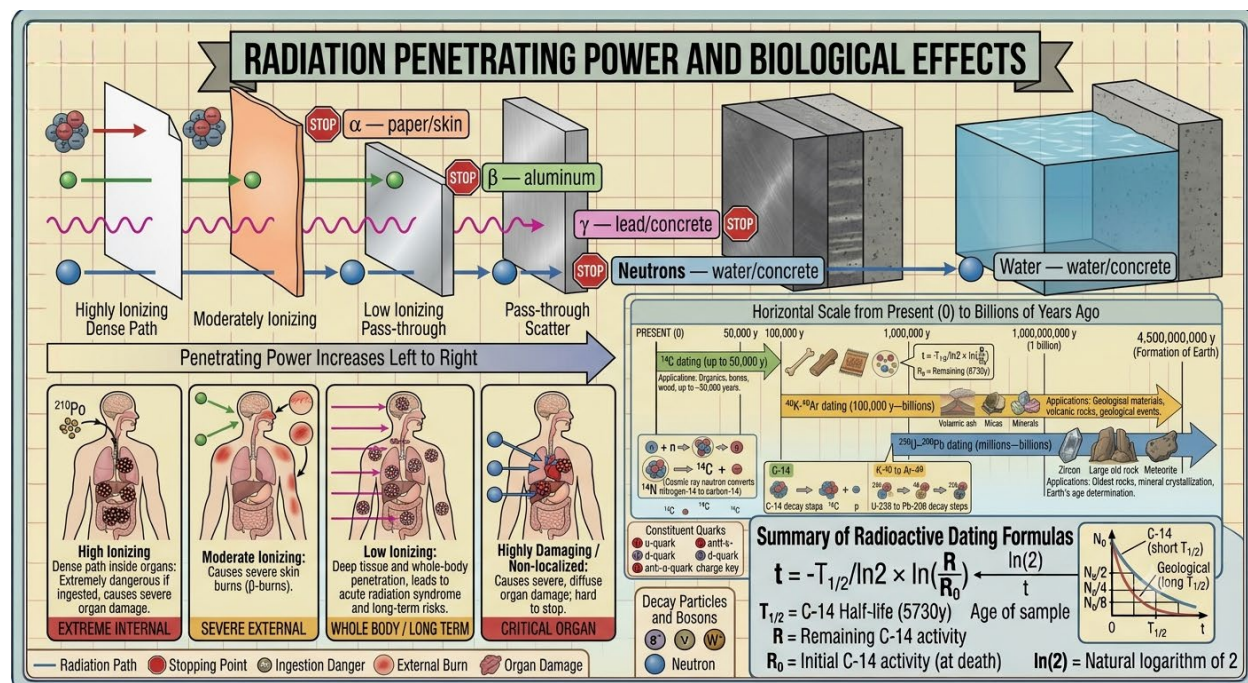
The Proton-Proton Chain (Solar Fusion)

The Sun's energy comes from the proton-proton chain, shown in **Figure**:

Radiation Dosimetry and Biological Effects

Dosimetric Quantities

Quantity	Unit	Definition
Activity	Becquerel (Bq)	Decays per second; $1 \text{ Bq} = 1 \text{ s}^{-1}$
Absorbed Dose	Gray (Gy)	Energy absorbed per kilogram; $1 \text{ Gy} = 1 \text{ J/kg}$
Equivalent Dose	Sievert (Sv)	Absorbed dose $\times$ RBE; $1 \text{ Sv} = 1 \text{ Gy} \times \text{RBE}$



RBE (Relative Biological Effectiveness):

- X-rays, γ -rays: 1
- Beta particles: 1.0-1.5
- Slow neutrons: 3-5
- Protons: 10
- Alpha particles: 20

Equivalent Dose (rem): $1 \text{ Sv} = 100 \text{ rem}$

Health Effects

- Natural background:** ~2-3 mSv/year
- Chest x-ray:** ~0.2-0.4 mSv
- Short-term whole-body dose of 5 Sv:** Usually fatal within days or weeks
- Occupational limit:** ~20 mSv/year

Fundamental Forces and the Standard Model

The Four Fundamental Forces

Force	Relative Strength	Range (m)	Exchange Particle
Strong	1	$\sim 10^{-15}$	Gluons
Electromagnetic	10^{-2}	Infinite	Photons

Weak	10^{-6}	$\sim 10^{-18}$	$W^{\pm}, Z^0$
Gravitational	10^{-43}	Infinite	Graviton (hypothetical)

Building Blocks of Matter

Quarks: Six flavors—up (u), down (d), strange (s), charm (c), bottom (b), top (t). Quarks have fractional charges (+2/3 or -1/3) and come in three "colors" (red, green, blue).

Leptons: Six types—electron, muon, tau, and their corresponding neutrinos. Leptons do not experience the strong force.

Hadrons: Particles made of quarks.

- **Baryons:** Three quarks (e.g., proton = uud, neutron = udd).
- **Mesons:** One quark and one antiquark.

Exchange Particles (Bosons): Mediate the fundamental forces.

- **Gluons:** Strong force
- **Photons:** Electromagnetic force
- $W^{\pm}, Z^0$: Weak force
- **Graviton:** Gravitational force (not yet observed)

Key Equations Summary

Concept	Equation
Nuclear radius	$R = R_0 A^{1/3}$
Mass defect	$\Delta m = [Zm_p + (A - Z)m_n] - m_{\text{nucleus}}$
Binding energy	$E_B = \Delta mc^2$
Binding energy (using atomic masses)	$E_B = [ZM_H + (A - Z)m_n - M_{\text{atom}}]c^2$
Decay law	$N = N_0 e^{-\lambda t}$
Activity	$R = \lambda N = R_0 e^{-\lambda t}$
Half-life	$T_{1/2} = \frac{\ln 2}{\lambda}$
Mean lifetime	$\tau = \frac{1}{\lambda}$
Reaction Q-value	$Q = (m_a + m_x - m_y - m_b)c^2$
Lawson's criterion	$n\tau > 10^{20} \text{ s/m}^3$

Nuclear Physics: One-Liners

Discovery of the Nucleus

1. The nucleus occupies only about 10^{-12} of the atom's volume but contains more than 99.9% of its mass.
2. Rutherford's gold foil experiment (1910) revealed that the atom is mostly empty space with a tiny, massive, positively charged nucleus.
3. Most alpha particles passed through the gold foil undeflected, indicating the atom is mostly empty space.
4. A small fraction of alpha particles were deflected through large angles, indicating a concentrated positive charge.
5. About 1 in 8000 alpha particles were scattered backward at angles approaching 180° .
6. Rutherford concluded that the positive charge and most of the mass are concentrated in a tiny central nucleus.
7. The distance of closest approach for an alpha particle is determined by equating initial kinetic energy to Coulomb potential energy.
8. Rutherford estimated the nuclear radius to be about 10^{-14} m, roughly 10^4 times smaller than the atomic radius.

Nuclear Terminology and Properties

9. Nucleons are the constituent particles of the nucleus—protons and neutrons.
10. The atomic number Z is the number of protons; it determines the chemical element.
11. The neutron number N is the number of neutrons in the nucleus.
12. The mass number $A = Z + N$ is the total number of nucleons.
13. A nuclide is a specific nuclear species characterized by its Z and A values.
14. Isotopes have the same Z but different A (same element, different neutrons).
15. Isobars have the same A but different Z .
16. Isotones have the same N but different Z .
17. Stable nuclides form a narrow band called the valley of stability.
18. No stable nuclides exist for $Z > 83$ (bismuth).
19. Magic numbers (2, 8, 20, 28, 50, 82, 126) correspond to particularly stable nuclear configurations.
20. Doubly magic nuclides include ${}^4\text{He}$ (2,2), ${}^{16}\text{O}$ (8,8), ${}^{40}\text{Ca}$ (20,20), and ${}^{208}\text{Pb}$ (82,126).

Nuclear Size and Density

21. Nuclear radius: $R = R_0 A^{1/3}$, where $R_0 = 1.2 \times 10^{-15}$ m = 1.2 fm.
22. Nuclear volume is proportional to mass number: $V \propto A$.
23. Nuclear density is approximately constant for all nuclei: $\rho \approx 2.3 \times 10^{17}$ kg/m³.
24. Nuclear density is about 2×10^{14} times the density of water.
25. Neutron stars have densities comparable to nuclear matter.

Nuclear Angular Momentum and Magnetic Moment

26. Nuclear spin is the total angular momentum of a nucleus, quantized as $J = \sqrt{j(j+1)}\hbar$.
27. If A is even, j is an integer; if A is odd, j is a half-integer.
28. If both Z and N are even, $j = 0$ (nucleons pair with opposite spins).

126. The weak force has relative strength 10^{-6} and range $\sim 10^{-18}$ m; mediated by $W^{\pm}$ and Z^0 bosons.
127. The gravitational force has relative strength 10^{-43} and infinite range; mediated by gravitons (hypothetical).
128. Quarks have fractional charges (+2/3 or -1/3) and come in three colors.
129. Baryons are composed of three quarks (e.g., proton = uud, neutron = udd).
130. Mesons are composed of one quark and one antiquark.

Practice MCQs

M 1. Rutherford's gold foil experiment revealed that:

- K** A) The atom is mostly empty space with a tiny, massive, positively charged nucleus
- B) The atom is a solid sphere
- P** C) Electrons orbit in fixed paths
- D) The nucleus contains neutrons

R Correct Answer: A

E 2. In Rutherford's experiment, most alpha particles passed through the gold foil because:

- P** A) They were attracted to the nucleus
- B) The atom is mostly empty space
- A** C) The gold foil was very thin
- D) Alpha particles have low energy

R Correct Answer: B

A 3. A small fraction of alpha particles were deflected through large angles because:

- T** A) They hit electrons
- B) They experienced the repulsive force of a concentrated positive charge
- I** C) They were absorbed by the foil
- D) They scattered from multiple nuclei

O Correct Answer: B

N 4. About 1 in 8000 alpha particles were scattered backward because:

- S** A) They hit the nucleus head-on
- B) They hit electrons
- C) The foil was thick
- D) Alpha particles are heavy

Correct Answer: A

5. Rutherford concluded that the nucleus:

- A) Is negatively charged
- B) Contains the positive charge and most of the

mass

C) Is the same size as the atom

D) Contains electrons

Correct Answer: B

6. The nuclear radius is approximately:

A) 10^{-10} m

B) 10^{-14} m

C) 10^{-6} m

D) 10^{-15} m

Correct Answer: B

7. The atomic number Z is:

A) The number of neutrons

B) The number of protons

C) The total number of nucleons

D) The mass number

Correct Answer: B

8. The mass number A =:

A) $Z + N$

B) $Z - N$

C) ZN

D) Z/N

Correct Answer: A

9. Isotopes have:

A) Same Z , different A

B) Same A , different Z

C) Same N , different Z

D) Same Z , same A

Correct Answer: A

10. Isobars have:

A) Same Z , different A

B) Same A , different Z

C) Same N , different Z

Chapter 19: Quantum Mechanics

Introduction

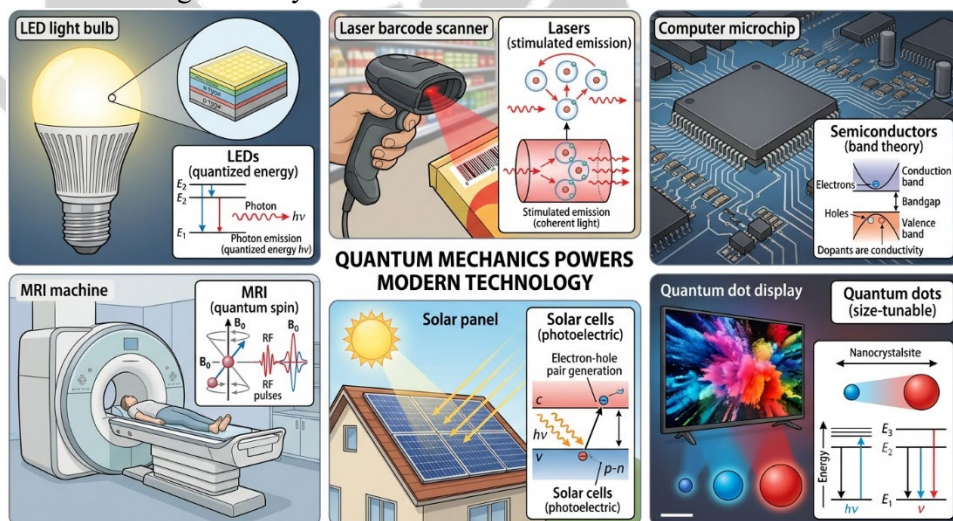
The history of physics up to the end of the nineteenth century presented a remarkably coherent picture of the physical world. Newtonian mechanics, with its deterministic laws of motion, described the behavior of particles from falling apples to orbiting planets. Maxwell's theory of electromagnetism unified electricity, magnetism, and optics, demonstrating conclusively that light is an electromagnetic wave—a continuous, oscillating disturbance of electric and magnetic fields propagating through space at speed $c = 3.00 \times 10^8 \text{ m s}^{-1}$. This classical worldview seemed complete and unassailable.

Yet, as physicists peered more deeply into the nature of light and matter, cracks began to appear in this elegant edifice. Phenomena such as the photoelectric effect, atomic spectra, and blackbody radiation defied explanation within the classical framework. For centuries, the debate over the nature of light oscillated between two seemingly contradictory descriptions: the wave model, which explained interference and diffraction, and the particle model, which accounted for rectilinear propagation and reflection. The resolution of these puzzles required a radical departure from classical thinking—one that would forever change our understanding of reality.

In this chapter, we develop the foundational principles of quantum mechanics, beginning with the particle-like behavior of light and the wave-like behavior of matter, and culminating in the full quantum-mechanical description of atomic structure. We shall see

that quantization—the restriction of physical quantities to discrete values—is not an exotic mathematical curiosity but a fundamental principle governing the behavior of confined systems, from electrons in atoms to artificial nanostructures like quantum dots. The journey from Planck's quantum hypothesis to Schrödinger's wave mechanics and Heisenberg's uncertainty principle represents one of the greatest intellectual achievements in human history.

We will discover that light and matter exhibit a profound wave-particle duality: under some circumstances, electromagnetic radiation behaves as if it consists of discrete particles (photons); under other circumstances, particles such as electrons behave as if they are waves. This duality is not a mere curiosity but a fundamental aspect of nature that underlies the stability of atoms, the operation of lasers, the behavior of semiconductors, and ultimately all of modern physics and chemistry.



The Particle Nature of Light: The Photon

Quantization of Energy

The foundational concept of quantum physics is quantization: many physical quantities are found only in discrete, indivisible units called quanta (singular: quantum). In 1900, Max Planck proposed that the energy of atomic oscillators in the walls of a hot cavity is quantized. Specifically, an oscillator of frequency f can only have energies that are integer multiples of a fundamental unit:

$$E_n = nhf, n = 1, 2, 3, \dots \quad (3.1)$$

where $h = 6.626 \times 10^{-34} \text{ J s}$ is Planck's constant. This radical proposal successfully explained the observed spectrum of blackbody radiation—a feat that classical physics could not achieve.

In 1905, Albert Einstein extended Planck's idea by proposing that electromagnetic radiation itself is quantized. According to Einstein, light consists of discrete packets of energy called photons. Each photon of frequency f carries energy

$$E = hf. \quad (3.2)$$

Using the relationship between frequency, wavelength λ , and speed of light c ($f = c/\lambda$), we may also write

$$E = \frac{hc}{\lambda}. \quad (3.3)$$

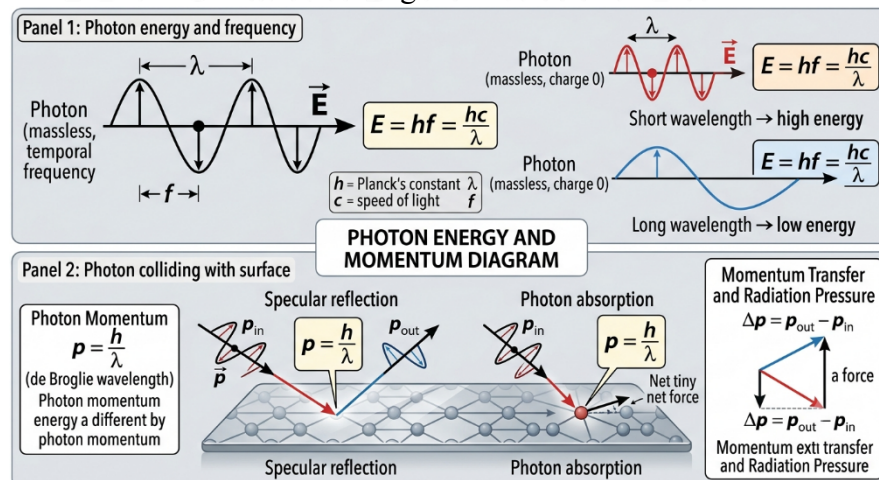
The photon is not a classical particle; it is massless, has no electric charge, and travels at speed c in vacuum. However, when light interacts with matter—during absorption or emission—the interaction occurs at a point and transfers the entire photon energy hf to a single electron or atom.

Photon Momentum

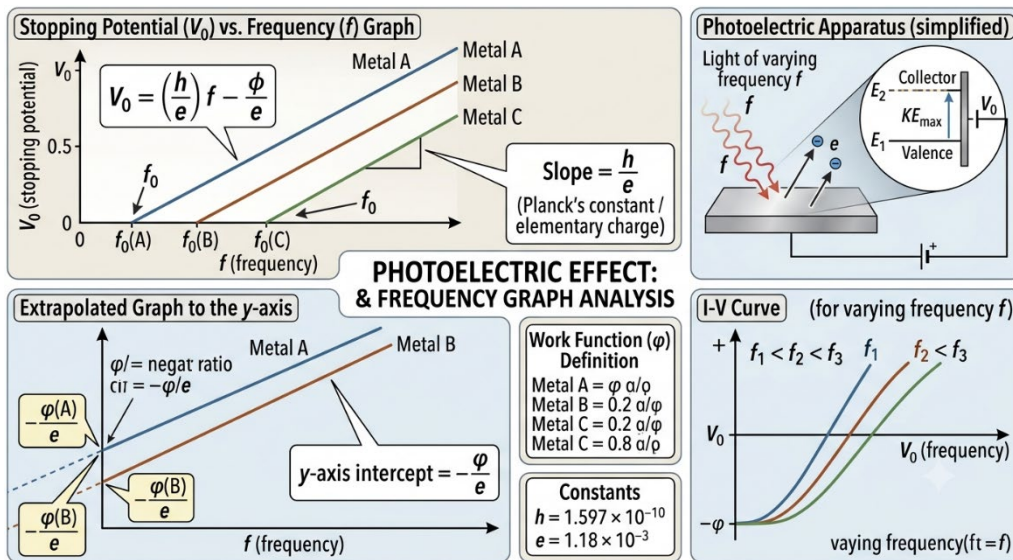
Although a photon has no rest mass, it carries momentum. From special relativity, for a particle with zero rest mass, the energy and momentum are related by $E = pc$. Therefore,

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}. \quad (3.4)$$

Thus, momentum is inversely proportional to wavelength: shorter-wavelength photons (e.g., gamma rays) carry more momentum than longer-wavelength photons (e.g., radio waves). This momentum can be transferred to matter, exerting radiation pressure—a phenomenon with practical applications from solar sails to laser cooling.



Instantaneous emission: Emission occurs without measurable time delay (less than 10^{-9} s), even at very low intensities.

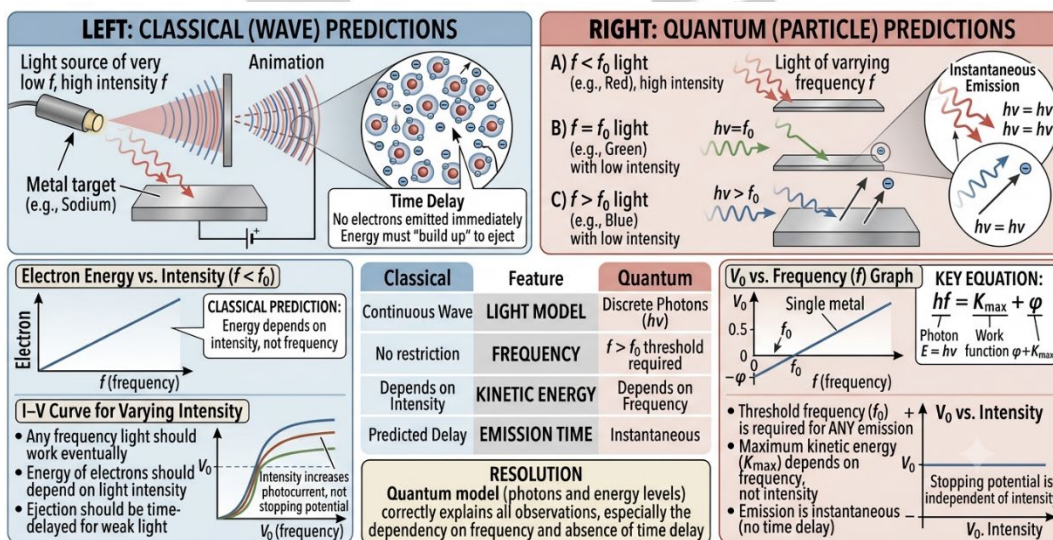


Classical Physics Fails

The wave theory of light predicts:

- Electrons should be ejected at any frequency if the light is sufficiently intense (energy accumulates over time).
- The kinetic energy of ejected electrons should increase with intensity (brighter light $\rightarrow$ larger electric field amplitude $\rightarrow$ more energetic "kick").
- There should be a time delay for low-intensity light (energy takes time to accumulate).

All three predictions contradict experiment. Classical physics is utterly unable to explain the photoelectric effect.



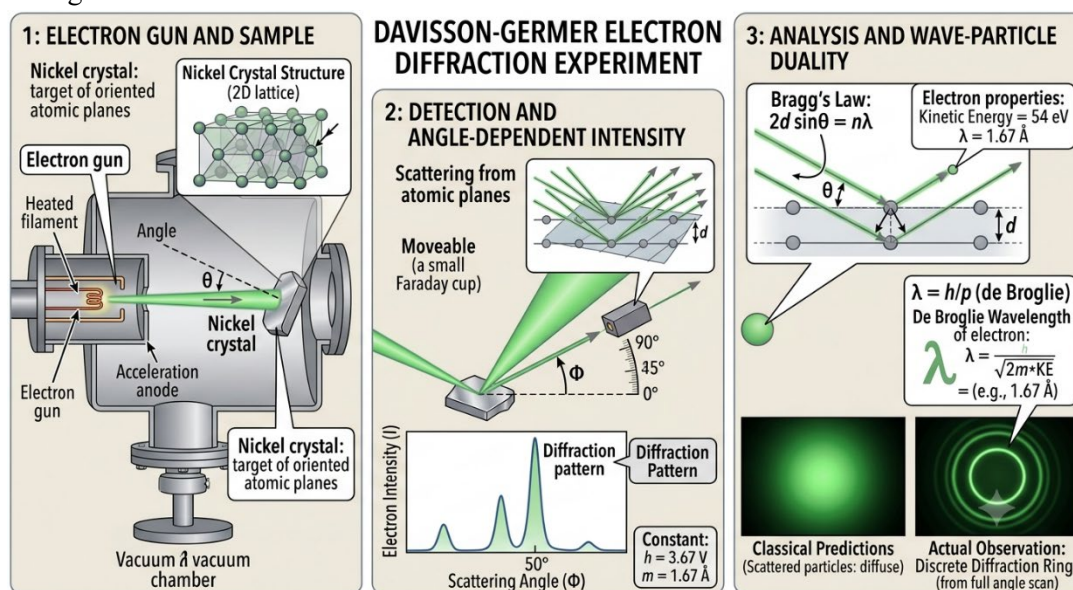
Experimental Confirmation: Electron Diffraction

In 1927, Davisson and Germer (and independently G. P. Thomson) demonstrated that a beam of electrons directed at a nickel crystal produced a diffraction pattern identical to that of X-rays of the same wavelength, as predicted by de Broglie. The observed diffraction angles agreed precisely with the de Broglie wavelength calculated from the electron's momentum.

Later experiments with electrons passing through a double slit produced interference fringes—even when electrons were sent one at a time. The fringes build up statistically, confirming that each electron interferes with itself as a wave.

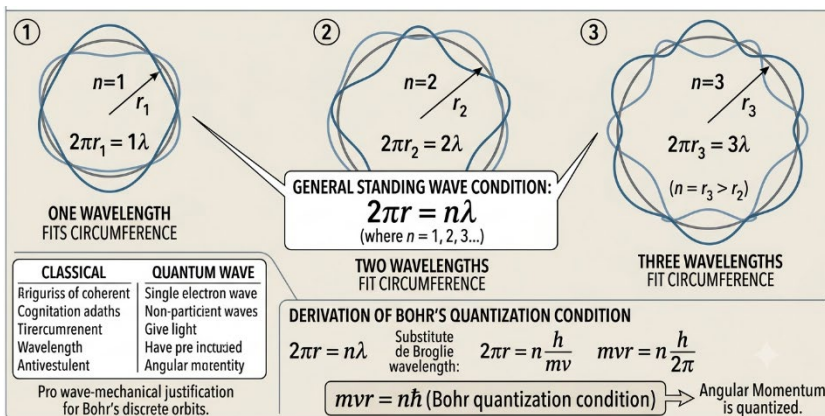
The wave nature of matter is now routinely exploited in electron microscopes, which achieve far higher resolution than optical microscopes because the de Broglie wavelength of accelerated electrons is much shorter than that of visible light.

Figure shows a schematic of a transmission electron microscope (TEM). Electrons are accelerated, focused by magnetic lenses, passed through a thin specimen, and projected onto a fluorescent screen to form an image.



The De Broglie Interpretation of Bohr's Quantization

De Broglie's wave hypothesis provides a physical picture for Bohr's angular momentum quantization (which we shall encounter in Section 3.7). For an electron wave to fit around a circular orbit of circumference $2\pi r_n$, an integer number of wavelengths must fit:



$$2\pi r_n = n\lambda_n, n = 1, 2, 3, \dots \quad (3.18)$$

Using $\lambda_n = h/mv_n$, we obtain:

$$2\pi r_n = n \frac{h}{mv_n} \Rightarrow mv_n r_n = n \frac{h}{2\pi} = n\hbar. \quad (3.19)$$

Thus, the quantization of angular momentum emerges naturally from the requirement that the electron wave be a standing wave around the nucleus.

The Heisenberg Uncertainty Principle

Position-Momentum Uncertainty

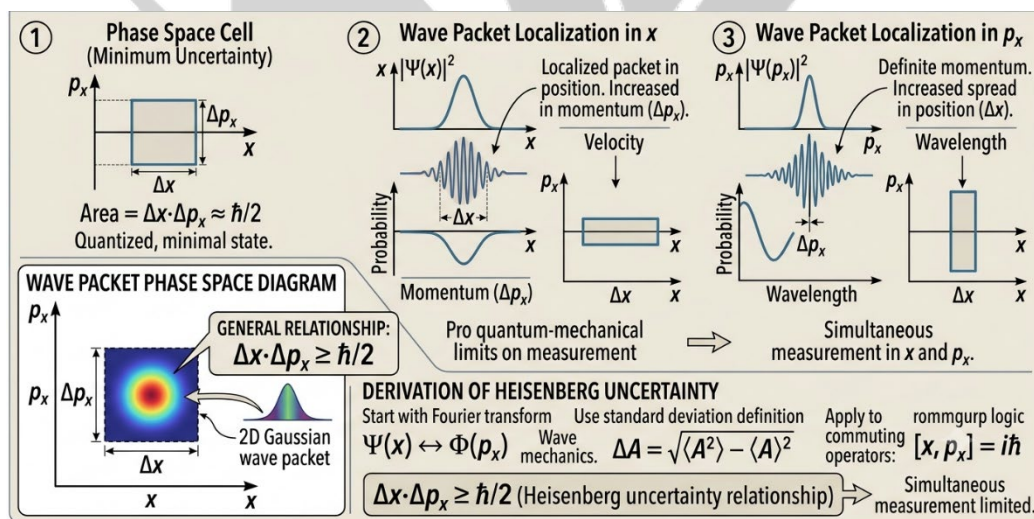
The wave nature of particles imposes fundamental limitations on our ability to measure physical quantities simultaneously. Consider an electron passing through a slit of width a . The electron's position in the y -direction is uncertain by $\Delta y \approx a$. The diffraction pattern indicates that the electron's momentum in the y -direction has an uncertainty Δp_y related to the slit width by:

$$\Delta p_y \cdot a \geq h. \quad (3.20)$$

This leads to the general Heisenberg uncertainty principle:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}, \quad \Delta y \Delta p_y \geq \frac{\hbar}{2}, \quad \Delta z \Delta p_z \geq \frac{\hbar}{2}, \quad (3.21)$$

where $\hbar = h/2\pi = 1.055 \times 10^{-34} \text{ J s}$.



The interpretation is profound: it is impossible to simultaneously know both the position and momentum of a particle with arbitrary precision. The more precisely we know one quantity, the less precisely we can know the other. This is not a limitation of our measuring instruments but a fundamental property of nature.

Figure illustrates the uncertainty principle in phase space (position-momentum space). The product $\Delta x \Delta p_x$ cannot be less than $\hbar/2$.

Energy-Time Uncertainty

A similar uncertainty relation exists between energy and time:

$$\Delta E \Delta t \geq \frac{\hbar}{2}. \quad (3.22)$$

Quantized Wavelengths and Energies

For a standing matter wave, the boundary conditions require an integer number of half-wavelengths to fit in the well:

$$L = n \frac{\lambda_n}{2}, n = 1, 2, 3, \dots \quad (3.32)$$

Thus, $\lambda_n = 2L/n$. The de Broglie relation $p = h/\lambda$ gives the momentum magnitude $p_n = h/\lambda_n = nh/(2L)$. The kinetic energy (since $U = 0$ inside) is $E_n = p_n^2/(2m)$. Therefore,

$$E_n = \frac{n^2 h^2}{8mL^2} = \frac{n^2 \pi^2 \hbar^2}{2mL^2}, n = 1, 2, 3, \dots \quad (3.33)$$

Several crucial points emerge:

- **Energy quantization:** Only discrete energies are allowed. The electron cannot have energies between E_n and E_{n+1} .
- **Zero-point energy:** The lowest energy state ($n = 1$) has $E_1 > 0$. A confined particle can never be at rest. As $L \rightarrow \infty$ (free particle), $E_1 \rightarrow 0$ and quantization disappears.
- **Energy spacing:** $\Delta E = E_{n+1} - E_n = \frac{h^2}{8mL^2} (2n + 1)$. For macroscopic L , the spacing is negligible; quantization becomes observable only at atomic scales.

Wave Functions and Probability Density

Solving the time-independent Schrödinger equation for this system yields the wave functions:

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), 0 < x < L, \quad (3.34)$$

and $\psi_n(x) = 0$ elsewhere. The factor $\sqrt{2/L}$ ensures normalization: $\int_0^L |\psi_n(x)|^2 dx = 1$, which means the probability of finding the electron somewhere in the well is exactly 1.

The probability density $P(x) = |\psi_n(x)|^2$ gives the probability per unit length of detecting the electron at position x . For $n = 1$, $P(x)$ is maximum at the center ($x = L/2$) and zero at the walls. For $n = 2$, $P(x)$ has two maxima (at $L/4$ and $3L/4$) and a node (zero) at the center. These patterns are standing matter waves.

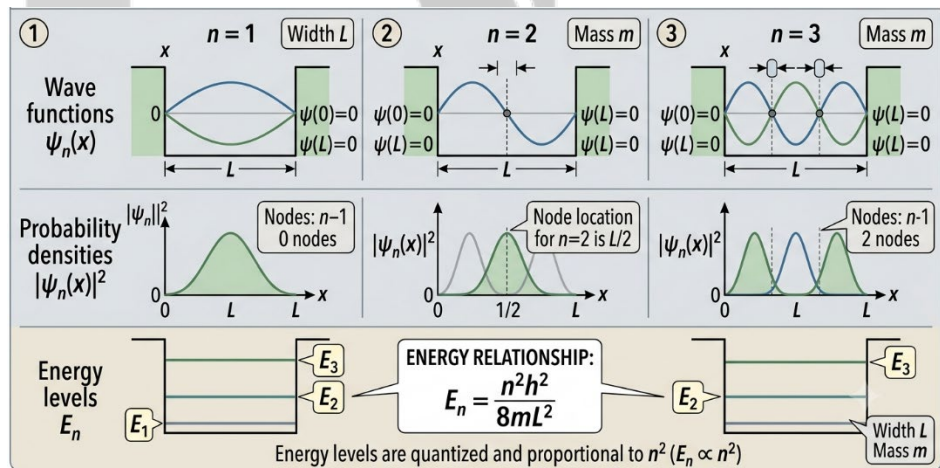


Figure shows the first

three wave functions $\psi_n(x)$ and probability densities $|\psi_n(x)|^2$ for a particle in a one-dimensional box. Note that the probability density is zero at certain points for $n > 1$ —the particle cannot be found at these nodes. The number of nodes in ψ_n is $n - 1$; the number of nodes in $|\psi_n|^2$ is also $n - 1$.

- $\ell = 3$: f orbitals (fundamental)

Wave Function of the Ground State

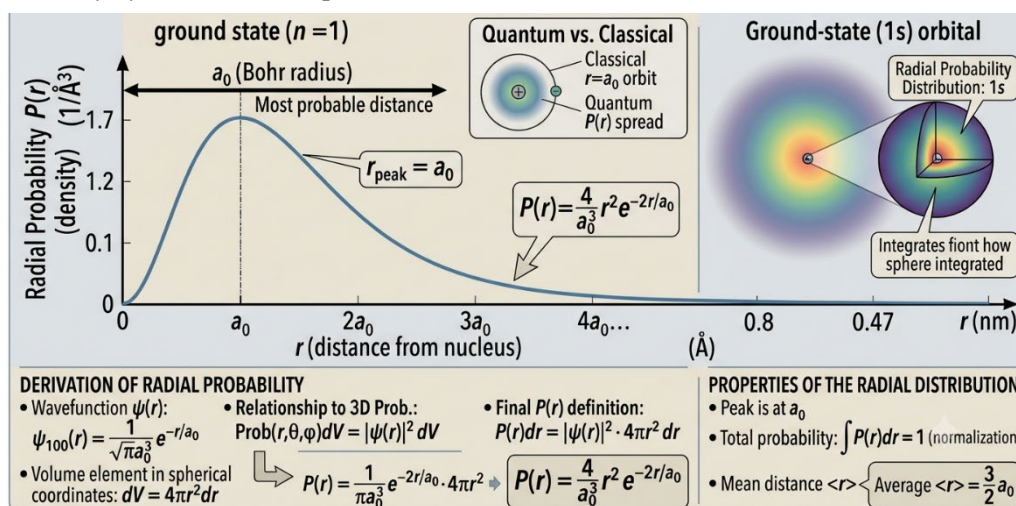
For $n = 1$, $\ell = 0$, the normalized wave function is

$$\psi_{100}(r) = \frac{1}{\sqrt{\pi}} \left(\frac{1}{a_0} \right)^{3/2} e^{-r/a_0}. \quad (3.44)$$

The radial probability density $P(r)$ is defined such that $P(r) dr$ is the probability of finding the electron between spherical shells of radius r and $r + dr$. Since the volume of a spherical shell is $4\pi r^2 dr$,

$$P(r) = 4\pi r^2 |\psi|^2 = \frac{4}{a_0^3} r^2 e^{-2r/a_0}. \quad (3.45)$$

Figure shows $P(r)$ peaks at $r = a_0$. This is the Bohr radius—the most probable distance of the electron from the nucleus in the ground state. The electron is not in a definite orbit; it exists as a probability cloud. Figure shows cross-sectional views of the probability distributions for hydrogen atom orbitals. The 1s orbital is spherically symmetric. The 2p orbitals have two lobes with a node at the nucleus.



Hydrogen Spectrum

When an electron makes a transition from a higher energy level n_{high} to a lower level n_{low} , the emitted photon wavelength is given by Eq. (3.30). Spectral lines are grouped into series:

Series	n_{low}	Wavelength Range
Lyman	1	Ultraviolet
Balmer	2	Visible and near UV
Paschen	3	Infrared
Brackett	4	Far infrared

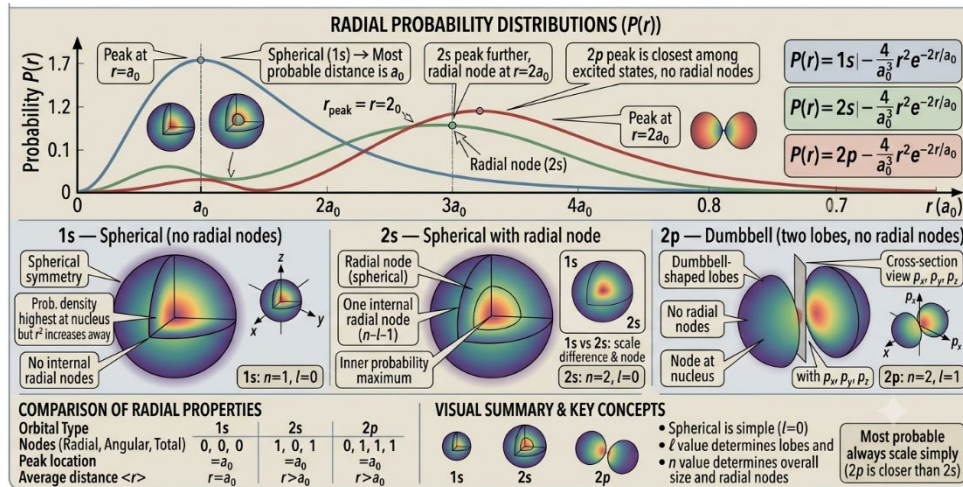
Degeneracy

A crucial feature of the hydrogen atom is degeneracy—different quantum states can have the same energy. In hydrogen, all states with the same n have the same energy (ignoring fine structure). For the n th shell, the number of distinct states (including electron spin, which we introduce next) is $2n^2$.

For example, the $n = 2$ shell has:

- $\ell = 0$: $m_\ell = 0$ (1 orbital) $\rightarrow$ with spin: 2 states
- $\ell = 1$: $m_\ell = -1, 0, +1$ (3 orbitals) $\rightarrow$ with spin: 6 states

Total: 8 states = 2×2^2



Perturbation Theory

The Schrödinger equation can be solved exactly for only a handful of systems: the harmonic oscillator, the hydrogen atom, and a few others. For most real-world problems—atoms with multiple electrons, atoms in external fields, or molecules—the Schrödinger equation cannot be solved exactly. **Perturbation theory** provides a systematic method for finding approximate solutions when the Hamiltonian differs only slightly from a solvable one.

The Basic Idea

The Unperturbed and Perturbed Hamiltonians

We write the full Hamiltonian as:

$$H = H_0 + \lambda H'$$

where:

Term	Meaning	Known?
H_0	Unperturbed Hamiltonian	Exactly solvable
H'	Perturbation (small correction)	Small compared to H_0
λ	Expansion parameter ($0 \leq \lambda \leq 1$)	Tracks the perturbation

The Perturbation Expansion

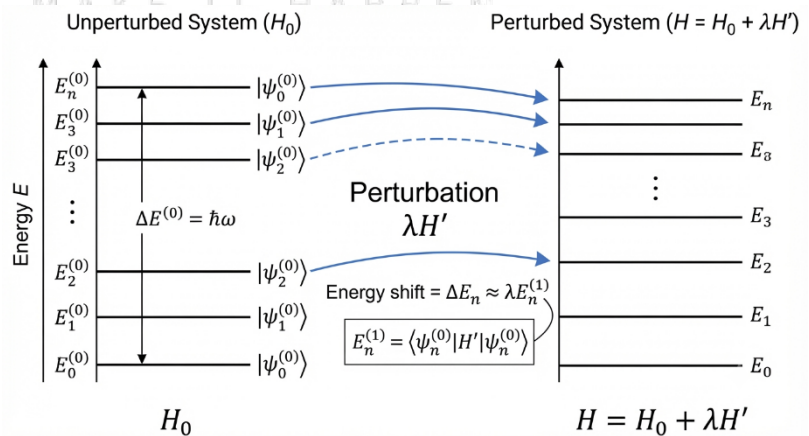
We expand the energy eigenvalues and eigenfunctions as power series in λ :

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \dots$$

$$|\psi_n\rangle = |\psi_n^{(0)}\rangle + \lambda |\psi_n^{(1)}\rangle + \lambda^2 |\psi_n^{(2)}\rangle + \dots$$

where:

- $E_n^{(0)}$ and $|\psi_n^{(0)}\rangle$ are the **unperturbed** energy and state
- $E_n^{(1)}$ is the **first-order correction** to the energy



Let $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ (momentum transfer). Then:

$$f(\theta) = -\frac{2m}{4\pi\hbar^2} \tilde{V}(\mathbf{q})$$

where $\tilde{V}(\mathbf{q})$ is the Fourier transform of the potential.

Worked Example: Yukawa Potential

Given Data: $V(r) = -V_0 \frac{e^{-\mu r}}{\mu r}$

Solution:

The Fourier transform gives:

$$f(\theta) = \frac{2mV_0}{\hbar^2} \frac{1}{\mu^2 + q^2}$$

where $q = |\mathbf{k} - \mathbf{k}'| = 2k \sin(\theta/2)$.

Physical Interpretation

Feature	Meaning
Forward scattering	$q = 0 \rightarrow f \propto 1/\mu^2$
Rutherford scattering	$\mu \rightarrow 0 \rightarrow f \propto 1/q^2$ (Coulomb)
Short-range potential	Larger $\mu \rightarrow$ smaller scattering cross section

Electron Spin

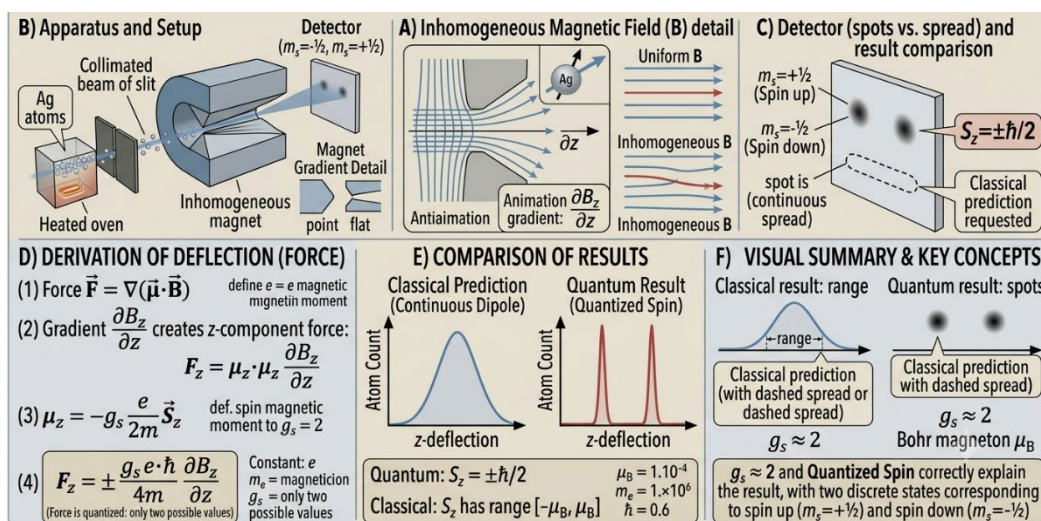
The Concept of Spin

In 1925, Samuel Goudsmit and George Uhlenbeck proposed that the electron possesses an intrinsic spin angular momentum $\mathbf{S}$, independent of its orbital motion. This spin has the following properties:

- The magnitude of spin is $S = \sqrt{s(s+1)} \hbar$ with $s = \frac{1}{2}$.
- The z-component can take two values: $S_z = m_s \hbar$ with $m_s = +\frac{1}{2}$ (spin up) or $m_s = -\frac{1}{2}$ (spin down).

The electron also has an intrinsic magnetic moment μ_s associated with its spin:

$$\mu_z \approx -2 \frac{e}{2m} S_z = -2\mu_B m_s, \quad (3.46)$$



Neon	10	$1s^2 2s^2 2p^6$
------	----	------------------

The Periodic Table

The periodic table emerges naturally from the Pauli exclusion principle:

- Noble gases (He, Ne, Ar, Kr, Xe, Rn) have completely filled shells or subshells and are chemically inert.
- Alkali metals (Li, Na, K, Rb, Cs, Fr) have one electron beyond a noble gas configuration and are highly reactive, tending to lose that electron.
- Halogens (F, Cl, Br, I, At) have one electron less than a noble gas configuration and tend to gain an electron.

The similarity in chemical properties within each group arises from identical outer (valence) electron configurations.

X-Ray Spectra and Moseley's Law

When an inner-shell electron is knocked out of a heavy atom (e.g., by a high-energy electron in an X-ray tube), an electron from an outer shell can fall into the vacancy, emitting an X-ray photon. The K series

corresponds to transitions to the $n = 1$ (K) shell; the K_α line is from the $n = 2 \rightarrow n = 1$ transition.

For a target with atomic number Z , the effective nuclear charge seen by the falling electron is approximately $Z - 1$

(due to screening by the remaining K-shell electron). The photon energy is:

$$E_{K_\alpha} \approx (Z - 1)^2 \left(\frac{13.6 \text{ eV}}{1^2} - \frac{13.6 \text{ eV}}{2^2} \right) = (Z - 1)^2 (10.2 \text{ eV}). \quad (3.49)$$

Thus, the frequency follows Moseley's law:

$$f_{K_\alpha} = (2.48 \times 10^{15} \text{ Hz})(Z - 1)^2. \quad (3.50)$$

This relationship allows the identification of elements by their characteristic X-ray spectra and was crucial in the discovery of new elements.

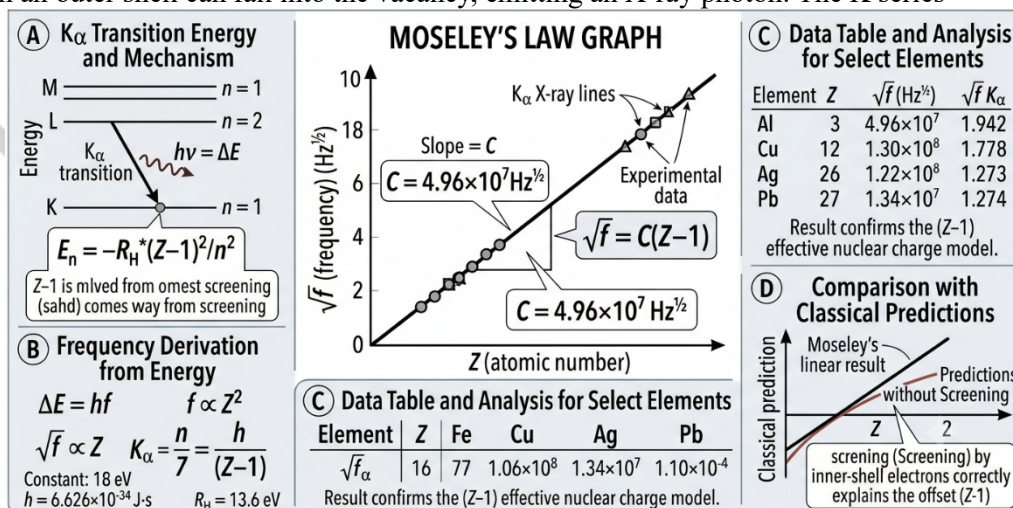
Pair Production and Pair Annihilation

Pair Production

A high-energy photon (gamma ray) passing near a heavy nucleus can spontaneously convert into an electron-positron pair. The positron is the antiparticle of the electron—identical mass but opposite charge.

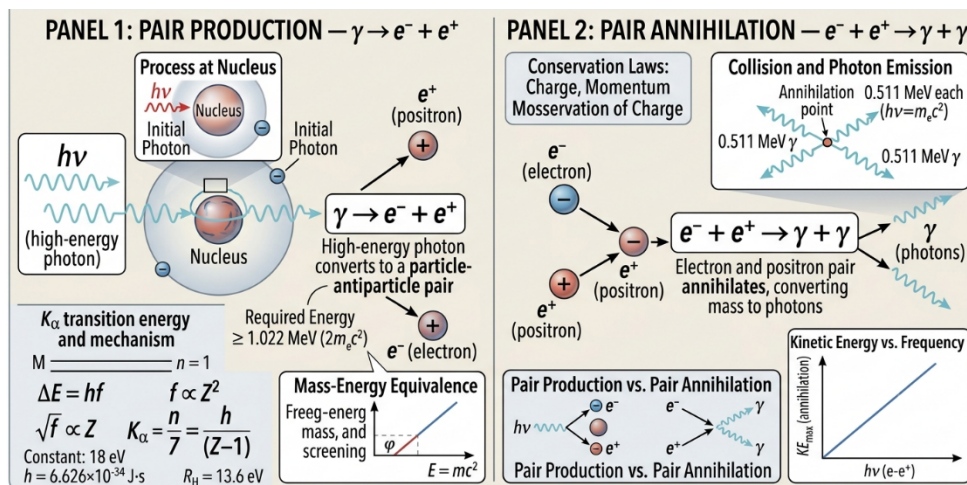
This process, called pair production, requires:

- Conservation of energy: $hf \geq 2m_e c^2 = 1.022 \text{ MeV}$.
- Conservation of momentum: a nearby nucleus recoils to absorb excess momentum.
- Conservation of charge: net charge before (0) equals net charge after ($-e + e = 0$).



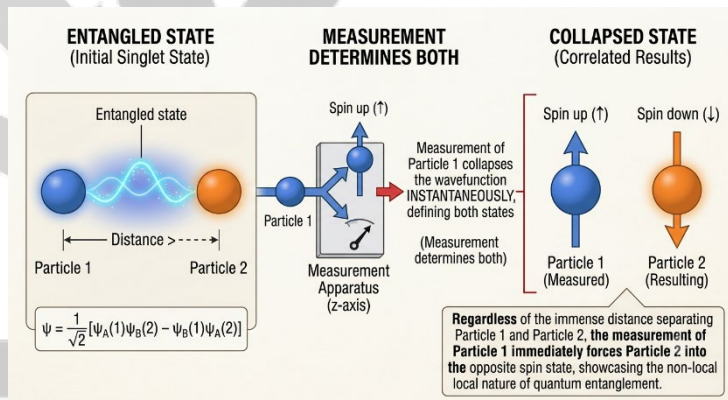
Pair Annihilation

The reverse process occurs when an electron and a positron meet. They annihilate, converting their entire rest mass and kinetic energy into two gamma-ray photons (to conserve momentum). Each photon typically has energy ≥ 0.511 MeV. Annihilation is the ultimate demonstration of Einstein's mass-energy equivalence $E = mc^2$.



Quantum Entanglement

When two particles are in an entangled state, their wave function cannot be written as a product of single-particle wave functions. For two identical fermions (e.g., two electrons), the total wave function must be antisymmetric under exchange:



$$\psi(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2}} [\psi_A(\mathbf{r}_1)\psi_B(\mathbf{r}_2) - \psi_B(\mathbf{r}_1)\psi_A(\mathbf{r}_2)]. \quad (3.51)$$

This wave function has remarkable properties. If we measure the state of particle 1 and find it to be in state A , the wave function collapses, and particle 2 is determined to be in state B —even if the particles are far apart. This "spooky action at a distance" (Einstein's phrase) has been experimentally verified and is a key resource for emerging technologies such as quantum computing and quantum cryptography.

Quantum entanglement is one of the most profound and counterintuitive aspects of quantum mechanics. It describes correlations between particles that cannot be explained by classical physics.

The EPR Paradox

The Einstein-Podolsky-Rosen (EPR) paradox (1935) argued that quantum mechanics is incomplete.

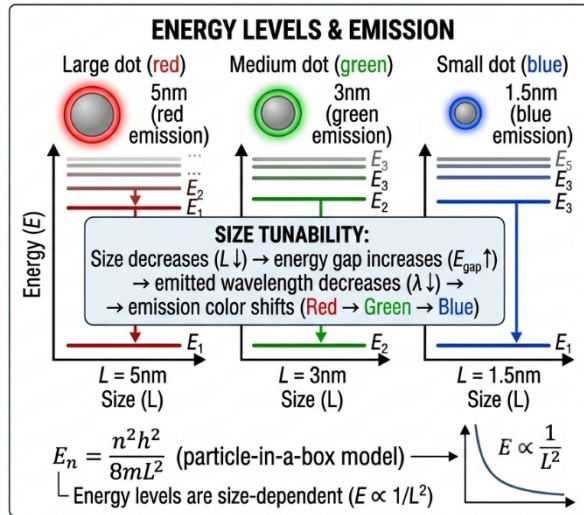
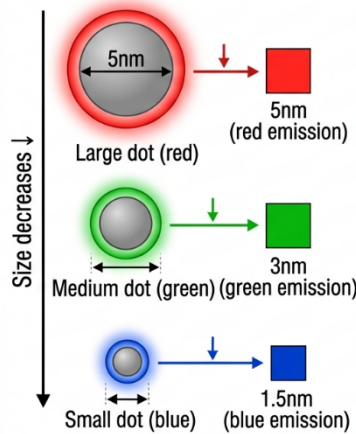
The EPR Argument

1. Consider two entangled particles (e.g., two spin-1/2 particles in a singlet state)
2. Measure the spin of particle 1 along the z-axis $\rightarrow$ you instantly know the spin of particle 2

Quantum Dots and Nanocrystals

When semiconductor crystals are reduced to nanometer dimensions, electrons become confined in all three directions—a "quantum box." The energy levels depend strongly on particle size (Eq. 3.33 with L as the particle diameter). Smaller crystals have larger energy gaps, shifting their absorption and emission colors toward the blue. This size-tunable fluorescence is exploited in biological imaging, quantum dot displays, and solar cells.

QUANTUM DOT SIZE-TUNABLE FLUORESCENCE



Quantum Computing

Quantum computing harnesses quantum phenomena (superposition and entanglement) to perform calculations that would be impossible for classical computers.

The Qubit

A **qubit** (quantum bit) is the quantum analog of a classical bit.

Qubit State

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

where α and β are complex numbers satisfying:

$$|\alpha|^2 + |\beta|^2 = 1$$

Bloch Sphere Representation

State	Bloch Sphere Position	Meaning
$ 0\rangle$	North pole	Qubit in state 0
$ 1\rangle$	South pole	Qubit in state 1
$ \psi\rangle = \frac{1}{\sqrt{2}}(0\rangle + 1\rangle)$	Equator	Equal superposition

Quantum Gates

Gate	Symbol	Matrix	Action
Hadamard (H)	H	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	Creates superposition
Pauli-X	X	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	NOT gate
Pauli-Z	Z	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Phase flip

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \frac{L}{\sqrt{12}} \approx 0.289L$$

And the momentum uncertainty is approximately:

$$\Delta p_x \approx \frac{\hbar}{2\Delta x} \approx \frac{\hbar}{0.577L} \approx \frac{1.732\hbar}{L}$$

Then:

$$E_{\text{est}} \approx \frac{(\Delta p_x)^2}{2m} \approx \frac{3\hbar^2}{2mL^2}$$

$$\frac{E_1}{E_{\text{est}}} \approx \frac{\pi^2 \hbar^2 / (2mL^2)}{3\hbar^2 / (2mL^2)} = \frac{\pi^2}{3} \approx 3.29$$

This is a much more reasonable comparison. The uncertainty principle estimate is within a factor of about 3 of the exact result.

Final Comparison:

Using the simple estimate $\Delta x \approx L$:

$$E_{\text{est}} \approx \frac{\hbar^2}{8mL^2} \approx \frac{h^2}{32\pi^2 mL^2}$$

$$E_1 = \frac{h^2}{8mL^2}$$

$$\frac{E_1}{E_{\text{est}}} = 4\pi^2 \approx 39.5$$

Using the more accurate estimate $\Delta x = L/\sqrt{12}$:

$$E_{\text{est}} \approx \frac{3\hbar^2}{2mL^2}$$

$$\frac{E_1}{E_{\text{est}}} = \frac{\pi^2}{3} \approx 3.29$$

Physical Interpretation:

The uncertainty principle predicts that a confined particle cannot have zero energy. The ground-state (zero-point) energy arises from the inherent position-momentum uncertainty. The exact result from the Schrödinger equation gives $E_1 = h^2/(8mL^2)$. The uncertainty principle estimate gives the correct order of magnitude and demonstrates that the energy scales as $\hbar^2/(mL^2)$, but it cannot give the exact numerical factor. The exact factor $\pi^2/2 \approx 4.93$ is of the same order as the uncertainty principle estimate when using the proper position uncertainty.

This problem illustrates two important points:

1. The uncertainty principle is a fundamental consequence of wave-particle duality.
2. While the uncertainty principle provides a useful estimate, the Schrödinger equation is required for precise predictions.

Quantum Mechanics: One-Liners

The Particle Nature of Light

1. Planck's quantum hypothesis (1900): Energy of atomic oscillators is quantized as $E_n = nhf$, where $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$.
2. Einstein (1905) proposed that light consists of discrete packets called photons, each with energy $E = hf = hc/\lambda$.
3. A photon has zero rest mass, no electric charge, and travels at speed c in vacuum.
4. Photon momentum: $p = E/c = hf/c = h/\lambda$.
5. The electronvolt (eV) is $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$.
6. Planck's constant in $\text{eV} \cdot \text{s}$ is $h = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$.
7. The product $hc = 1240 \text{ eV} \cdot \text{nm}$.
8. Photon energy in eV: $E(\text{eV}) = 1240/\lambda(\text{nm})$.

The Photoelectric Effect

9. The photoelectric effect is the emission of electrons from a metal surface when illuminated by sufficiently high-frequency light.
10. Each metal has a threshold frequency f_0 below which no electrons are emitted regardless of intensity.
11. The maximum kinetic energy of photoelectrons increases linearly with frequency and is independent of intensity.
12. Increasing light intensity increases the number of photoelectrons but not their maximum kinetic energy.
13. Photoelectric emission is instantaneous (less than 10^{-9} s), even at very low intensities.
14. Classical wave theory fails to explain the photoelectric effect because it predicts intensity-dependent kinetic energy and time delays.
15. Einstein's photoelectric equation: $hf = K_{\max} + \phi$, where ϕ is the work function.
16. The work function ϕ is the minimum energy required to liberate an electron from the metal.
17. The threshold frequency is $f_0 = \phi/h$.
18. The stopping potential satisfies $K_{\max} = eV_0$.
19. A plot of stopping potential vs. frequency has slope h/e and intercept $-\phi/e$.
20. Millikan (1916) experimentally determined Planck's constant from photoelectric measurements.

Compton Scattering

21. Compton scattering (1923) demonstrated that photons possess momentum.
22. The Compton shift formula: $\Delta\lambda = \lambda' - \lambda = (h/m_e c)(1 - \cos \theta)$.
23. The Compton wavelength of the electron is $\lambda_c = h/(m_e c) = 2.426 \text{ pm}$.
24. Compton scattering provides direct evidence for the particle nature of electromagnetic radiation.
25. The unshifted peak in Compton scattering arises from scattering by tightly bound electrons (scattering off the entire atom).
26. Compton received the Nobel Prize in 1927 for this discovery.

Wave-Particle Duality

27. Light exhibits both wave properties (interference, diffraction) and particle properties (photoelectric effect, Compton scattering).

90. The Pauli exclusion principle applies to all fermions (half-integer spin particles).
91. The maximum number of electrons in a subshell is $2(2\ell + 1)$: s=2, p=6, d=10, f=14.
92. Noble gases have completely filled shells and are chemically inert.
93. Moseley's law: $f_{K\alpha} = (2.48 \times 10^{15} \text{ Hz})(Z - 1)^2$.

Pair Production and Annihilation

94. Pair production: a high-energy photon converts into an electron-positron pair.
95. Pair production requires photon energy $\geq 2m_e c^2 = 1.022 \text{ MeV}$.
96. Pair annihilation: an electron and positron annihilate, producing two gamma-ray photons.

Quantum Entanglement

97. Quantum entanglement describes correlations between particles that cannot be explained classically.
98. The singlet state: $|\psi\rangle = (1/\sqrt{2})(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2)$.
99. Bell's inequality: $|E(a, b) - E(a, b')| + |E(a', b) + E(a', b')| \leq 2$.
100. Quantum mechanics violates Bell's inequality; experimental tests (Aspect, 1982) confirmed quantum predictions.
101. The 2022 Nobel Prize was awarded to Aspect, Clauser, and Zeilinger for entanglement experiments.
102. Applications of entanglement include quantum computing, quantum cryptography, and quantum teleportation.
103. The no-cloning theorem states that an arbitrary unknown quantum state cannot be cloned.

Quantum Computing

104. A qubit is the quantum analog of a classical bit: $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$, with $|\alpha|^2 + |\beta|^2 = 1$.
105. The Bloch sphere represents qubit states geometrically.
106. Quantum gates include Hadamard (superposition), Pauli-X (NOT), Pauli-Z (phase flip), and CNOT (entangling).
107. Quantum parallelism allows evaluating a function for all inputs simultaneously.
108. Shor's algorithm factors integers exponentially faster than classical algorithms, threatening RSA encryption.
109. Grover's algorithm searches unstructured databases quadratically faster.
110. Key challenges in quantum computing: decoherence, error correction, scaling, and cryogenics.

Practice MCQs

1. Planck's quantum hypothesis states that:

- A) Energy is continuous
- B) Energy is emitted in discrete quanta $E = nhf$
- C) Light is a wave only
- D) Matter is continuous

Correct Answer: B

2. Planck's constant h is approximately:

- A) $6.626 \times 10^{-34} \text{ J} \cdot \text{s}$
- B) $6.626 \times 10^{-34} \text{ J}$
- C) $3.00 \times 10^8 \text{ J} \cdot \text{s}$

- D) $1.055 \times 10^{-34} \text{ J} \cdot \text{s}$

Correct Answer: A

3. A photon has energy:

- A) $E = hf$
- B) $E = hc/\lambda$
- C) Both A and B
- D) $E = mc^2$

Correct Answer: C

4. Photon momentum is:

- A) $p = hf$

B) $p = h/\lambda$

C) $p = E/c$

D) Both B and C

Correct Answer: D

5. The electronvolt (eV) is:

A) $1.602 \times 10^{-19} \text{ J}$

B) $1.602 \times 10^{-19} \text{ J} \cdot \text{s}$

C) $1.602 \times 10^{19} \text{ J}$

D) $1.602 \times 10^{-19} \text{ C}$

Correct Answer: A

6. hc in $\text{eV} \cdot \text{nm}$ is approximately:

A) 1240

B) 124

C) 12.4

D) 12400

Correct Answer: A

7. The photoelectric effect demonstrates:

A) The wave nature of light

B) The particle nature of light

C) The wave nature of matter

D) The particle nature of matter

Correct Answer: B

8. Einstein's photoelectric equation is:

A) $hf = K_{\max} + \phi$

B) $hf = K_{\max} - \phi$

C) $hf = \phi - K_{\max}$

D) $hf = K_{\max}$

Correct Answer: A

9. The work function ϕ is:

A) The maximum kinetic energy of electrons

B) The minimum energy to liberate an electron

C) The photon energy

D) The stopping potential

Correct Answer: B

10. The threshold frequency is:

A) $f_0 = \phi/h$

B) $f_0 = h/\phi$

C) $f_0 = \phi h$

D) $f_0 = 1/\phi$

Correct Answer: A

11. The stopping potential satisfies:

A) $eV_0 = hf - \phi$

B) $eV_0 = hf + \phi$

C) $eV_0 = \phi - hf$

D) $eV_0 = hf$

Correct Answer: A

12. A plot of stopping potential vs. frequency has slope:

A) h

B) h/e

C) e/h

D) ϕ

Correct Answer: B

13. Compton scattering demonstrates:

A) The wave nature of light

B) The particle nature of light (momentum conservation)

C) The wave nature of matter

D) The particle nature of matter

Correct Answer: B

14. The Compton shift formula is:

A) $\Delta\lambda = (h/mc)(1 - \cos \theta)$

B) $\Delta\lambda = (h/mc)(1 + \cos \theta)$

C) $\Delta\lambda = (mc/h)(1 - \cos \theta)$

D) $\Delta\lambda = (h/mc)\sin \theta$

Correct Answer: A

15. The Compton wavelength of the electron is:

A) $2.426 \times 10^{-12} \text{ m}$

B) $2.426 \times 10^{-10} \text{ m}$

C) $2.426 \times 10^{-15} \text{ m}$

D) $2.426 \times 10^{-9} \text{ m}$

Correct Answer: A

16. de Broglie's hypothesis states:

A) Light is a particle

B) Particles have wave-like properties: $\lambda = h/p$

C) Matter is continuous

D) Energy is quantized

Correct Answer: B

17. The de Broglie wavelength of an electron accelerated through voltage V is:

A) $\lambda = h/\sqrt{2meV}$

B) $\lambda = h\sqrt{2meV}$



Part VII: Mathematical Foundations



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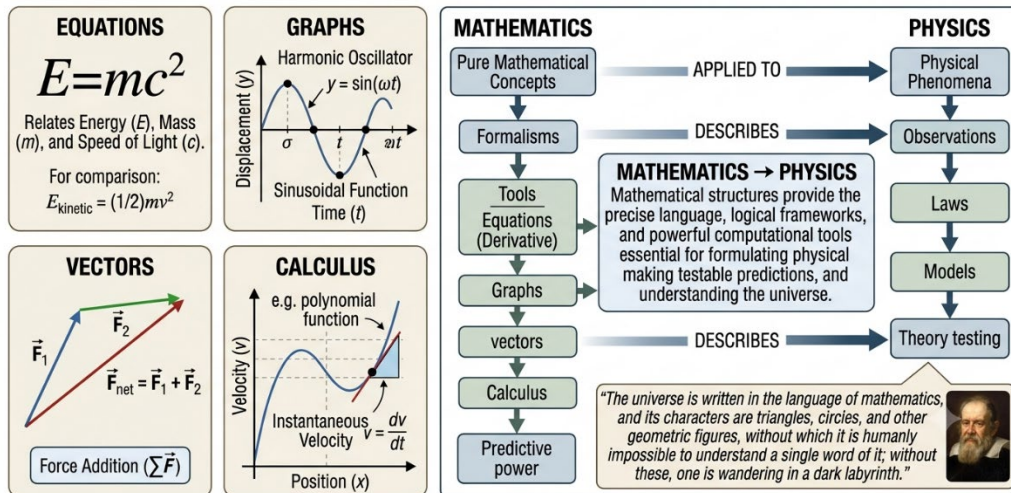


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Chapter 20: Applied And Mathematical Physics

Introduction

Physics is fundamentally a quantitative science. The relationship between physical phenomena and mathematical description is so profound that the great physicist Galileo Galilei famously stated: *"The universe is written in the language of mathematics."* Applied and Mathematical Physics serves as the bridge between abstract mathematical structures and tangible physical reality. This chapter develops the essential mathematical tools that physicists employ to model, analyze, and predict physical behavior, from the motion of planets to the behavior of subatomic particles.



This chapter is structured to build your understanding progressively. We begin with the most fundamental tool—dimensional analysis—and advance through vectors, calculus, differential equations, complex numbers, and finally vector calculus. Each section provides clear definitions, step-by-step derivations, physical interpretations, and worked examples. By the end, you should be able to confidently apply these mathematical methods to any physics problem you encounter.

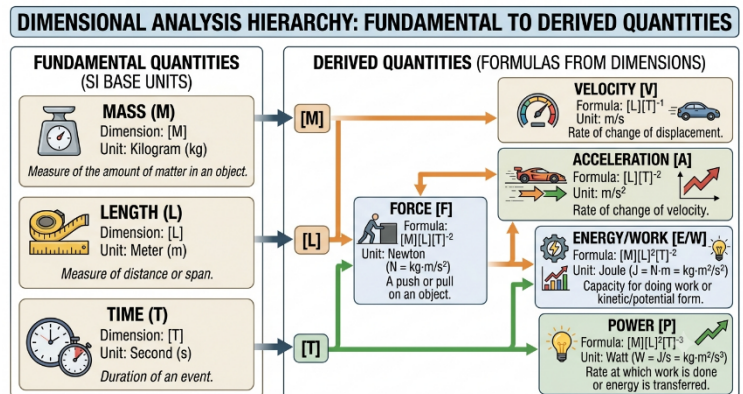
DIMENSIONAL ANALYSIS

Fundamental and Derived Quantities

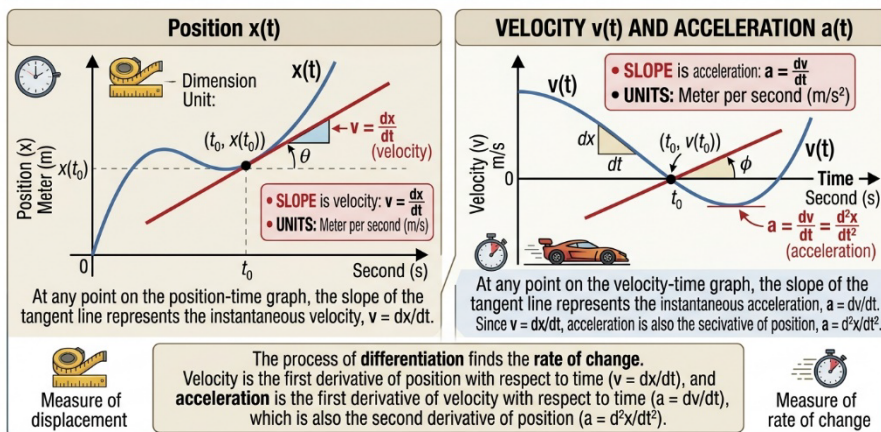
Every physical quantity can be expressed in terms of a small set of **fundamental quantities**. The International System (SI) recognizes seven base quantities, but for most of mechanics and electromagnetism, we need only three:

Fundamental Quantity	Symbol	SI Unit
Mass	M	kilogram (kg)
Length	L	metre (m)
Time	T	second (s)

Derived quantities are expressed as combinations of these fundamentals. For example:



DIFFERENTIATION IN PHYSICS: POSITION AND VELOCITY GRAPHS



Rules for Differentiation (Essential for Physics)

Rule	Mathematical Form	Physical Example
Power rule	$\frac{d}{dx}(x^n) = nx^{n-1}$	Position $x = t^2 \rightarrow v = 2t$
Product rule	$\frac{d}{dt}(fg) = f \frac{dg}{dt} + g \frac{df}{dt}$	Force from momentum $p = mv$
Chain rule	$\frac{df}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt}$	Acceleration in terms of position

Worked Example

The position of a particle is given by $x(t) = 4t^3 - 2t^2 + 5t - 3$ (x in metres, t in seconds). Find:

- Velocity as a function of time
- Acceleration as a function of time
- Velocity at $t = 2$ s

Solution:

- $v(t) = \frac{dx}{dt} = 12t^2 - 4t + 5$ m/s
- $a(t) = \frac{dv}{dt} = 24t - 4$ m/s²
- $v(2) = 12(4) - 4(2) + 5 = 48 - 8 + 5 = 45$ m/s

Partial Derivatives

When a function depends on multiple variables, the **partial derivative** measures the rate of change with respect to one variable while holding others constant.

Notation: $\frac{\partial f}{\partial x}$ means differentiate f with respect to x, treating y, z, etc. as constants.

Physical Applications:

- Pressure as a function of temperature and volume
- Potential energy surfaces in molecular physics
- Thermodynamic state functions

Example 3.2: For $f(x, y) = 3x^2y + 2y^3$:

$$\frac{\partial f}{\partial x} = 6xy \frac{\partial f}{\partial y} = 3x^2 + 6y^2$$

LC circuit	$L \frac{d^2 Q}{dt^2} + \frac{Q}{C} = 0$	$Q(t) = Q_0 \cos(\omega t), \omega = 1/\sqrt{LC}$
Wave equation	$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$	$y(x, t) = f(x - vt) + g(x + vt)$
Schrödinger equation	$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + V\psi = E\psi$	Depends on potential V

COMPLEX NUMBERS IN PHYSICS

Definition and Basic

Operations

A complex number is written as $z = a + ib$, where $i = \sqrt{-1}$, a is the real part, and b is the imaginary part.

Addition: $(a + ib) + (c + id) = (a + c) + i(b + d)$

Multiplication: $(a + ib)(c + id) = (ac - bd) + i(ad + bc)$

Complex conjugate: $z^* = a - ib$

Magnitude: $|z| =$

$$\sqrt{a^2 + b^2} = \sqrt{zz^*}$$

Exponential Form (Euler's Formula)

Euler's formula is one of the most important equations in applied mathematics:

$$e^{i\theta} = \cos \theta + i \sin \theta$$

Therefore:

$$\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \quad \sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

A complex number can be written in polar form:

$$z = re^{i\theta} = r(\cos \theta + i \sin \theta)$$

where $r = |z|$ and $\theta = \tan^{-1}(b/a)$.

Physical Applications of Complex Numbers

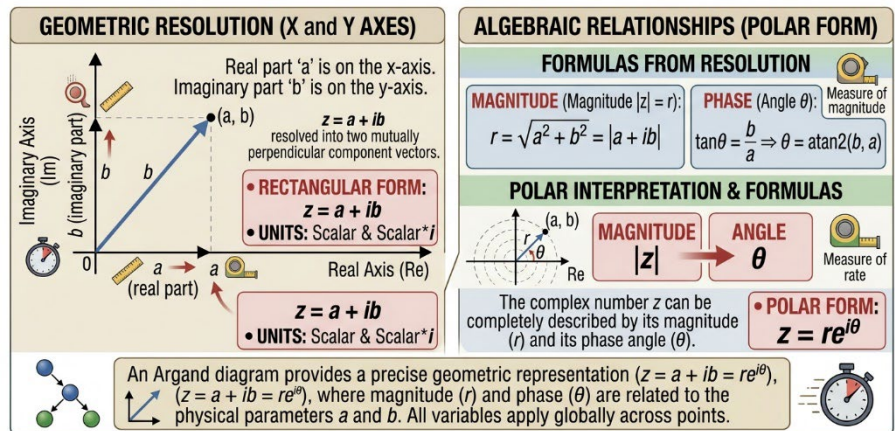
1. Representing Oscillations

Instead of writing $x(t) = A \cos(\omega t + \phi)$, we can use:

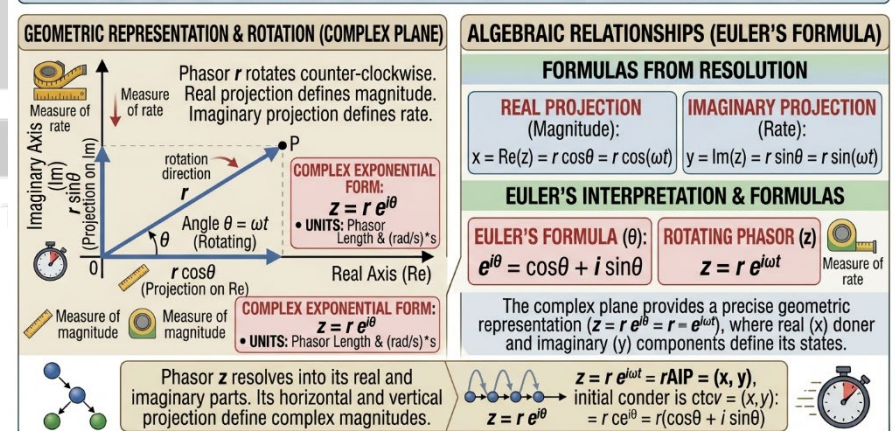
$$x(t) = \text{Re}[Ae^{i(\omega t + \phi)}] = \text{Re}[Ae^{i\phi} e^{i\omega t}]$$

Let $\tilde{A} = Ae^{i\phi}$ (complex amplitude). Then $x(t) = \text{Re}[\tilde{A}e^{i\omega t}]$.

ARGAND DIAGRAM (COMPLEX PLANE): GEOMETRIC REPRESENTATION



PHASOR REPRESENTATION IN COMPLEX PLANE



$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos(n \omega_0 t) dt \quad (n = 0, 1, 2, \dots)$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin(n \omega_0 t) dt \quad (n = 1, 2, 3, \dots)$$

Physical Interpretation

Coefficient	Physical Meaning
$a_0/2$	Average value (DC component)
a_n	Amplitude of cosine component at frequency $n\omega_0$
b_n	Amplitude of sine component at frequency $n\omega_0$
$\sqrt{a_n^2 + b_n^2}$	Total amplitude of the n -th harmonic

Worked Example: Square Wave

Given Data

A square wave with period $T = 2\pi$ and amplitude A :

$$f(t) = \begin{cases} +A, & 0 < t < \pi \\ -A, & \pi < t < 2\pi \end{cases}$$

Solution

Step 1: Calculate a_0

$$a_0 = \frac{2}{T} \int_{-T/2}^{T/2} f(t) dt$$

$$= \frac{1}{\pi} \left[\int_0^{\pi} A dt + \int_{\pi}^{2\pi} (-A) dt \right] = 0$$

Step 2: Calculate a_n

Since the function is odd, all $a_n = 0$.

Step 3: Calculate b_n

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin(n \omega_0 t) dt$$

$$b_n = \frac{1}{\pi} \left[\int_0^{\pi} A \sin(nt) dt + \int_{\pi}^{2\pi} (-A) \sin(nt) dt \right]$$

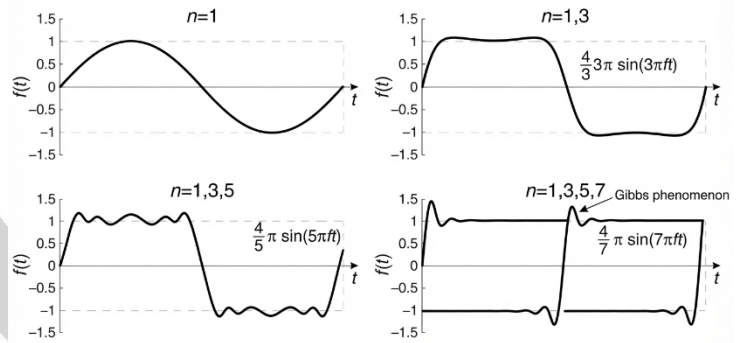
$$b_n = \frac{2A}{n\pi} (1 - \cos(n\pi)) = \begin{cases} \frac{4A}{n\pi}, & n \text{ odd} \\ 0, & n \text{ even} \end{cases}$$

Step 4: Write the Fourier Series

$$f(t) = \frac{4A}{\pi} \left[\sin t + \frac{1}{3} \sin 3t + \frac{1}{5} \sin 5t + \dots \right]$$

Physical Interpretation

Feature	Description
Only odd harmonics	Square wave contains only odd multiples of fundamental frequency



Amplitude decay	Amplitude decays as $1/n$
Gibbs phenomenon	Overshoot near discontinuities (about 9% of the jump)

Applications of Fourier Series

Application	Description
Signal processing	Analyzing and filtering signals
AC circuit analysis	Analyzing non-sinusoidal waveforms
Quantum mechanics	Expanding wavefunctions in basis sets
Heat conduction	Solving the heat equation
Vibrations	Analyzing modes of oscillation

THE FOURIER TRANSFORM

While Fourier series apply to periodic functions, the **Fourier transform** extends the idea to non-periodic functions by taking the limit as the period goes to infinity.

Definition

The Fourier Transform

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

The Inverse Fourier Transform

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

Key Properties

Property	Formula	Description
Linearity	$\mathcal{F}[af + bg] = aF + bG$	Transform of sum = sum of transforms
Time shift	$\mathcal{F}[f(t - t_0)] = e^{-i\omega t_0} F(\omega)$	Time shift $\rightarrow$ phase shift
Frequency shift	$\mathcal{F}[e^{i\omega_0 t} f(t)] = F(\omega - \omega_0)$	Modulation $\rightarrow$ frequency shift
Scaling	$\mathcal{F}[f(at)] = \frac{1}{a} F(\omega/a)$	Time scaling $\rightarrow$ inverse frequency scaling
Derivative	$\mathcal{F}[f'(t)] = i\omega F(\omega)$	Differentiation $\rightarrow$ multiplication by $i\omega$
Convolution	$\mathcal{F}[f * g] = F(\omega)G(\omega)$	Convolution $\rightarrow$ multiplication

Worked Example: Fourier Transform of a Gaussian

Given Data

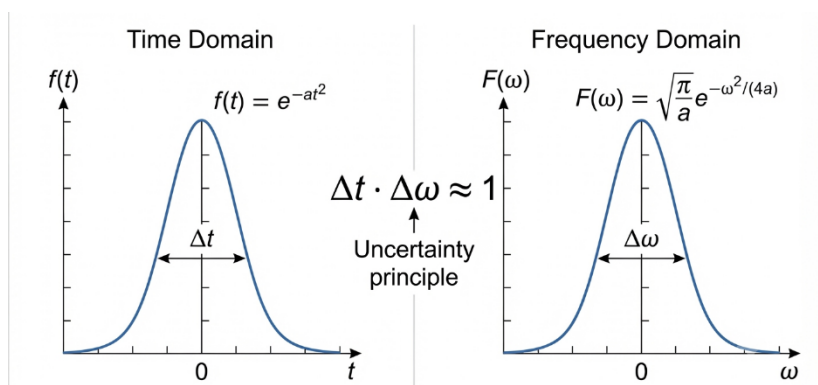
$$f(t) = e^{-at^2} (a > 0)$$

Solution

$$F(\omega) = \int_{-\infty}^{\infty} e^{-at^2} e^{-i\omega t} dt$$

Completing the square:

$$F(\omega) = \sqrt{\frac{\pi}{a}} e^{-\omega^2/(4a)}$$



Trapezoidal Rule:

$$\int_a^b f(x)dx \approx \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

Physical application: Computing work from experimental force-displacement data, or distance from velocity-time data.

Iterative Methods: Newton-Raphson

To solve $f(x) = 0$:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Physical application: Finding equilibrium positions where net force is zero, or energy minima in molecular dynamics.

NUMERICAL METHODS (Extended)

(Insert after your existing Numerical Methods section)

Numerical methods are essential when analytical solutions are impossible. Modern physics relies heavily on computational techniques for solving complex problems.

Root Finding Methods

Method	Description	Convergence
Bisection method	Iteratively narrow down interval containing root	Linear
Newton-Raphson	$x_{n+1} = x_n - f(x_n)/f'(x_n)$	Quadratic
Secant method	Approximation using two previous points	Superlinear

Worked Example: Newton-Raphson

Given: Find the root of $f(x) = x^2 - 2$ (i.e., $\sqrt{2}$).

Solution:

$$x_{n+1} = x_n - \frac{x_n^2 - 2}{2x_n} = \frac{x_n}{2} + \frac{1}{x_n}$$

Starting with $x_0 = 1$:

n	x_n	Error
0	1	0.4142
1	1.5	0.0858
2	1.4167	0.0025
3	1.4142	2.1×10^{-6}

Key Insight: Newton-Raphson converges quadratically—error roughly squares each iteration!

Numerical Integration

Method	Formula	Error
Trapezoidal rule	$I \approx \frac{h}{2} [f_0 + 2f_1 + \dots + 2f_{n-1} + f_n]$	$O(h^2)$
Simpson's rule	$I \approx \frac{h}{3} [f_0 + 4f_1 + 2f_2 + 4f_3 + \dots + f_n]$	$O(h^4)$
Gaussian quadrature	$I \approx \sum_i w_i f(x_i)$	Very accurate

Comparison of Methods

Method	Accuracy	When to Use
Trapezoidal	Low	Simple functions, rough estimates
Simpson's	Good	Smooth functions
Gaussian	Excellent	High accuracy needed

NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS

Euler's Method

The simplest method for solving $\frac{dy}{dx} = f(x, y)$:

$$y_{n+1} = y_n + hf(x_n, y_n)$$

where h is the step size.

Runge-Kutta Methods

Method	Order	Accuracy	Formula
Euler	1st	$O(h)$	$y_{n+1} = y_n + hf(x_n, y_n)$
RK2 (Heun)	2nd	$O(h^2)$	$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2)$
RK4	4th	$O(h^4)$	Most widely used

RK4 Algorithm

For $\frac{dy}{dx} = f(x, y)$:

$$k_1 = f(x_n, y_n)$$

$$k_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right)$$

$$k_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right)$$

$$k_4 = f(x_n + h, y_n + hk_3)$$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Worked Example: RK4 for Simple Harmonic Oscillator

Given Data

$$\frac{d^2x}{dt^2} = -\omega^2x$$

Convert to first-order system:

$$\frac{dx}{dt} = v, \frac{dv}{dt} = -\omega^2x$$

Solution

Using RK4 with $\omega = 1$, $x(0) = 1$, $v(0) = 0$, $h = 0.1$:

Time (t)	Analytical $x = \cos t$	Numerical (RK4)	Error
0.0	1.00000	1.00000	0
0.1	0.99500	0.99500	$<10^{-8}$
0.5	0.87758	0.87758	$<10^{-7}$
1.0	0.54030	0.54030	$<10^{-6}$

Key Insight: RK4 is extremely accurate—errors grow slowly even for large step sizes.

$$\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$$

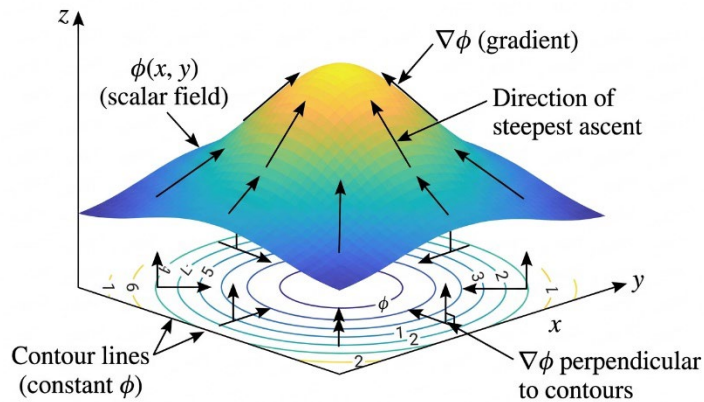
The Gradient (grad)

The gradient of a scalar field $\phi(x,y,z)$ is a vector:

$$\nabla \phi = \frac{\partial \phi}{\partial x} \hat{i} + \frac{\partial \phi}{\partial y} \hat{j} + \frac{\partial \phi}{\partial z} \hat{k}$$

Physical interpretation: The gradient points in the direction of steepest increase of ϕ . Its magnitude gives the rate of increase. In physics:

- Electric field $\vec{E} = -\nabla V$ (minus sign: field points downhill in potential)
- Force from potential energy $\vec{F} = -\nabla U$

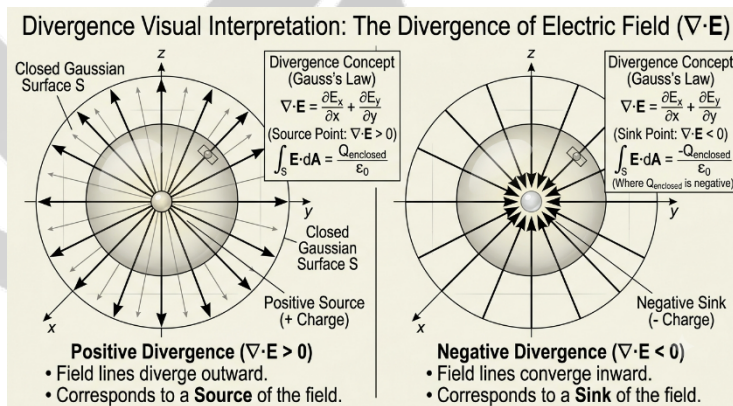


The Divergence (div)

The divergence of a vector field $\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$ is a scalar:

$$\nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Physical interpretation: Divergence measures the net "outflow" of a vector field from a point. Positive divergence indicates a source; negative divergence indicates a sink.



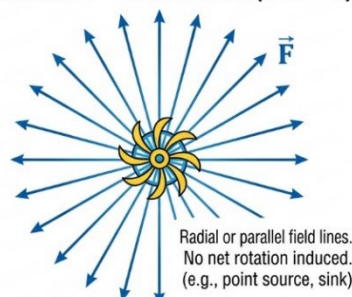
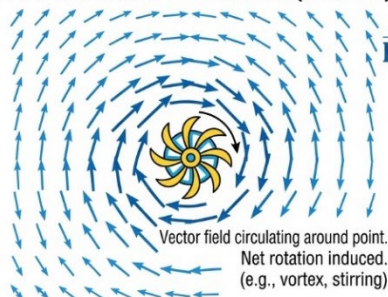
Gauss's Law (Electricity): $\nabla \cdot \vec{E} =$

$\frac{\rho}{\epsilon_0}$ — The divergence of the electric field equals the charge density divided by ϵ_0 . This is the differential form of Gauss's law.

The Curl

The curl of a vector field is a vector:

1. NON-ZERO CURL — ROTATION ($\nabla \times \vec{F} \neq 0$)
2. ZERO CURL — IRROTATIONAL ($\nabla \times \vec{F} = 0$)



FARADAY'S LAW EXAMPLE ($\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$)
Changing Magnetic Field ($\vec{B}$) induces a Circulating Electric Field ($\vec{E}$).



- $[T] = T$
- $[L] = L$
- $[m] = M$
- $[g] = LT^{-2}$

Substituting:

$$T = L^a \cdot M^b \cdot (LT^{-2})^c$$

$$T = L^{a+c} \cdot M^b \cdot T^{-2c}$$

Equating powers:

Dimension	Equation	Solution
M	$b = 0$	Mass does not appear!
T	$1 = -2c$	$c = -\frac{1}{2}$
L	$0 = a + c$	$a = -c = \frac{1}{2}$

Therefore:

$$T = kL^{1/2}g^{-1/2} = k\sqrt{\frac{L}{g}}$$

The actual derivation using Newton's laws gives $k = 2\pi$, so:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Problem 2

Check the dimensional consistency of the equation $v^2 = u^2 + 2gh$ for an object falling under gravity.

Dimensions of each term:

Term	Dimensions
v^2	$(LT^{-1})^2 = L^2T^{-2}$
u^2	$(LT^{-1})^2 = L^2T^{-2}$
$2gh$	$(LT^{-2})(L) = L^2T^{-2}$

Conclusion: All terms have the same dimensions (L^2T^{-2}). The equation is **dimensionally consistent**.

Note: This does NOT prove the equation is correct — it only shows it is not obviously wrong. The factor 2 would need to be derived from physics principles.

Problem 3

The force F between two charges q_1 and q_2 separated by distance r is given by $F = k\frac{q_1q_2}{r^2}$. Find the dimensions of k .

Given equation:

$$F = k\frac{q_1q_2}{r^2} \Rightarrow k = \frac{Fr^2}{q_1q_2}$$

Known dimensions:

- $[F] = MLT^{-2}$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 5 & -2 & 0 \end{vmatrix}$$

Determinant expansion:

$$\vec{\tau} = \hat{i}[(4)(0) - (0)(-2)] - \hat{j}[(3)(0) - (0)(5)] + \hat{k}[(3)(-2) - (4)(5)]$$

$$\vec{\tau} = \hat{i}(0) - \hat{j}(0) + \hat{k}(-6 - 20)$$

$$\vec{\tau} = -26\hat{k} \text{ N} \cdot \text{m}$$

$$\boxed{\vec{\tau} = -26\hat{k} \text{ N} \cdot \text{m}}$$

Physical interpretation: The torque is 26 N·m in the **negative z-direction** (clockwise when viewed from above). The magnitude is 26 N·m, and the direction is perpendicular to the plane containing $\vec{r}$ and $\vec{F}$.

CALCULUS IN PHYSICS

Problem 7

The position of a particle is $x(t) = 2t^3 - 9t^2 + 12t + 5$. Find (a) velocity, (b) acceleration, (c) time when velocity is zero.

(a) Velocity:

Velocity is the first derivative of position:

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}(2t^3 - 9t^2 + 12t + 5)$$

Using the power rule: $\frac{d}{dt}(t^n) = nt^{n-1}$

$$v(t) = 6t^2 - 18t + 12$$

$$\boxed{v(t) = 6t^2 - 18t + 12 \text{ m/s}}$$

(b) Acceleration:

Acceleration is the derivative of velocity (or second derivative of position):

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}(6t^2 - 18t + 12)$$

$$a(t) = 12t - 18$$

$$\boxed{a(t) = 12t - 18 \text{ m/s}^2}$$

(c) Time when velocity is zero:

Set $v(t) = 0$:

$$6t^2 - 18t + 12 = 0$$

$$6(t^2 - 3t + 2) = 0$$

$$t^2 - 3t + 2 = 0$$

$$(t - 1)(t - 2) = 0$$

$$t = 1 \text{ or } t = 2 \text{ s}$$

$$\boxed{t = 1, 2 \text{ s}}$$

Physical interpretation: The particle is instantaneously at rest at $t = 1 \text{ s}$ and $t = 2 \text{ s}$. At these moments, the particle changes direction. To determine whether it is turning around or just pausing, check the acceleration: $a(1) = -6 \text{ m/s}^2$ (decelerating through zero), $a(2) = 6 \text{ m/s}^2$ (accelerating through zero).

Problem 8

A force varies with position as $F(x) = 4x^2 - 2x + 3$. Calculate the work done from $x = 0$ to $x = 2 \text{ m}$.

Find the impedance of a series RLC circuit with $R = 10 \Omega$, $L = 0.1 \text{ H}$, $C = 100 \mu\text{F}$ at $\omega = 100 \text{ rad/s}$.

Formulas:

- Inductive reactance: $X_L = \omega L$
- Capacitive reactance: $X_C = \frac{1}{\omega C}$
- Impedance: $Z = R + i(X_L - X_C)$

Step 1: Calculate X_L

$$X_L = \omega L = 100 \times 0.1 = 10 \Omega$$

Step 2: Calculate X_C

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 100 \times 10^{-6}} = \frac{1}{100 \times 10^{-4}} = \frac{1}{0.01} = 100 \Omega$$

Step 3: Calculate impedance

$$Z = R + i(X_L - X_C)$$

$$Z = 10 + i(10 - 100)$$

$$Z = 10 - i90$$

$$\boxed{Z = 10 - i90 \Omega}$$

Magnitude and phase (optional):

$$|Z| = \sqrt{10^2 + (-90)^2} = \sqrt{100 + 8100} = \sqrt{8200} = 90.55 \Omega$$

$$\phi = \tan^{-1}\left(\frac{-90}{10}\right) = -83.66^\circ$$

Physical interpretation: The impedance is $10 - i90 \Omega$. The negative imaginary part means the circuit is **capacitive** (current leads voltage). The magnitude of the impedance is about 90.55Ω , and the phase difference between voltage and current is -83.66° .

VECTOR CALCULUS

Problem 13

For the scalar field $\phi = x^2y + yz^2$, find $\nabla\phi$ at the point $(1, 2, 1)$.

Formula: Gradient of a scalar field:

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

Step 1: Compute partial derivatives.

$$\frac{\partial\phi}{\partial x} = \frac{\partial}{\partial x}(x^2y + yz^2) = 2xy$$

$$\frac{\partial\phi}{\partial y} = \frac{\partial}{\partial y}(x^2y + yz^2) = x^2 + z^2$$

$$\frac{\partial\phi}{\partial z} = \frac{\partial}{\partial z}(x^2y + yz^2) = 2yz$$

Step 2: Evaluate at $(1, 2, 1)$.

$$\frac{\partial\phi}{\partial x}|_{(1,2,1)} = 2(1)(2) = 4$$

$$\frac{\partial\phi}{\partial y}|_{(1,2,1)} = (1)^2 + (1)^2 = 1 + 1 = 2$$

$$\frac{\partial\phi}{\partial z}|_{(1,2,1)} = 2(2)(1) = 4$$

Applied And Mathematical Physics: One-Liners

Dimensional Analysis

1. Dimensions are expressed in terms of fundamental quantities: Mass (M), Length (L), Time (T), and others.
2. The SI base quantities relevant to mechanics are mass (kg), length (m), and time (s).
3. Velocity has dimensions LT^{-1} ; acceleration has dimensions LT^{-2} .
4. Force has dimensions MLT^{-2} ; energy and work have dimensions ML^2T^{-2} .
5. Every valid physical equation must be dimensionally consistent—both sides must have the same dimensions.
6. Dimensional consistency is necessary but not sufficient for a correct equation.
7. Dimensional analysis can derive relationships when the relevant quantities are known.
8. Dimensional analysis cannot determine dimensionless constants (e.g., 2π , $1/2$).
9. Dimensional analysis cannot distinguish between quantities with the same dimensions (e.g., work and torque).
10. Arguments of trigonometric, exponential, and logarithmic functions must be dimensionless.

Vector Algebra

11. Scalars have magnitude only; vectors have both magnitude and direction.
12. Vectors are added using the triangle law or parallelogram law.
13. Vector subtraction: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$.
14. Components of a vector: $A_x = A \cos \theta$, $A_y = A \sin \theta$.
15. Magnitude: $A = \sqrt{A_x^2 + A_y^2}$; direction: $\tan \theta = A_y/A_x$.
16. Unit vectors $\hat{i}, \hat{j}, \hat{k}$ have magnitude 1 and point along the positive x, y, z axes.
17. Dot product: $\vec{A} \cdot \vec{B} = AB \cos \theta = A_x B_x + A_y B_y + A_z B_z$.
18. Dot product is commutative and yields a scalar.
19. If $\vec{A} \perp \vec{B}$, then $\vec{A} \cdot \vec{B} = 0$.
20. Cross product: $\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$.
21. Cross product is anti-commutative: $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$.
22. If $\vec{A} \parallel \vec{B}$, then $\vec{A} \times \vec{B} = 0$.
23. Cross product magnitude equals the area of the parallelogram formed by the two vectors.
24. Work: $W = \vec{F} \cdot \vec{d}$; only the component of force parallel to displacement does work.
25. Torque: $\vec{\tau} = \vec{r} \times \vec{F}$; direction given by the right-hand rule.
26. Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$.
27. Magnetic force: $\vec{F} = q\vec{v} \times \vec{B}$.

Calculus in Physics

28. Velocity is the derivative of position: $v = dx/dt$.
29. Acceleration is the derivative of velocity: $a = dv/dt = d^2x/dt^2$.
30. Force is the rate of change of momentum: $F = dp/dt$.
31. Current is the rate of flow of charge: $I = dQ/dt$.
32. Power is the rate of energy transfer: $P = dE/dt$.

103. Curl measures the rotation or circulation of a vector field.
104. Faraday's law: $\nabla \times \vec{E} = -\partial \vec{B} / \partial t$.
105. Ampère's law: $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \partial \vec{E} / \partial t$.
106. The Laplacian: $\nabla^2 \phi = \partial^2 \phi / \partial x^2 + \partial^2 \phi / \partial y^2 + \partial^2 \phi / \partial z^2$.
107. Laplace's equation: $\nabla^2 \phi = 0$.
108. Poisson's equation: $\nabla^2 V = -\rho / \epsilon_0$.
109. Wave equation: $\nabla^2 \psi = (1/v^2) \partial^2 \psi / \partial t^2$.
110. Heat equation: $\nabla^2 T = (1/\alpha) \partial T / \partial t$.

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Practice MCQs

1. The dimensions of velocity are:

- A) LT
- B) LT^{-1}
- C) $L^{-1}T$
- D) L^2T^{-1}

Correct Answer: B

2. The dimensions of acceleration are:

- A) LT^{-1}
- B) LT^{-2}
- C) L^2T^{-2}
- D) $L^{-1}T^2$

Correct Answer: B

3. The dimensions of force are:

- A) MLT^{-1}
- B) MLT^{-2}
- C) ML^2T^{-2}
- D) $M^{-1}L^3T^{-2}$

Correct Answer: B

4. The dimensions of energy are:

- A) MLT^{-2}
- B) ML^2T^{-2}
- C) ML^2T^{-1}
- D) $M^{-1}L^2T^{-2}$

Correct Answer: B

5. Dimensional analysis can:

- A) Determine dimensionless constants
- B) Check the correctness of equations
- C) Distinguish between work and torque
- D) Handle trigonometric functions of dimensional quantities

Correct Answer: B

6. Arguments of trigonometric functions must be:

- A) Dimensional
- B) Dimensionless
- C) Either dimensional or dimensionless
- D) Always in radians

Correct Answer: B

7. The dot product of two vectors is:

- A) A vector
- B) A scalar
- C) A tensor
- D) A complex number

Correct Answer: B

8. The dot product of perpendicular vectors is:

- A) Maximum
- B) Zero
- C) Negative
- D) Equal to the product of magnitudes

Correct Answer: B

9. The cross product of two vectors is:

- A) A scalar
- B) A vector
- C) A tensor
- D) A complex number

Correct Answer: B

10. The cross product of parallel vectors is:

- A) Maximum
- B) Zero
- C) Equal to the product of magnitudes
- D) Perpendicular to both vectors

Correct Answer: B



Solved Past Papers



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1. If a positive charged particle of mass "m" is projected parallel to uniform electric field E. The acceleration of the particle is:

- a) Option A
- b) Option B
- c) Option C
- d) Option D

Answer: B

2. If no fourth band is present on a carbon resistor then its tolerance will taken:

- a) $\pm 20\%$
- b) $\pm 10\%$
- c) $\pm 5\%$
- d) ± 0

Answer: A

3. To display the given voltage along Y—axis on C.R.O, connected to it:

- a) Option A
- b) Option B
- c) Option C
- d) Option D

Answer: B

4. If we want to increase the measuring range of voltmeter, the series high resistance value should be:

- a) Increased
- b) Decreased
- c) Kept constant
- d) Zero

Answer: A

5. The direction of induced current in a circuit is determined by:

- a) Ohm's law
- b) Faraday's law
- c) Guass's law
- d) Lenz's law

Answer: D

6. For alternating current in a circuit, the inductor behaves like:

- a) Thermistor

- b) Resistor
- c) Capacitor
- d) Rectifier

Answer: B

7. Metal detectors consist of:

- a) R-C circuit
- b) R-L circuit
- c) RLC series circuit
- d) L-C circuit

Answer: D

8. At high frequency, RLC series circuit shows the behaviour as:

- a) R-L circuit
- b) Pure inductive circuit
- c) R-C circuit
- d) Pure capacitive circuit

Answer: A

9. High temperature super conductors have a critical temperature greater than:

- a) 55K
- b) 77K
- c) 125K
- d) 163K

Answer: B

10. The SI unit of current gain are:

- a) Ampere
- b) Ohm
- c) Gray
- d) No unit

Answer: D

11. Which photon of light has least energy?

- a) Red
- b) Yellow
- c) Blue
- d) Green

Answer: A

12. The rest mass of photon is:

- a) 9.1×10^{-31} kg
- b) 1.67×10^{-27} kg
- c) Zero

d) Infinity

Answer: C

13. X-rays are:

- a) High energy electrons
- b) High energy neutrons
- c) High energy protons
- d) High energy photons

Answer: D

14. Heat produced due to fission reaction taking place in the core of nuclear reactor is about:

- a) 500°C
- b) 900°C
- c) 1100°C
- d) 1300°C

Answer: A

15. Subatomic particles are divided into:

- a) Six groups
- b) Five groups
- c) Four groups
- d) Three groups

Answer: D

16. A 3KΩ resistor is connected in series with a capacitor of capacitance 2mF.

The time constant for capacitor is:

- a) 1 sec
- b) 5 sec
- c) 6 sec
- d) 1.33 sec

Answer: C

17. The concept of an electric field was introduced by:

- a) Henry
- b) Faraday
- c) Watt
- d) Oersted

Answer: B

18. Electric field intensity due to an infinite sheet of charge is:

- a) Option A
- b) Option B
- c) Option C

d) Option D

Answer: C

19. The value of drift velocity of electrons is of the order of:

- a) 103ms⁻¹
- b) 102ms⁻¹
- c) 10⁻³ms⁻¹
- d) 10⁻²ms⁻¹

Answer: C

20. Formula for shunt resistance R_s is:

- a) Option A
- b) Option B
- c) Option C
- d) Option D

Answer: A

21. Voltmeter is connected in the circuit in:

- a) Perpendicular
- b) Parallel
- c) Series
- d) Anti parallel

Answer: B

22. The principle of an A.C. generator is based on:

- a) Mutual Induction
- b) Lenz's law
- c) Self induction
- d) Faraday's law of electromagnetic induction

Answer: D

23. When the motor is just started back emf always:

- a) Becomes zero
- b) Decreases
- c) Remains same
- d) Increases

Answer: A

24. Root mean square value of an alternating voltage is:

- a) Option A
- b) Option B
- c) Option C



143. During an isothermal process, the temperature:

- a) Remains constant during the initial phase
- b) Remains constant throughout the process
- c) Alters throughout the process
- d) Increases throughout the process

Answer: b

144. What is the value of heat energy Q in an adiabatic process?

- a) +1
- b) -1
- c) 0
- d) +2

Answer: c

145. Coulomb's law states:

- a) Force between two point charges is inversely proportional to the product of the charges and directly proportional to the square of the distance between them
- b) Force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them
- c) Force between two point charges is directly proportional to the sum of the charges and inversely proportional to the square of the distance between them
- d) Force between two point charges is directly proportional to the product of the charges and the square of the distance between them

Answer: b

146. The formula $V = W/q_0$ represents:

- a) Electric intensity
- b) Electric power
- c) Electric potential
- d) Electric field gradient

Answer: c

147. The SI unit of capacitance of a capacitor is:

- a) Coulomb
- b) Volt
- c) Farad
- d) Ampere

Answer: c

148. Electric intensity between two oppositely charged plates in the middle region is:

- a) Non-uniform
- b) Uniform
- c) Cannot be predicted
- d) Variable

Answer: b

149. Find the potential difference in moving 2 C of charge which requires 600 J of work between two points.

- a) 1200 V
- b) 300 V
- c) 150 V
- d) 2400 V

Answer: b

150. Which one of the following is not a feature of electric forces?

- a) They act on charges
- b) They act on masses
- c) They can be attractive
- d) They can be repulsive

Answer: b

151. A charge of 90 C passes through a wire for 30 seconds. Then the current in the wire will be:

- a) 3 A
- b) 0.3 A
- c) 3 mA
- d) 0.3 mA

Answer: a

152. The magnitude of current in metals is proportional to the potential difference across it as long as the temperature of the conductor is kept constant. This is known as:

- a) Joule's law



- a) Conducts in one direction only
- b) Conducts in both directions
- c) Amplifies signals
- d) Generates AC

Answer: a

206. The Earth's radius is R and the acceleration due to gravity on its surface is g . At a height h above the surface, the acceleration due to gravity is:

- a) g
- b) gh
- c) $gR/(R+h)^2$
- d) $gR^2/(R+h)^2$

Answer: d

207. A body of mass m is at a height h from the Earth's surface. If its height is increased, the increase in potential energy is:

- a) mgh
- b) $2mgh$
- c) $4.5mgh$
- d) $5.6mgh$

Answer: a

208. Objects in the universe are receding from us with a speed that is proportional to:

- a) reciprocal of distance
- b) square of distance
- c) their distance from us
- d) inverse square of distance

Answer: c

209. For a satellite moving in orbit around the Earth, the ratio of kinetic energy to potential energy is:

- a) $1/2$
- b) $1/\sqrt{2}$
- c) 2
- d) $\sqrt{2}$

Answer: a

210. For a particle inside a uniform spherical shell, the gravitational force

is:

- a) infinite
- b) zero
- c) $-Gm_1m_2/r^2$
- d) Gm_1m_2/r^2

Answer: b

211. A body falls freely under gravity. Which of the following remains constant?

- a) Potential energy
- b) Kinetic energy
- c) Total mechanical energy
- d) Velocity

Answer: c

212. Orbital velocity of a satellite depends on:

- a) mass of satellite
- b) radius of orbit
- c) shape of earth
- d) time period only

Answer: b

213. The value of acceleration due to gravity near Earth surface is:

- a) 9.81 m/s^2
- b) 9.82 m/s^2
- c) 9.83 m/s^2
- d) 9.84 m/s^2

Answer: a

214. The gravitational force between two masses varies as:

- a) r
- b) $1/r$
- c) $1/r^2$
- d) r^2

Answer: c

215. In spacecraft, astronauts feel weightless because:

- a) astronaut and spacecraft are in free fall
- b) mass of astronaut becomes zero
- c) gravity ceases to act

d) None of these

Answer: a

514. A static charge distribution produces a radial electric field $E = a r e^{(-br^2)}$.

Find the charge density.

a) $\rho = \epsilon_0 a e^{(-br^2)} (3 - 2br^2)$

b) $\rho = \epsilon_0 a e^{(-br^2)} (2 - 3br^2)$

c) $\rho = \epsilon_0 a e^{(-br^2)} (1 - br^2)$

d) $\rho = \epsilon_0 a e^{(-br^2)} (3 + 2br^2)$

Answer: a

515. A sphere of radius R carries a charge Q uniformly distributed throughout its volume. What is the electric field at a distance r inside the sphere?

a) $E = Q/(4\pi\epsilon_0 r^2)$

b) $E = Qr/(4\pi\epsilon_0 R^3)$

c) $E = 0$

d) $E = Q/(4\pi\epsilon_0 R^2)$

Answer: b

516. Four identical charges q are placed at the corners of a square of side a. Find the work required to assemble the configuration.

a) $U = (kq^2/a)(4 + \sqrt{2})$

b) $U = (kq^2/a)(4 - \sqrt{2})$

c) $U = (kq^2/a)(2 + \sqrt{2})$

d) $U = (kq^2/a)(2 - \sqrt{2})$

Answer: a

517. Electrical potential is the potential energy per unit:

a) Charge

b) Voltage

c) Force

d) None of these

Answer: a

518. If the distance between two charges is doubled, the Coulomb force becomes:

a) One fourth

b) Double

c) Half

d) Same

Answer: a

519. The law that governs the force between electric charges is:

a) Newton's law

b) Bernoulli's law

c) Coulomb's law

d) Ampere's law

Answer: c

520. Electric dipole moment vector is directed from charge:

a) + to -

b) - to +

c) + to +

d) - to -

Answer: b

521. Two charges of 1 nC and 4 nC are placed a certain distance apart. The ratio of Coulomb forces experienced by the two charges is:

a) 1:1

b) 1:9

c) 9:1

d) 1:81

Answer: a

522. The separation between the plates of a parallel-plate capacitor whose original capacitance was C is doubled. The new capacitance becomes:

a) C/4

b) C/2

c) 2C

d) 4C

Answer: b

523. The force of repulsion between two electrons at a certain distance is F. The force between two protons separated by the same distance is:

a) 2F

b) F

c) 1836F

d) None of these

Answer: a

577. A certain power-supply filter produces an output with a ripple of 100 mV peak-to-peak and a dc value of 20 V. The ripple factor is:

- a) 0.05
- b) 0.005
- c) 0.00005
- d) 0.02

Answer: b

578. A 60 V peak full-wave rectified voltage is applied to a capacitor-input filter. If $f = 120$ Hz, $R_L = 10$ k Ω , and $C = 10$ μ F, the ripple voltage is:

- a) 0.6 V
- b) 6 mV
- c) 5.0 V
- d) 2.88 V

Answer: d

579. If the load resistance of a capacitor-filtered full-wave rectifier is reduced, the ripple voltage:

- a) Increases
- b) Decreases
- c) Is not affected
- d) Has a different frequency

Answer: a

580. Line regulation is determined by:

- a) Load current
- b) Zener current and load current
- c) Changes in load resistance and output voltage
- d) Changes in output voltage and input voltage

Answer: d

581. Load regulation is determined by:

- a) Changes in load current and input voltage
- b) Changes in load current and output voltage
- c) Changes in load resistance and input

voltage

d) Changes in Zener current and load current

Answer: b

582. A 10 V peak-to-peak sinusoidal voltage is applied across a silicon diode and series resistor. The maximum voltage across the diode is:

- a) 9.3 V
- b) 5 V
- c) 0.7 V
- d) 10 V

Answer: c

583. In a certain biased limiter, the bias voltage is 5 V and the input is a 10 V peak sine wave. If the positive terminal of the bias voltage is connected to the cathode of the diode, the maximum voltage at the anode is:

- a) 10 V
- b) 5 V
- c) 5.7 V
- d) 0.7 V

Answer: c

584. The cathode of a Zener diode in a voltage regulator is normally:

- a) More positive than the anode
- b) More negative than the anode
- c) At +0.7 V
- d) Grounded

Answer: a

585. If a certain Zener diode has a Zener voltage of 3.6 V, it operates in:

- a) Regulated breakdown
- b) Zener breakdown
- c) Forward conduction
- d) Avalanche breakdown

Answer: b

586. For a certain 12 V Zener diode, a 10 mA change in Zener current produces a 0.1 V change in Zener voltage. The Zener impedance for this current

d) Doping

Answer: b

727. Recombination is when:

- a) An electron falls into a hole
- b) A positive and a negative ion bond together
- c) A valence electron becomes a conduction electron
- d) A crystal is formed

Answer: a

728. The current in a semiconductor is produced by:

- a) Electrons only
- b) Holes only
- c) Negative ions
- d) Both electrons and holes

Answer: d

729. In an intrinsic semiconductor:

- a) There are no free electrons
- b) The free electrons are thermally produced
- c) There are as many electrons as there are holes
- d) Both (b) and (c)

Answer: d

730. The process of adding an impurity to an intrinsic semiconductor is called:

- a) Doping
- b) Recombination
- c) Atomic modification
- d) Ionization

Answer: a

731. A trivalent impurity is added to silicon to create:

- a) Germanium
- b) A p-type semiconductor
- c) An n-type semiconductor
- d) A depletion region

Answer: b

732. The purpose of a pentavalent impurity is to:

- a) Reduce the conductivity of silicon

b) Increase the number of holes

c) Increase the number of free electrons

d) Create minority carriers

Answer: c

733. The majority carriers in an n-type semiconductor are:

- a) Holes
- b) Valence electrons
- c) Conduction electrons
- d) Protons

Answer: c

734. Holes in an n-type semiconductor are:

- a) Minority carriers that are thermally produced
- b) Minority carriers that are produced by doping
- c) Majority carriers that are thermally produced
- d) Majority carriers that are produced by doping

Answer: a

735. A pn junction is formed by:

- a) The recombination of electrons and holes
- b) Ionization
- c) The boundary of a p-type and an n-type material
- d) The collision of a proton and a neutron

Answer: c

736. The depletion region is created by:

- a) Ionization
- b) Diffusion
- c) Recombination
- d) All of these

Answer: d

737. The depletion region consists of:

- a) Nothing but minority carriers
- b) Positive and negative ions
- c) No majority carriers